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Short Communication

On the Trace of a Permuting Tri-additive Mapping in Left s_Γ-unital Γ-rings

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Abstract

Let M be 2 and 3 torsion-free left s_Γ-unital Γ-rings. Let D: $M \times M \times M \to M$ be a permuting tri-additive mapping with the trace $d(x) = D(x,x,x)$. Let σ: $M \to M$ be an endomorphism and τ: $M \to M$ an epimorphism. The objective of this paper is to prove the following: a) If d is (σ,τ)-skew commuting on M, then $D = 0$; b) If d is (τ,τ)-skew-centralizing on M, then d is (τ,τ)-commuting on M; c) If d is $2\text{–}(\sigma,\tau)$-commuting on M, then d is (σ,τ)-commuting on M.

Keywords: Permuting tri-additive mappings; Skew-commuting mappings; Skew-centralizing mappings; Commuting mappings.

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1. Introduction

In this paper, we consider M as a Γ-ring in the sense of Barnes [1]. It is obvious that every ring is a Γ-ring. Ceven and Ozturk [2] worked on the trace of a permuting tri-additive mapping in left s-unital rings. Some characterizations of the left s-unital rings were obtained by means of the trace of the permuting tri-additive mappings. Ozturk [3] proved some properties of prime and semiprime rings by using the permuting tri-additive derivations. Ozturk *et al.* [4] worked on symmetric bi-derivations on prime Γ-rings. They obtained some remarkable results on prime Γ-rings.

Ozden and Ozturk [3] studied on permuting tri-derivations in prime and semiprime Γ-rings. They obtained some fruitful results. An example of a permuting tri-derivation is given here.

In this paper, we develop some results of Ceven and Ozturk [2] in Γ-rings. Here we prove the following:

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Let d be the trace of a permuting tri-additive mapping D on 2 and 3 torsion-free left s_Γ-unital Γ-rings M and σ be an endomorphism on M and τ an epimorphism on M. Then

(i) If d is (σ,τ)-skew commuting on M, then $D = 0$.

(ii) If d is (τ,τ)-skew-centralizing on M, then d is (τ,τ)-commuting on M.

(iii) If d is 2–(σ,τ)-commuting on M, then d is (σ,τ)-commuting on M.

2. Preliminaries

Throughout this paper, all rings M will be a Γ-ring and the center of a ring will be denoted by Z. Let σ,τ be additive mappings of M into M and $x, y \in M$. As usual, we introduce the following notations

$[x, y]_\alpha = x\alpha y - y\alpha x,$ $\quad < x, y >_\alpha = x\alpha y + y\alpha x,$

$[x, y]_\alpha^{(\sigma,\tau)} = x\alpha\sigma(y) - \tau(y)\alpha x,$ $\quad < x, y >_\alpha^{(\sigma,\tau)} = x\alpha\sigma(y) + \tau(y)\alpha x.$

Let d be a mapping from M into M, and S a nonempty subset of M. Then d is called (σ,τ)-skew-commuting (respectively, (σ,τ)-skew-centralizing) on S if $<d(x),x>_\alpha^{(\sigma,\tau)} = 0$ (respectively, $< d(x), x >_\alpha^{(\sigma,\tau)} \in Z$) for all $x \in S$. Similarly f is said to be (σ,τ)-commuting on S if $[f(x), x]_\alpha^{(\sigma,\tau)} = 0$ for all $x \in S$. If $\sigma = \tau = 1$ (the identity map on M), then d is called simply skew-commuting, skew-centralizing and commuting on S, respectively. A mapping $D : M\times M \to M$ is said to be symmetric if $D(x, y) = D(y, x)$ for all $x, y \in M$.

A mapping $d : M\to M$ defined by $d(x) = D(x, x)$ for all $x \in M$, where $D: M \times M \to M$ is a symmetric mapping, is called the trace of D.

A mapping $D: M\times M\times M\to M$ is called tri-additive if

$D(x+w, y, z) = D(x, y, z)+D(w, y, z),$

$D(x, y + w, z) = D(x, y, z) + D(x, w, z),$

$D(x, y, z + w) = D(x, y, z) + D(x, y,w)$ holds for all $x, y, z, w\in M$.

A tri-additive mapping $D : M\times M\times M\to M$ is called permuting tri-additive if $D(x,y,z) = D(x, z, y) = D(y, x, z) = D(y, z, x) = D(z, x, y) = D(z, y, x)$ holds for all $x, y, z \in M$. A mapping $d : M\to M$ defined by $d(x) = D(x, x, x)$ is called the trace of the permuting tri-additive mapping D. It is obvious that, if $D : M \times M \times M\to M$ is a permuting tri-additive mapping then the trace of D satisfies the relation $d(x+y) = d(x)+d(y)+3D(x, x, y)+3D(x, y, y)$ for all $x, y \in M$. The mapping $d : M\to M$ defined by $d(x) = D(x, x, x)$ is an odd function.

M is called a left s_Γ-unital (resp. s_Γ-unital) Γ-ring if for each $x \in M$ there holds $x \in M\Gamma x$ (resp. $x \in M\Gamma x\cap x\Gamma M$). If M is a left s_Γ-unital (resp. s_Γ-unital) Γ-ring then for any finite subset F of M there exists an element e in M such that $e\alpha x = x$ (resp. $e\alpha x = x\alpha e = x$)

for all $x \in F$, $\alpha \in \Gamma$. Such an element e will be called a left pseudo-identity (resp. pseudo-identity) of F.

Throughout this paper e will be a left pseudo-identity of the set

$E = \{x, d(x), d(e), \sigma(x), D(x, x, e), D(x, e, e)\} \subseteq M$,

where x is an arbitrary element of M.

In this paper, we investigate permuting tri-additive mapping and the trace of its with (σ,τ)-skew-commuting and (σ,τ)-skew-centralizing maps in left s_Γ-unital Γ-rings.

3. Some Results on the Trace of a Permuting Tri-additive Mapping

The first result is the following.

Theorem 3.1. Let M be 2 and 3-torsion-free left s_Γ-unital Γ-ring. Let $\sigma: M\to M$ be an endomorphism and $\tau : M\to M$ an epimorphism. Let $D: M\times M\times M \to M$ be a permuting tri-additive mapping and d the trace of D. If d is (σ,τ)-skew-commuting on M, then $D = 0$.

Proof. It is given that, for all $x \in M$,

$$< d(x), x >_\alpha^{(\sigma,\tau)} = d(x)\alpha\sigma(x) + \tau(x)\alpha d(x) = 0 \text{ for all } \alpha\in\Gamma. \quad (1)$$

$\tau(e)$ is also a left pseudo-identity of M since τ is an epimorphism. So from (1), we have

$$< d(e), e >_\alpha^{(\sigma,\tau)} = d(e)\alpha\sigma(e) + d(e) = 0 \text{ , for all } \alpha\in\Gamma. \quad (2)$$

and right-multiplying by σ(e) gives d(e)$\alpha\sigma$(e) = 0 since M is 2-torsion-free.
Hence, by (2), we get d(e) = 0.
Substituting x + e for x in (1), we obtain, for all $x \in M$,

$$< d(x), e>_\alpha^{(\sigma,\tau)} +3 < P, x>_\alpha^{(\sigma,\tau)} +3 < P, e >_\alpha^{(\sigma,\tau)} +3 < Q, x>_\alpha^{(\sigma,\tau)} + 3 < Q, e>_\alpha^{(\sigma,\tau)} = 0 \text{ ,} \quad (3)$$

where $P = D(x, x, e)$, $Q = D(x, e, e)$.

Putting $-x$ instead of x in (3) and comparing (3) with the obtained equation, we have

$$P\alpha\sigma(e) + P + Q\alpha\sigma(e) + Q = 0 \text{ ,} \quad (4)$$

since d is odd function, M is 2 and 3-torsion-free and τ(e) is a left pseudo-identity. Right multiplication of (4) by σ(e) gives $P\alpha\sigma(e) + Q\alpha\sigma(e) = 0$.

Using the last relation and (4), we obtain $P + Q = 0$. Hence, we arrive at $d(x + e) = d(x)$ for all $x \in M$.

Then, we have

$$0 = < d(x + e), x + e>_\alpha^{(\sigma,\tau)} = < d(x), e>_\alpha^{(\sigma,\tau)} = d(x)\alpha\sigma(e) + d(x). \quad (5)$$

Multiplying $\sigma(e)$ from the right, we get $d(x)\alpha\sigma(e) = 0$. So from (5), we obtain

$$d(x) = D(x, x, x) = 0 \quad (6)$$

for all $x \in M$. Then it follows that, for all $x, y \in M$,

$$D(x, x, y) + D(x, y, y) = 0\,, \tag{7}$$

since $D(x + y, x + y, x + y) = 0$, D is permuting tri-additive mapping and M is 3-torsion-free ring. Since $D(x + y + z, x + y + z, x + y + z) = 0$ and M is 2 and 3-torsion free, and using (7), we obtain $D(x, y, z) = 0$ for all $x, y, z \in M$ which gives the conclusion.

Theorem 3.2. Let M be 2 and 3-torsion-free left s_Γ-unital Γ-ring. Let τ: $M \to M$ be an epimorphism. Let $D : M \times M \times M \to M$ be a permuting tri-additive mapping and d the trace of D. If d is (τ,τ)-skew-centralizing on M, then d is (τ,τ)-commuting on M.

Proof. Since d is (τ,τ)-skew-centralizing on M, we know that

$$< d(x), x >_\alpha^{(\sigma,\tau)} = d(x)\alpha\tau(x) + \tau(x)\alpha d(x) \in Z \qquad \text{for all } x \in M. \tag{8}$$

Hence $d(e)\alpha\tau(e) + d(e) \in Z$, since $\tau(e)$ is a left pseudo-identity (9)

Commuting with $\tau(e)$ gives $d(e) = d(e)\alpha\tau(e)$ and we get $2d(e) \in Z$ by (9). Hence $d(e) \in Z$.

Let us replace $x + e$ by e in (8). We get

$$2\tau(x)\alpha d(e) + 3\tau(x)\alpha P + 3\tau(x)\alpha Q + d(x) + 3P + 3Q + d(x)\alpha\tau(e) + 3P\alpha\tau(x)$$

$$+3P\alpha\tau(e) + 3Q\alpha\tau(x) + 3Q\alpha\tau(e) \in Z, \tag{10}$$

using (8), (9) and $d(e) \in Z$, where $P = D(x, x, e)$, $Q = D(x, e, e)$.

Substituting $-x$ for x in (10) and comparing (10) with the new one, we have

$$\tau(x)\alpha Q + P + P\alpha\tau(e) + Q\alpha\tau(x) \in Z\,, \tag{11}$$

or, $2\tau(x)\alpha d(e) + 3\tau(x)\alpha P + d(x) + 3P + d(x)\alpha\tau(e) + 3P\alpha\tau(x) + 3Q\alpha\tau(e) \in Z$, (12)

since M is 2 and 3 torsion-free ring.

Let us put $x + e$ instead of x in (10). Since $d(e) \in Z$ and $\tau(e)$ is left pseudo-identity, we obtain $\tau(x)\alpha Q + 2\tau(x)\alpha d(e) + 3Q + P + P\alpha\tau(e) + 3Q\alpha\tau(e) + Q\alpha\tau(x) \in Z$.

Using (5), we get

$$2\tau(x)\alpha d(e) + 3Q + 3Q\alpha\tau(e) \in Z \tag{13}$$

and commuting with $\tau(e)$, we obtain $Q\alpha\tau(e) = Q$. Writing this in (13), and using 2-torsion free, we have $\tau(x)\alpha d(e) + 2Q \in Z$. Commuting with $\tau(x)$, using $d(e) \in Z$,we get

$$Q = D(x, e, e) \in Z, \tag{14}$$

since τ is an epimorphism.

Let us commute with $\tau(e)$ the equation (11). We obtain $P\alpha\tau(e) = P$ since $Q \in Z$. Hence from (11), we have $Q\alpha\tau(x) + P \in Z$ and commuting again with $\tau(x)$, we obtain

$$P = D(x, x, e) \in Z. \tag{15}$$

Using the equations (14) and (15) in Eq. (12), we get

$$2\tau(x)\alpha d(e) + 6\tau(x)\alpha P + 6Q + d(x) + d(x)\alpha\tau(e) \in Z \qquad (16)$$

Commuting with $\tau(e)$ in (16), we obtain, for all $x \in M$, $d(x)\alpha\tau(e) = d(x)$. Using this equality in (16), we have $\tau(x)\alpha d(e) + 3\tau(x)\alpha P + 3Q + d(x) \in Z$.

Commuting with $\tau(x)$, it is obtained that $d(x)\alpha\tau(x) = \tau(x)\alpha d(x)$. Hence d is (τ,τ)-commuting.

Theorem 3.3. Let M be 2 and 3−torsion free left s_Γ-unital Γ-ring. Let $\sigma: M\to M$ be an endomorphism and $\tau : M\to M$ an epimorphism. Let $D : M\times M\times M\to M$ be a permuting tri-additive mapping and d the trace of D. If d is $2-(\sigma,\tau)$−commuting on M, then d is (σ,τ)-commuting on M.

Proof. Let us define a mapping $h : M\to M$ by $h(x) = [d(x), x]_\alpha^{(\sigma,\tau)}$ for all $x \in M$, $\alpha\in\Gamma$. Note that h is even function. From the hypothesis, we can write

$$< h(x), x >_\alpha^{(\sigma,\tau)}= [d(x), x\alpha x]_\alpha^{(\sigma,\tau)} = 0 , \text{ for all } x\in M,\ \alpha\in\Gamma. \qquad (24)$$

Since τ is an epimorphism, $\tau(e)$ is also a left pseudo-identity. So, we have

$$h(e)\alpha\sigma(e) + h(e) = 0, \text{ for all } x \in M,\ \alpha\in\Gamma. \qquad (25)$$

Right multiplying by $\sigma(e)$ gives $h(e)\alpha\sigma(e) = 0$ since M is 2-torsion free. Hence, by (25), we get

$$h(e) = [g(e), e]_\alpha^{(\sigma,\tau)} = 0. \qquad (26)$$

Since $d(x+e) = d(x)+d(e)+3M+3N$, where $M = G(x, x, e)$ and $N = G(x, e, e)$,
we obtain

$$h(x + e) = h(x) + [d(x), e]_\alpha^{(\sigma,\tau)} + [d(e), x]_\alpha^{(\sigma,\tau)} + 3[M, x]_\alpha^{(\sigma,\tau)}$$
$$+ 3[M, e]_\alpha^{(\sigma,\tau)} + 3[N, x]_\alpha^{(\sigma,\tau)} + 3[N, e]_\alpha^{(\sigma,\tau)} \qquad (27)$$

If we replace x by $x+e$ in (24) and using (24), (26) and permuting tri-additivity of D, we have, for all $x \in M$, $\alpha\in\Gamma$.

$$h(x)\alpha\sigma(e) + [d(x), e]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) + [d(x), e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + [d(e), x]_\alpha^{(\sigma,\tau)}\sigma(x)+$$
$$[d(e), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3[M, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) + 3[M, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e)+ 3[M, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(x)$$
$$+ 3[M, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3[N, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) + 3[N, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e)+ 3[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) +$$
$$3[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e)+ h(x) +\tau(x)\alpha[d(x), e]_\alpha^{(\sigma,\tau)} + [d(x), e]_\alpha^{(\sigma,\tau)} + \tau(x)\alpha[d(e), x]_\alpha^{(\sigma,\tau)} +$$
$$[d(e), x]_\alpha^{(\sigma,\tau)} + 3\tau(x)\alpha[M, x]_\alpha^{(\sigma,\tau)} + 3[M, x]_\alpha^{(\sigma,\tau)} + 3\sigma(x)\alpha[M, e]_\alpha^{(\sigma,\tau)}+ 3[M, e]_\alpha^{(\sigma,\tau)} +$$
$$3\tau(x)\alpha[N, x]_\alpha^{(\sigma,\tau)} + 3[N, x]_\alpha^{(\sigma,\tau)} + 3\tau(x)\alpha[N, e]_\alpha^{(\sigma,\tau)}+ 3[N, e]_\alpha^{(\sigma,\tau)} = 0 . \qquad (28)$$

Substituting $-x$ for x in (28) and comparing (28) with the obtained result, we get, for all $x \in M$,

$$[d(x), e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + [d(e), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3[M, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3[M, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) +$$
$$[N, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) + 3[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(\mathrm{e}) + [d(x), e]_\alpha^{(\sigma,\tau)} + [d(e), x]_\alpha^{(\sigma,\tau)} + 3[M, x]_\alpha^{(\sigma,\tau)} +$$
$$3\sigma(x)\alpha[M, \mathrm{e}]_\alpha^{(\sigma,\tau)} + 3\sigma(x)\alpha[N, x]_\alpha^{(\sigma,\tau)} + 3[N, \mathrm{e}]_\alpha^{(\sigma,\tau)} = 0 \qquad (29)$$

since h and M are even, d and N are odd, M is 2-torsion free ring.

Right multiplication of (29) by $\sigma(e)$ gives

$$2[d(x), e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 2[d(\mathrm{e}), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 6[M, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e)$$
$$+ 6[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3[M, e]_\alpha^{(\sigma,\tau)}\alpha(x)\alpha\sigma(e) + 3[N, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(x)\alpha\sigma(e) +$$
$$3\sigma(x)\alpha[M, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3\sigma(x)[N, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) = 0\,. \qquad (30)$$

Substituting again x + e instead of x in (30) and using (30), we obtain

$$4[d(e), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 12[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 6[M, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 6[N, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) +$$
$$3[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(x)\alpha\sigma(e) + [d(e), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(x)\alpha\sigma(e) + 3\tau(x)\alpha[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) +$$
$$\tau(x)\alpha[d(e),x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) = 0, \qquad (31)$$

since M is 2-torsion free ring.

Putting $-x$ for x and comparing (31), we get

$$[d(e), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) = 0\,. \qquad (32)$$

Furthermore we get

$$[d(e), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) + 3[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) = [d(e), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e\alpha x) + 3[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e\alpha x)$$
$$= ([d(e), x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3[N, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e))\alpha\sigma(x) = 0 \qquad (33)$$

According to Eqs. (32) and (33), the relation (31) becomes

$$[M, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + [N, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) = 0. \qquad (34)$$

With similar process as obtaining of Eq. (33), we have

$$[M, e]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) + [N, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(x) = 0. \qquad (35)$$

Using the obtained Eqs. (32), (34) and (35) in (30), we get

$$[d(x), e]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) + 3[M, x]_\alpha^{(\sigma,\tau)}\alpha\sigma(e) = 0.$$

Therefore Eq. (29) becomes

$$[d(x), e]_{\alpha}^{(\sigma,\tau)} + [d(e), x]_{\alpha}^{(\sigma,\tau)} + 3[M, x]_{\alpha}^{(\sigma,\tau)} + \tau(x)[M, e]_{\alpha}^{(\sigma,\tau)}$$
$$+ 3\tau(x)\alpha[N, x]_{\alpha}^{(\sigma,\tau)} + 3[N, e]_{\alpha}^{(\sigma,\tau)} = 0. \quad (36)$$

If we put $x + e$ instead of x in Eq. (36), and compare with Eq. (36), we get

$$2[d(e), x]_{\alpha}^{(\sigma,\tau)} + 3[M, e]_{\alpha}^{(\sigma,\tau)} + 6[N, e]_{\alpha}^{(\sigma,\tau)} + 3[N, x]_{\alpha}^{(\sigma,\tau)}$$
$$+ 3\tau(x)\alpha[N, e]_{\alpha}^{(\sigma,\tau)} + \tau(x)[d(e), x]_{\alpha}^{(\sigma,\tau)} = 0. \quad (37)$$

Substituting $-x$ for x and comparing Eq. (36) we write

$$[d(e), x]_{\alpha}^{(\sigma,\tau)} + 3[N, e]_{\alpha}^{(\sigma,\tau)} = 0. \quad (38)$$

So, the Eq. (37) becomes

$$[M, e]_{\alpha}^{(\sigma,\tau)} + [N, x]_{\alpha}^{(\sigma,\tau)} = 0. \quad (39)$$

Hence from Eq. (36), we have

$$[g(x), e]_{\alpha}^{(\sigma,\tau)} + 3[M, x]_{\alpha}^{(\sigma,\tau)} = 0. \quad (40)$$

Using Eqs. (38), (39) and (40) in (27), we obtain $h(x + e) = h(x)$. Since $< h(x), x >_{\alpha}^{(\sigma,\tau)} = 0$ for all $x \in M$, the relation $h(x + e)\alpha\sigma(x + e) + \tau(x + e)\alpha h(x + e) = 0$ becomes

$$h(x)\alpha\sigma(e) + h(x) = 0 \quad (41)$$

for all $x \in M$. Right multiplying Eq. (41) by $\sigma(e)$ we have $h(x)\alpha\sigma(e) = 0$ since M is 2-torsion free. Hence from Eq. (41), we obtain $h(x) = 0$ for all $x \in M$ which gives the conclusion.

References

1. W. E. Barnes, Pacific J. Math. 18, 411 (1966).
2. Y. Ceven and M. A. Ozturk, International J. Pure and Appl. Math. **23** (4), 465 (2005).
3. M. A. Ozturk, East Asian Math. J. **15** (2), 177 (1999).
4. M. A. Ozturk, M. Sapanci, M. Soyturk, and K. H. Kim, Sci. Math. Jpn. **53** (3), 495 (2001).
5. D. Ozden and M. A. Ozturk, Kyungpook Math. J. **46**, 153 (2006).
6. M. A. Ozturk, M. Sapanci, and Y. B. Jun, East Asian Math. J. **15** (1), 105 (1999).
7. M. A. Ozturk and M. Sapanci, East Asian Math. J. **15** (2), 165 (1999).
8. J. Vukman, Aequationes Math. **38** (2), 245 (1989). doi:10.1007/BF01840009
9. J. Vukman, Aequationes Math. **40** (2-3), 181 (1990). doi:10.1007/BF02112294
10. H. Yasret, M. A. Ozturk and Y. B. Jun, Commun. Fac. Sci. Univ. Ank. Series A1**54**, (1), 1 (2005).