

**Review article**

**Mathematical Analysis of Side effects of HIV/AIDS Medication**

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**Abstract:**

Acquired Immune deficiency syndrome (AIDS) is caused by Human immunodeficiency virus (HIV); More than 40 Million people live with HIV world wide. Various antiretroviral classes of drugs are used to treat HIV infection and the prolonged treatment has to be taken for the rest of life. It is very difficult to continue prolonged treatment because of problem of adherence and side effects. The basic purpose of antiretroviral treatment is to maintain the quality of life of a patient. However, these side effects vary from drug to drug and patient to patient. The objective of this research is to be modeled some major drug induced side effects by using fuzzy matrix theory which is one of the best tools to analyze unsupervised data involving imprecision. This study is of interest for HIV Specialist and physician who care for HIV/AIDS patients with an aim to concentrate more on safety profile of HIV/AIDS drugs.

**Key Words:** Mathematical Analysis, HIV/AIDS, Medication, Fuzzy matrix theory.

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**Introduction:**

HIV/AIDS is one of the greatest threats to the mankind of the world. The immune system of our body protects us from various kinds of infections and diseases. HIV virus suppresses the immune system and any small infection can cause severe health problem even death. From 1981, more than 25 million people were killed with this disease. Five millions people are becoming infected with this virus every year and presently 40 million people have HIV that causes Aids. HIV/Aids acquired through unprotected sexual contact, blood transfusion, syringes and during child birth and breast-feeding through mother to child.

Antiretroviral toxicity is becoming an important subject in the management of HIV-infected persons. HIV infection is a chronic disease, and the drugs are being used in the patients for longer period of time. A study indicates that although the mortality and morbidity rates have been minimized with advanced Combination antiretroviral drugs treatment but major adverse effects is being increased and up to

25% of patients stop their initial HAART (Highly active antiretroviral therapy) regimen because of lethal<sup>1</sup>. The drugs, which are available in the world market, can be divided into five groups and their table is given below with mode of action.

**Table I: (Drugs available in the market with mode of action)**

ARV drugs class	Abbreviations	First approved HIV treatment (year)	Mode of action
Nucleoside/Nucleotide Reverse Transcriptase Inhibitors	NRTIs, nucleoside analogues, nukes	1987	NRTIs inhibit reverse transcriptase enzyme which is helpful in virus replication.
Non-Nucleoside Reverse Transcriptase Inhibitors	NNRTIs, non-nucleosides, non-nukes	1997	NNRTIs also inhibit the reverse transcriptase prot ein which prevents HIV from replicating within cells.
Protease Inhibitors	PIs	1995	PIs is protease inhibitor
Fusion or Entry Inhibitors		2003	Fusion or entry inhibitors stop HIV from entering and binding with human immune cells.
Integrase inhibitors		2007	Integrase inhibitors inhibits enzyme, which is the requirement of HIV to pop in its genetic material into human cells.

The following data is taken from AIDS info ‘A service of the U.S. Department of Health and Human Services’, which is helpful in our study. These are some most frequent reported serious side effects with HIV medication.

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**Table II: (Drugs with their brand names and possible serious side effects)**

Drug Class	Brand names available in market	Possible Serious Side Effects
Nucleoside Reverse Transcriptase Inhibitors. (NRTIs)	<ul style="list-style-type: none"> <li>Abacavir sulfate</li> <li>tavudine</li> <li>Tenofovir disoproxil fumarate</li> <li>Zidovudine</li> </ul>	The most frequently reported serious side effects include peripheral paresthesia, hypercholesterolemia, hypertriglyceridemia, hyperglycemia, skin rash, and mood disorders. Other less serious side effects include headache, oral paresthesia, GI disturbances, fatigue, and taste disorders
Non-Nucleoside Reverse Transcriptase Inhibitors. (NNRTIs)	<ul style="list-style-type: none"> <li>Delavirdine</li> <li>Efavirenz</li> <li>Etravirine</li> <li>Nevirapine</li> </ul>	Hepatic failure, acute kidney failure. Pancreatitis, hemolytic anemia, rhabdomyolysis. Substantial increases in liver enzymes and hepatic failure, Skin rashes, suicidal ideation. Severe depression, Granulocytopenia (occurring more frequently in children), Stevens -Johnson syndrome and psychiatric symptoms.
Protease Inhibitors (PIs)	<ul style="list-style-type: none"> <li>Amprenavir</li> <li>Darunavir</li> <li>Indinavir sulfate</li> <li>Ritonavir</li> <li>Saquinavir</li> <li>Tipranavir</li> </ul>	PR Interval Prolongation, hyperglycemia, hypercholesterolemia, hypertriglyceridemia, skinrash, nephrolithiasis/uroolithiasis, peripheral paresthesia, acute hemolytic anemia and mood disorders
Fusion or Entry Inhibitors	<ul style="list-style-type: none"> <li>Enfuvirtide</li> <li>Maraviroc</li> </ul>	Hepatotoxicity, hypocoellularity, pancreatitis, Bone marrow, skin papilloma, anxiety, decreased appetite, asthenia, herpes simplex, cough expression, pruritis, insomnia, myalgia, and weight loss. Heart rate elevations and blood pressure, Lymphoid atrophy local injection site reactions, sinusitis, conjunctivitis, peripheral neuropathy.
Integrase inhibitors	<ul style="list-style-type: none"> <li>Raltegravir</li> </ul>	Psychiatric Disorders, depression (mainly in patients with a previous history of psychiatric problem) with suicidal ideation. Muscle spasm, Skin and Subcutaneous Tissue Disorders, rash, Stevens -Johnson syndrome.

**Description and Methodology:**

In this research paper we are going to implement fuzzy matrix theory which is one of the best tool to analyzed raw data with imprecision. These mathematical matrices can be applied to social and natural situations to predict likely outcomes. It was first introduced by W. B. Vasantha and V. Indira to study the passenger transportation problem. They used fuzzy optimization techniques to solve this problem. In this present study we used this fuzzy model for the analysis of adverse effects of HIV/AIDS medication. We have conducted a study on 195 reported cases of HIV/AIDS from Agha Khan Medical University and Hospital and Baqai Medical University and Hospital Karachi (Pakistan). These are very selective patients who are already under regular treatment with antiretroviral medications and facing severe adverse effects of HIV/AIDS medication. As the problem faced by them we feel it can be modeled using fuzzy matrix theory to identify the intensity of adverse effects in different intervals of time of treatment. We transform the collected data into matrix form by taking side effects along rows and duration of treatment along the column. The average time dependent matrix (ATD) is obtained by dividing each entry of the matrix by the time period and taking the average  $\mu_j$  and standard deviation of each column of the ATD

matrix. We select a parameter  $\alpha$  from the interval  $[0,1]$  and form the refined Time dependent matrix by using the formula

$$\text{if } a_{ij} \leq (\mu_j - \alpha * \sigma_j) \text{ then } e_{ij} = -1$$

$$\text{else if } a_{ij} \in (\mu_j - \alpha * \sigma_j, \mu_j + \alpha * \sigma_j) \text{ then } e_{ij} = 0$$

$$\text{else if } a_{ij} > (\mu_j + \alpha * \sigma_j) \text{ then } e_{ij} = 1.$$

Where  $a_{ij}$  are the entries of the ATD matrix. In this way, we form the Refined Time Dependent (RTD) fuzzy matrix. The entries of this matrix are  $-1, 0,$  or  $1$ . Entries and the sum of rows gives the matrix of intensity of adverse effects. We also combine these matrices for different values of  $\alpha$  select from the interval  $[0,1]$ , in this way we get the Combined Effective Time Dependent Data (CETD) matrix and the row sum is obtained for CETD matrix. Conclusions are derived based on the row sums. Its graphical presentation is very simple and could be understood by a layman.

**ESTIMATION OF SIDE EFFECTS OF HIV/AIDS MEDICATION USING 4X8 FUZZY MATRIX**

On the basis of history of patients we have taken the following eight major side effects ( $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ ) to study in divided four intervals of time which is given below.

- $A_1$  - Liver disease
- $A_2$  - Heart disease
- $A_3$  - Kidney disease
- $A_4$  - Septicemia
- $A_5$  - Osteoporosis
- $A_6$  - hyperglycemia
- $A_7$  - Severe depression
- $A_8$  - Pancreatitis

**Table III: Initial data based on history of patients (n=195)**

Treatment time duration (years)	$A_1$ Liver diseases	$A_2$ Heart diseases	$A_3$ Kidney diseases	$A_4$ Septicemia	$A_5$ Osteoporosis	$A_6$ Hyperglycemia	$A_7$ Severe depression	$A_8$ Pancreatitis
0 - 2.9	13	17	04	19	04	09	22	17
3 - 6.9	69	44	52	78	17	17	35	35
7 - 9.9	48	61	39	45	30	30	39	61
10 - 15	51	70	86	51	41	10	30	72

Average Time Dependent Data (ATD) Matrix

$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
4.48	5.86	1.37	6.55	1.37	3.10	7.59	5.86
17.69	11.28	13.33	20	4.36	4.36	8.97	8.97
16.55	21.03	13.45	15.52	10.34	10.34	13.45	21.03
10.2	14	17.2	10.2	8.2	2	6	14.4
4.48	5.86	1.37	6.55	1.37	3.10	7.59	5.86
17.69	11.28	13.33	20	4.36	4.36	8.97	8.97
16.55	21.03	13.45	15.52	10.34	10.34	13.45	21.03
10.2	14	17.2	10.2	8.2	2	6	14.4

Table IV: The Average and the Standard Deviation of the above ATD Matrix

	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
Average	12.23	13.04	11.33	13.07	6.07	4.95	9	12.56
S.D	6.13	6.31	6.88	5.90	3.99	3.72	3.20	6.66

The RTD matrix for  $\alpha = 0.1$  the row sum matrix

The RTD matrix for  $\alpha = 0.1$  the row sum matrix

$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	1	1	-1	-1	0	-1	1	-1	1	1	-1	-1	0	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1

$$\begin{bmatrix} -8 \\ -1 \\ 8 \\ 8 \\ 0 \end{bmatrix}$$

The RTD matrix for  $\alpha = 0.25$  the row sum matrix

The RTD matrix for  $\alpha = 0.25$  the row sum matrix

$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	1	1	0	0	0	-1	1	-1	1	1	0	0	0	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	0	1	-1	1	-1	-1	1	-1	0	1	-1	1	-1	-1	1

$$\begin{bmatrix} -8 \\ 1 \\ 8 \\ 8 \\ -1 \end{bmatrix}$$

The RTD matrix for  $\alpha=0.45$  the row sum matrix

The RTD matrix for  $\alpha=0.45$  the row sum matrix

$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1	-1	0	0	-1
1	0	0	1	0	0	0	-1	1	0	0	1	0	0	0	-1
1	1	0	0	1	1	1	1	1	1	0	0	1	1	1	1
0	0	1	-1	1	-1	-1	0	0	0	1	-1	-1	-1	0	

$$\begin{bmatrix} -7 \\ 1 \\ 6 \\ 6 \\ -1 \end{bmatrix}$$

The RTD matrix for  $\alpha=0.75$  the row sum matrix

The RTD matrix for  $\alpha=0.75$  the row sum matrix

$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
-1	-1	-1	-1	-1	0	0	-1	-1	-1	-1	-1	0	0	0	-1
1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0
0	1	0	0	1	1	1	1	0	1	0	0	1	1	1	1
0	0	1	0	0	-1	-1	0	0	0	1	0	0	-1	-1	0

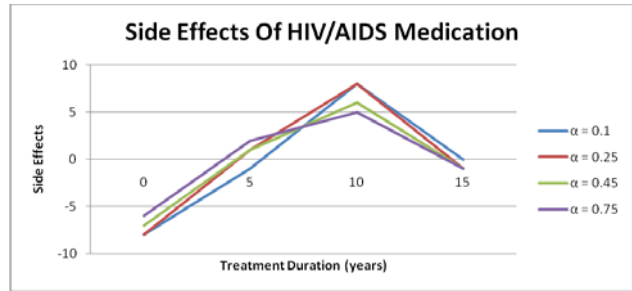
$$\begin{bmatrix} -6 \\ 2 \\ 5 \\ 5 \\ -1 \end{bmatrix}$$


Fig 1: The graph depicting the side effects of HIV/ AIDS medication with the passage of time for different parametric values.

(CETD) matrix

$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$
$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$

The Combined Effective Time Dependent Data the row sum matrix

Discussion:

Side effects of antiretroviral medicine have been shown to be a major cause for non-adherence to medications. Understanding of the use and side effects of antiretroviral medicine are therefore essential in managing HIV-infected patient.

Hepatotoxicity are associated with antiretroviral agents PIs in a potential cohort study it was observed that patients who were treated with ritonavir (RTV) and also the patients who were receiving saquinavir, nelfinavir (NFV) or indinavir (IDV) were faced severe hepatotoxicity<sup>2</sup>. Severe hepatotoxicity was also observed in patient using Ritonavir. Hepatotoxicity incidence is observed in other treatment groups, i.e., nucleoside analogs, nelfinavir, saquinavir, and indinavir as well<sup>3</sup>. In another study, highly active antiretroviral therapy (HAART) containing protease inhibitors (PIs) and ritonavir with saquinavir, drug-induced hepatotoxicity was observed with administration of its full dose<sup>4</sup>.

HIV-infected patients also show risk factors for cardiovascular problems. Most recently, studies showed that NRTI, NNRTI and PI drug classes (alone and in combination) are increasing the risk of heart attack in HIV patients<sup>8</sup>. The Food and Drug Administration (FDA) has approved the DAD study (Data collection on Adverse events of anti-HIV Drugs), which indicates that those HIV patient treating with abacavir and didanosine have a maximum chance of heart attack than patients on another medications<sup>5</sup>.

Drug-induced renal failure is a common incident in patients with HIV. The antiviral like acyclic nucleoside phosphonates *cidofovir* and *adefovir* causes worse nephrotoxicity<sup>6</sup>. A study showed that acute renal failure developed in a patient with HIV using *tenofovir*<sup>25</sup>. A study showed that Urological complications were associated in majority of patients with the *indinavir* treatment because of elevated *indinavir* plasma concentration<sup>1</sup>.

Some special metabolic complications like *osteopenia* and *osteoporosis* are associated with *protease*<sup>27</sup>. In the opinion of some researchers *lactic acidemia* and proposed mitochondrial toxicity are associated with nucleoside analog (NRTI) which become the main cause of withdrawal from treatment. The PI therapy can cause cumulative incidence of *hyperglycemia*, *hypercholesterolemia*, *hypertriglyceridemia*, and *lipodystrophy*<sup>7</sup>. Antiretroviral medications may cause pathogenesis of *hyperglycemia* in HIV-infected patients<sup>8,9,10</sup>. In vitro evidence, it is mentioned that *ritonavir* can caused both *insulin resistance*<sup>5</sup> and *impaired  $\beta$ -cell function*<sup>11</sup>. In some studies, it is also reported that administration of *ritonavir*-containing regimens can cause *glucose homeostasis*<sup>8,9,10-16</sup>.

*Acute pancreatitis* is a life-threatening problem<sup>3</sup>. The annual incidence of *acute pancreatitis* in the US HIV population is considerably higher than in the general non-HIV-infected population<sup>5</sup>. Some studies show that the use of *hydroxyurea* with *didanosine* should probably be discouraged because the risk of *pancreatitis* is increased<sup>17</sup>.

Although, with all complications, we can not ignore the importance of medication. Different numerous studies have reported that there is always a need of treatment with vision of risk-benefit. The net median survival time after infection with HIV is estimated to be 9 to 11 years, depending on the HIV subtype<sup>18</sup> and the median survival rate after diagnosis of AIDS in resource-limited settings where treatment is not available ranges between 6 to 19 months, depending on the study<sup>19</sup>. But HAART therapy reduced the death rate by 80%, and raised the life expectancy for a newly diagnosed HIV-infected person to about 20 years<sup>20</sup>.

#### **Conclusion and Suggestion:**

We have observed from the above graphs that adverse effects start from the first day of the treatment and getting peak level after 5-10 years during the treatment. Then some supporting medication is needed to maximize the adherence with the drug. As we know efficacy and tolerability, these are the factors which make a drug 'the drug of choice'. The efficacy of combination ARV regimen is increased but adverse effects of these therapies may cause worse effects on the body organs, we can say its mix blessing because of prolonged treatment and less tolerability, it is very difficult for the HIV positive person to continue the treatment that results in treatment failure. There is a need to continue the efforts in the improvement of medication and better understanding the side effects of ARV regimen for HIV Specialist and physician who care for HIV/AIDS patients.

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