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Affect on Several Control Strategies of A Model for Malaria in an Endemic Region Like Bangladesh

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Abstract

Malaria is infectious disease caused by single-celled Plasmodium parasites, including P. falciparum, P. vivax, P. malariae, and P. ovale. The parasites are usually transmitted from infected to non infected people via the bite of female Anopheles mosquitoes about 60 species of Anopheles can serve as vectors. We have seen that malaria is endemic in developing countries like Bangladesh where inadequate drainage provides water logging which are the suitable breeding sectors of the disease vector. In this paper a simple SIS model is formulated between human and mosquito population which are constant. Also try to formulate a model for Malaria from Ghosh, M. & Pugliese, A. (2003) in view of Bangladesh where adding a new control strategy with others strategies and including a delay due to the incubation period. Here discussed several controls of strategies on the transmission of malaria. The control strategies incorporated here were: (i) biological control of mosquito larvae, (ii) chemical control of malaria vectors-adult mosquito, (iii) use of bed net and (iv) use of electric instruments. This model is tried to analyze by differential equations, include proof of a theorem and computer imitation. Various control strategies present dissimilar outputs nevertheless to get a successful malaria control plan, also recommended to crack down on combinations of all control strategies.

Key words: Mathematical modeling, Endemic region and SIS Model.

Introduction

Malaria is an acute flu-like illness caused by one of four species of parasite of the genus Plasmodium (Discussed above). The malaria disease is most commonly transmitted to humans through a bite of an infected female Anopheles mosquito. Malaria is most common in tropical and subtropical lands, particularly (More than 80% cases) in Africa and south Asia: Bangladesh, India, Sri Lanka and etc. Also malaria is a curable disease if promptly and adequately treated. When an Anopheles mosquito ingests blood from a malaria-infected person, malaria parasites develop in the mosquito and migrate into the mosquito's salivary glands. While rare, the malaria parasite can also be transmitted by transfusion with infected blood, by shared needle use, or from a mother to her unborn child. By Malaria about 300-500 million peoples are affected in the world and kill more than 1.5 million People annually. For details we can be seen WHO (world malaria report 2005). There are various control methods to maintain the spread of malaria implemented earlier in different parts of the world. Though every control strategy is not valid for every region, there are several other factors which influence disease transmission and also the control measures. For details one can see Walker (1992). Now few control measures are discussed:

(i) Biological control of mosquito larvae

The larval is the first stage of mosquitoes which can be controlled by biologically. The principal biological control agents that have been successfully employed against Anopheles are predators, particularly fish and the some bacterial pathogen that attack the larval stage of the mosquito. For detail we can be seen Das, P.K., Amalraj, D.D. (1997).

(ii) Chemical control of malaria vectors adult mosquito

Chemical control of adult female mosquito has been the most flourishing vector control methods since 1940. The most common practice is indoor residual house spraying of insecticides. DDT is the most widely used insecticide for vector control. For a long time the people in endemic region use mosquito coil as in house control strategies.

(iii) Use of bed net

Mosquitoes which are start biting seriously after sunset. Actually they bite in any dark place. In Africa, the malaria

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parasite accounts for at least 25% of all childhood mortality below age five. Bangladesh is one of the risky places for breeding mosquitoes because of our environment. By using bed net people can save them from malaria. Bed net is also ideal because it protect environment from any pollution.

(iv) Use of electric instruments

At the present time we have used different types of electric instrument which can control adult mosquitoes such as Electric net, Mega-catch, Mat which are used only at night. It is possible to die more then 1200 mosquito at a time by using Mega-catch. Now a day this control strategy is very effective all over the world. So by using this method we can control mosquitoes breeding.

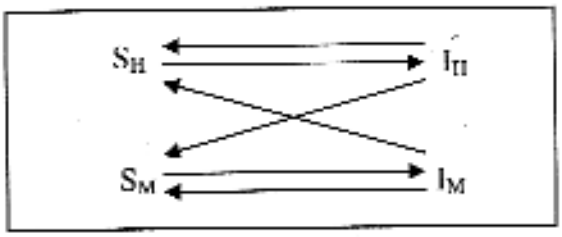
In Africa, southern part of Asia like India, Bangladesh, Sri-Lanka and etc persons who have been repeatedly infected by malaria a degree of immunity which suppresses most clinical symptoms. The people carry gametocytes in their blood that infect the mosquitoes biting them and form a separate class of reservoir population which helps in spreading malaria without getting affected themselves. For detail we can be seen Despommier, D.D., Gwadz, R.W., Hotez, P.J. (1994) & Nchinda, T.C. (1998). In this paper two models are constructed including this class of human population. Although there have been several experimental studies related to surveys of malaria in different regions. Das, S.C., Bhuyan, M., Baruah, I., Talukder, P.K. (1991) & Sharma, V.P. (1996, 1998) they discussed, spread of disease using mathematical models by considering different control strategies and delays due to the incubation period has not been conducted, particularly when the densities of the human and mosquito populations are variable. Here each control strategy is not valid for each region.

A Simple SIS Model

we take only susceptible and infective classes from the two species (Human & mosquitoes). This model is based on criss-cross interaction between this two species. Let, human and female mosquitoes both are constants which are shown in figure. For detail we can be seen Bailey, N.T J. (1975). Let the total number of human and mosquitoes are H and M respectively. Then

$$S_H(t) + I_H(t) = H$$

$$S_M(t) + I_M(t) = M \qquad (2.1.1)$$



First we consider, the rate of decrease of human susceptible (S_H) to be proportional to the human susceptible times the infectious mosquito (I_M) with a similar form for the mosquito rate. In this case, if some how any one infective human (I_H) or mosquitoes can be recovered, then they rejoin the susceptible class. Then the model (2.1.1) become,

$$\frac{dS_H}{dt} = -r_1 S_H I_M + \alpha_1 I_H$$

$$\&\ \frac{dI_H}{dt} = r_1 S_H I_M - \alpha_1 I_H$$

$$\frac{dS_M}{dt} = -r_2 S_M I_H + \alpha_2 I_H$$

$$\&\ \frac{dI_M}{dt} = r_2 S_M I_H - \alpha_2 I_H \qquad (2.1.2)$$

where r_1, α_1, r_2 and α_2 are positive parameters. At the initial condition we consider,

$$S_H(0) = S_{H_0} \ \&\ \ S_M(0) = S_{M_0}$$

$$I_H(0) = I_{H_0} \ \&\ \ I_M(0) = I_{M_0} \qquad (2.1.3)$$

But (2.1.3) is contain four equations *i.e.*
4th order system with (2.1.2) it reduces to a second order system in either S_H and S_M or I_H and I_M. Then

$$\frac{dI_H}{dt} = r_1(H - I_M)I_M - \alpha_1 I_H$$

$$\frac{dI_M}{dt} = r_2(M - I_M)I_H - \alpha_2 I_M \qquad (2.1.4)$$

Let, the equilibrium points (I_H, I_M) that are steady states of (2.1.4) are I_H=0=I_M. Then

$$\frac{dI_H}{dt} = 0 \ \ \&\ \ \frac{dI_M}{dt} = 0 \qquad (2.1.5)$$

From (2.1.5),

$$I_H = \frac{I_M H}{I_M + P_1} \quad [\because P_1 = \frac{\alpha_1}{r_1}] \qquad (2.1.6)$$

$$\text{and,} \quad I_M = \frac{I_H M}{I_H + P_2} [\because P_2 = \frac{\alpha_2}{r_2}] \qquad (2.1.7)$$

From (2.1.6) and (2.1.7)

$$MI_H^2 + P_1 I_H^2 + P_1 P_2 I_H = HIMI_H$$

$$\therefore I_H = 0 \text{ and } I_H = \frac{HM - P_1 P_2}{M + P_1} \quad (2.1.8)$$

Similarly,

$$I_M = 0 \text{ and } I_M = \frac{HM - P_1 P_2}{H + P_2} \quad (2.1.9)$$

Therefore, non-zero positive steady state levels of the infective population exist only if $\frac{HM}{P_1 P_2} > 1$, which is the threshold condition; if the positive steady state exists then the zero steady state is unstable. This is indeed the case. In metric form as a linearization of (2.1.5)

$$\frac{d}{dt}\begin{pmatrix} I_H \\ I_M \end{pmatrix} = \begin{pmatrix} -\alpha_1 & r_1 H \\ r_2 M & -\alpha_2 \end{pmatrix}\begin{pmatrix} I_H \\ I_M \end{pmatrix} \quad (2.1.10)$$

(Here, neglected non-linear term). Now find the Eigen values for the linearization

of (2.1.5) about $I_H = 0 = I_M$ are given by

$$\begin{vmatrix} -\alpha_1 & r_1 H \\ r_2 M & -\alpha_2 \end{vmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = 0$$

$$\therefore \lambda = \frac{-(\alpha_1 + \alpha_2) \pm \sqrt{(\alpha_1 + \alpha_2)^2 + 4\alpha_1\alpha_2\left(\frac{HM}{P_1 P_2} - 1\right)}}{2} \quad (2.1.11)$$

Here, if the threshold condition $\frac{HM}{P_1 P_2} > 1$ holds and $\lambda_1 < 0 < \lambda_2$ then the origin (0, 0) is a saddle point in the phase plane (I_H, I_M). If the threshold condition is not satisfied that is $\frac{HM}{P_1 P_2} < 1$ then the origin is stable since both $\lambda < 0$. In this case I_H is positive and I_M does not exist. Again if I_H and I_M exist, meaning in the context here that they are positive, then linearization (2.1.5) about it, if the Eigen values λ satisfy. In metric form of (2.1.5)

$$\frac{d}{dt}\begin{pmatrix} I_H \\ I_M \end{pmatrix} = \begin{pmatrix} -\alpha_1 - r_1 I_M & r_1 H - r_1 I_H \\ r_2 M - r_2 I_M & -\alpha_2 - r_2 I_H \end{pmatrix}\begin{pmatrix} I_H \\ I_M \end{pmatrix} \quad (2.1.12)$$

For, Eigen values

$$\lambda^2 + \lambda(\alpha_1 + \alpha_2 + r_1 I_M + r_2 I_H) + [\alpha_2 r_1 I_M + \alpha_1 r_2 I_H + \alpha_1\alpha_2(I_M H + I_H M) + \alpha_1\alpha_2 - r_1 r_2 HM] = 0 \quad (2.1.13)$$

is a characteristic equation where $\mathrm{Re}(\lambda) < 0$ and the positive steady state (I_H, I_M) is stable. So, the threshold condition for a non-zero steady state infected population is $\frac{HM}{P_1 P_2} > 1$. Now if every human is susceptible, then $\frac{r_1 H}{\alpha_1}$ is the average number of human contacted by a mosquito infective during its infectious period and vise versa for $\frac{r_2 M}{\alpha_2}$. Now $\frac{r_1 H}{\alpha_1}$ and $\frac{r_2 M}{\alpha_2}$ are the maximal human and mosquito contact rates respectively. This is not realistic for long term study of spread of disease. Thus in following article, a model is formulated where human and mosquito population is variable and including different control strategies.

The Mathematical Model

We consider, only two stages (larvae and adult) of mosquito population are considered in this article as it stays in pupa stage only for 4 days. So it is better to include delay in mosquito from larval stage to adult stage. Female mosquitoes of Anopheles species are the main culprit for breeding malaria in human population. So we take only female mosquitoes for this model. For details we can be seen Ghosh, M. & Pugliese, A. (2003). Suppose a fraction f of total mosquito density (D) is female then $fD = D_m$ (the density of female mosquitoes). The growth equations for the density of mosquito larvae (L) and for the density of adult mosquito (D) assuming density dependent mortality for both stages can be written as follows: (Here one can put some parameters zero to execute specific control methods suitable to that region).

$$\dot{L} = bfD - (\lambda + \lambda_1 L)L - cF(t)L$$

$$\dot{D} = cF(t)L - (\mu + \mu_1 D)D \quad (3.1.1)$$

Where, $b =$ is the population rate per individual, $c =$ is the settlement rate per unit of free space, $f =$ a fraction of total mosquito density is female, $\lambda =$ per capital death rates in

the larval of mosquito, λ_l = is constants corresponding to density dependent deaths in larvae , μ = per capital death rates in adult stages of mosquito, μ_1 = is constants corresponding to density dependent death in adult population(Breeding space) , $F(t)$ = is the free space at time t. From (3.1.1) we get

$$\dot{L} = bD_m - (\lambda + \lambda_1 L)L - cF(t)L$$

$$\dot{D}_m = fcF(t)L - (\mu + \mu_1 D_m)D_m \qquad (3.1.2)$$

we consider, female mosquito into two classes such as susceptible class(X_m) and infective class(Y_m). Now we are going to model the spread of malaria in epidemic region. Here the total human population density D_h into three classes such as susceptible class(X_h), infective class (Y_h), reservoir class (Z_h).Then the Model for the spread of malaria can be written, for details Ghosh, M. (2000).

$$\dot{X}_h = (b_h - f\frac{r_h}{k}D_h)D_h -$$

$$\left[d_h + (1-f)\frac{r_h}{K}D_h\right]X_h - \beta X_h Y_m + \nu Y_h$$

$$\dot{Y}_h = \beta X_h Y_m - \begin{bmatrix} \nu + \eta + d_h + \delta + \\ (1-f)\frac{r_h}{K}D_h \end{bmatrix} Y_h,$$

$$\dot{Z}_h = \delta Y_h - \{d_h + (1-f)\frac{r_h}{K}D_h\}Z_h,$$

$$\dot{D}_h = r_h(1-\frac{D_h}{K}) - \eta Y_h,$$

$$\dot{X}_m = fcFL - (\mu + \mu_1 D_m)X_m -$$

$$\beta' X_m Y_h - \gamma X_m Z_h,$$

$$\dot{Y}_m = \beta' X_m Y_h + \gamma Y_m Z_h - (\mu + \mu_1 D_m)Y_m,$$

$$\dot{D}_m = fcFL - (\mu + \mu_1 D_m)D_m \qquad (3.1.3)$$

where, b_h and d_h are natural birth rate & death rate respectively, $r_h = b_h - d_h$ is the growth rate, K = is the carrying capacity of the human population in the natural environment, where $0 < f < 1$ is the convex combination constant which governs the logistic birth and logistic death of the human population, when $f = 1$, then the model is logistic birth model as all of the restricted growth is due to decreasing birth rate and the death rate is constant, When $f = 0$ the model is logistic death model as all of the restricted growth is due to an increasing death rate and birth rate is constant (Gao, L.Q., Hethcote, H.W. 1992). ν = is the recovery rate constant, η = is the disease related death rates constant, β – is the interaction coefficient of the susceptible human with the infective mosquito population, β' = is the interaction coefficient of susceptible mosquito with the infective human class, γ = is the interaction coefficient of susceptible mosquito with the human reservoir class, δ = is the coefficient corresponding to the movement of human from infective class to reservoir class.

The Mathematical model with controls and delays

Let, $\kappa_1, \kappa_2, \kappa_3$ are the constants corresponding to the death of larvae due to the biological control, the chemical control, the death of adult mosquito due the electric instrument respectively. f_p = is the fraction of total population which uses bed net . Thus the fraction $(1-f_p)$ of each class of human are exposed to mosquito bites. Again we consider, τ_1 and τ_2 are the delay due to the incubation period in human & mosquito respectively and τ = is the delay corresponding to survival from larvae to adult stages. Thus the Model became:

$$\dot{L} = bD_m - cFL - (\lambda + \lambda_1 L)L - \kappa_1 L$$

$$\dot{Y}_m = \beta'(1-f)(D_m - Y_m)(t-\tau_1)Y_h(t-\tau_1) +$$
$$\gamma(1-f_p)Z_h(t-\tau_1)(D_m - Y_m)(t-\tau_1) -$$
$$(\mu + \mu_1 D_m)Y_m - \kappa_2 Y_m - \kappa_3 Y_m$$

$$\dot{D}_m = fcFL(t-\tau) - (\mu + \mu_1 D_m)D_m -$$
$$\kappa_2 D_m - \kappa_3 D_m$$

$$\dot{Y}_h = \beta(1-f_p)(D_h - Y_h - Z_h)(t-\tau_2)$$

$$Y_m(t-\tau_2) - (\nu + \eta + d_h + \delta + (1-f)\frac{r_h}{K}D_h)Y_h$$

$$\dot{Z}_h = \delta Y_h - \{d_h + (1-f)\frac{r_h}{K}D_h\}Z_h$$

$$\& \dot{D}_h = r_h(1-\frac{D_h}{K})D_h - \eta Y_h \qquad (3.2.1)$$

Here, the initial condition is $L(t)=I_0(t)\geq 0$, where $L_0(t)$ is given non negative function on $-\tau\leq t\leq 0$, $D_h(t)=D_{h0}(t)\geq 0$, where $D_{h0}(t)$ is given non-negative function on $-\tau_1\leq t\leq 0$, $Y_h(t)=Y_{h0}(t)\geq 0$, where $Y_{h0}(t)$ is given non-negative function on $-\tau_1\leq t\leq 0$, $Z_h(t)=Z_{h0}(t)\geq 0$, where $Z_{h0}(t)$ is given nonnegative function on $-\tau_1\leq t\leq 0$, $Y_m(t)=Y_{m0}(t)\geq 0$, where $Y_{m0}(t)$ is given nonnegative function on $-\tau_2\leq t\leq 0$, $D_m(t)=D_{m0}(t)\geq 0$, where $D_{m0}(t)$ is given nonnegative function on $-\tau_2\leq t\leq 0$, The steady state equilibrium of the system (3.2.1) can be derived as follows from the first and third equation we get, from first equation, $\dot{L}=0$

$$\therefore bD_m - cFL - (\lambda+\lambda_1 L)L - \kappa_1 L = 0 \tag{3.2.2}$$

From third equation of (3.2.1), $\dot{D}=0$;

$$\Rightarrow LfcF-(\mu+\mu_1 D_m)D_m-\kappa_2 D_m-\kappa_3 D_m=0$$

$$\therefore L=\frac{(\mu+\mu_1 D_m)D_m\kappa_2 D_m+\kappa_3 D_m}{fcF} \tag{3.2.3}$$

From (3.2.3) & (3.2.2),

$$D_m\{b-(cF+\lambda+\kappa_1)\frac{(\mu+\mu_1 D_m+\kappa_2+\kappa_3)}{fcF}- \lambda_1(\frac{\mu+\mu_1 D_m+\kappa_2+\kappa_3}{fcF})^2 D_m\}=0 \tag{3.2.4}$$

Thus $D_m=0$ and

$$\lambda_1(\frac{\mu_1}{\mu cF})^2 D_m{}^3+\frac{2(\mu+\kappa_2+\kappa_3)\mu_1\lambda_1}{(fcF)^2}D_m{}^2 \{(\frac{\mu+\kappa_2+\kappa_3}{fcF})^2+(\frac{cF+\lambda+\kappa_1}{fcF})\mu_1\}D_m+ (\frac{cF+\lambda+\kappa_1}{fcF})(\mu+\kappa_2+\kappa_3)-b=0 \tag{3.2.5}$$

where all the coefficients are positive except the constant term. For existence of unique positive root (say D_m^*) of (3.2.5), we need the following condition:

$$(\frac{\lambda+\kappa_1+cF}{fcF})(\mu+\kappa_2+\kappa_3)-b>0;$$

$$\therefore F>\frac{(\lambda+\kappa_1)(\mu+\kappa_2+\kappa_3)}{\{fb-(\mu+\kappa_2+\kappa_3)\}c} \tag{3.2.6}$$

At the steady state,

$$\dot{D}_h=r_h(1-\frac{D_h}{K})D_h-\eta Y_h=0;$$

$$\therefore Y_h=\frac{r_h}{\eta}(1-\frac{D_h}{K}) \tag{3.2.7}$$

$$\dot{Z}_h=\delta Y_h-\{d_h+(1-f)\frac{r_h}{K}D_h\}Z_h=0;$$

$$\therefore Z_h=\frac{\delta\frac{r_h}{\eta}(1-\frac{D_h}{K})}{d_h+(1-f)\frac{r_h}{K}D_h} \tag{3.2.8}$$

$$\dot{Y}_m=0,\Rightarrow \beta'(1-f_{ip})(D_m^*-Y_m)Y_h+ \gamma(1-f_{ip})Z_h(D_m^*-Y_m)-(\mu+\mu_1 D_m^*)Y_m -\kappa_2 Y_m-\kappa_3 Y_m=0$$

$$\therefore Y_m=\frac{(1-f_{ip})D_m^*\frac{r_h}{\eta}(1-\frac{D_h}{K})D_h}{\left[\beta'+\frac{\gamma\delta}{d_h+(1-f)\frac{r_h}{K}D_h}\right]} *(1-f_{ip})\frac{r_h}{\eta}(1-\frac{D_h}{K})D_h+ \left(\mu+\kappa_2+\kappa_3+\mu_1 D_m^*\right) \tag{3.2.9}$$

Again,

$$\beta(1-f_{ip})(D_h-Y_h-Z_h)Y_m- \{v+\eta+d_h+\delta+(1-f)\frac{r_h}{K}D_h\}Y_h=0$$

$$\beta(1-f_{ip})\left[1-\frac{r_h}{\eta}(1-\frac{D_h}{K})(1+\frac{\delta}{\{d_h+(1-f)\frac{r_h}{K}\}D_h})\right]*$$

$$D_hY_m=\{v+\eta+d_h+\delta+(1-f)\frac{r_h}{K}D_h\}\frac{r_h}{\eta}(1-\frac{D_h}{K})D_h \tag{3.2.10}$$

From (3.2.9) & (3.2.10), $D_h=0, D_h=K$ or

$$F(D_h) = R_1 D_h^5 + R_2 D_h^4 + R_3 D_h^3 + R_4 D_h^2 + R_5 D_h + R_6 = 0 \quad (3.2.11)$$

$$R_1 = \{(1-f)\frac{r_h}{K}\}^3 \beta'(1-f_{ip})\frac{r_h}{\eta K};$$

$$R_2 = \frac{\beta\beta' D_m}{\eta}(1-f)^2(1-f_{ip})^2(\frac{r_h}{K})^3 +$$

$$\{(1-f)\frac{r_h}{K}\}^2\{(\beta' d_h + \gamma\delta)(1-f_{ip})\frac{r_h}{\eta K} -$$

$$\beta'(1-f)(1-f_{ip})\frac{r_h^2}{\eta K}\} + \beta'(1-f)(1-f_{ip})$$

$$(\frac{r_h^2}{K^2\eta})\{\upsilon+\alpha+d_h+\delta(1-f)\frac{r_h}{K}+d_h(1-f)\frac{r_h}{K}\}.$$

$$R_3 = \frac{\beta(1-f_{ip})^2 D_m^* r_h^2(1-f)(\beta' d_h+\gamma\delta)}{K^2\eta} +$$

$$\beta(1-f_{ip})^2 D_m^* \beta'(1-f)\frac{r_h}{K}\{(1-\frac{r_h}{\eta})(1-f)\frac{r_h}{K} +$$

$$\frac{r_h}{K}(d_h+\delta)\} + (\upsilon+\eta+d_h+\delta)\beta'(1-f)\frac{r_h}{K} *$$

$$(1-f_{ip})\frac{r_h}{K\eta} + \{(\upsilon+\eta+2d_h+\delta)(1-f)\frac{r_h}{K}\} *$$

$$\{(\beta' d_h+\gamma\delta)(1-f_{ip})\frac{r_h}{K\eta} - \beta'(1-f) *$$

$$(1-f_{ip})\frac{r_h^2}{K\eta}\} - \{(1-f)\frac{r_h}{K}\}^2\{\mu+\kappa_2+\kappa_3 +$$

$$\mu_1 D_m^*(1-f)\frac{r_h}{K} + (\beta' d_h+\gamma\delta)(1-f_{ip})\frac{r_h}{\eta}\}$$

$$R_4 = \beta(1-f_{ip})^2 D_m^*(\beta' d_h+\gamma\delta)\{(1-\frac{r_h}{\eta}) *$$

$$(1-f)\frac{r_h}{K} + \frac{r_h}{\eta K}(d_h+\delta)\} + \beta(1-f_{ip})^2 *$$

$$D_m^*\beta'(1-f)\frac{r_h}{K}\{1-\frac{r_h}{K}(d_h+\delta)\}\{(1-f)\frac{r_h}{K}\}^2 *$$

$$(\mu+\kappa_2+\kappa_3+\mu_1 D_m^*)d_h - \{(\upsilon+\eta+2d_h+\delta) *$$

$$(1-f)\frac{r_h}{K}\}\{(\mu+\kappa_2+\kappa_3+\mu_1 D_m^*)(1-f)\frac{r_h}{K} +$$

$$(\beta' d_h+\gamma\delta)(1-f_{ip})\frac{r_h}{\eta}\} + d_h(\upsilon+\eta+2d_h+\delta) *$$

$$\{(\beta' d_h+\gamma\delta)(1-f_{ip})\frac{r_h}{\eta K} - \beta'(1-f)(1-f_{ip})\frac{r_h^2}{K^2\eta}\}.$$

$$R_5 = \beta(1-f_{ip})^2 D_m^*\{1-\frac{r_h}{\eta}(d_h+\delta)\} *$$

$$(\beta' d_h+\gamma\delta) - \{d_h(\upsilon+\eta+d_h+\delta) *$$

$$\{(\mu+\kappa_2+\kappa_3+\mu_1 D_m^*)(1-f)\frac{r_h}{K} +$$

$$(\beta' d_h+\gamma\delta)(1-f_{ip})\frac{r_h}{\eta}\} + (\mu+\kappa_2 +$$

$$\kappa_3+\mu_1 D_m^* d_h(\upsilon+\eta+2d_h+\delta)(1-f)\frac{r_h}{K}$$

$$R_6 = -\begin{pmatrix}\mu+\kappa_2+\\ \kappa_3+\mu_1 D_m^*\end{pmatrix} d_h^2 \begin{pmatrix}\upsilon+\eta+\\ d_h+\delta\end{pmatrix}$$

$$\text{Now, } F(0) = -\begin{pmatrix}\upsilon+\eta+\\ d_h+\delta\end{pmatrix}\begin{pmatrix}\mu+\mu_1 D_m^*+\\ \kappa_2+\kappa_3\end{pmatrix} d_h^2 < 0 \quad (3.2.12)$$

$$F(K) = \beta(1-f_{ip})^2 K D_m^*\{d_h+(1-f)r_h\}$$
$$[\{\beta' d_h+(1-f)r_h\}+\gamma\delta] - \{\upsilon+$$
$$\eta+d_h+\delta+(1-f)r_h\}(\mu+\mu_1 D_m^* +$$
$$\kappa_2+\kappa_3)\{d_h+(1-f)r_h\}^2 \quad (3.2.13)$$

Hence, the sufficient condition for the existence of at least one positive root of above (3.2.13) lying between 0 and K is $F(K) > 0$

$$R_0[say] = \frac{\left[\beta' + \frac{\gamma\delta}{d_h + (1-f)r_h}\right] * \beta(1-f_{ip})^2 K D_m^*}{\{\upsilon + \eta + d_h + \delta + (1-f)r_h\} * (\mu + \mu_1 D_m^* + \kappa_2 + \kappa_3)} > 1 \quad (3.2.14)$$

This condition (3.2.14) is the threshold condition (which we will see later too while showing the instability of disease free equilibrium, this is important in the sense that this is the key factor which control the persistence of disease in the population. Knowing other parameters values one can see what should be the value of f_{ip} and also what should the value be of κ_2 and κ_3 (when $\kappa_2 \uparrow, \kappa_3 \uparrow, D_m \downarrow$ & $(\frac{D_m^*}{\mu + \mu_1 D_m^* + \kappa_2 + \kappa_3}) \downarrow$) so that this R_0 can be made less than one and it will make disease free equilibrium to be stable. Here the larval control factor is not coming directly into the picture but it is associated with D_m^*. The fraction $\frac{D_m^*}{\mu + \mu_1 D_m^* + \kappa_2 + \kappa_3}$ is increasing with D_m^*. Hence control κ_1 regulates the total female mosquitoes D_m^* and thus affecting this threshold condition. We need to show that there exists only one positive root in the interval 0 and K. Here R_1 is positive and R_6 is negative, the sufficient condition for R_2 to be positive is $(\upsilon + \alpha + 2d_h + \delta) > r_h$. Hence for the existence of unique equilibrium point, we put following conditions:

$$R_2 > 0,\ R_3 > 0 \text{ and } R_5 < 0. \quad (3.2.15)$$

So that there will be only one change of sign in (3.2.11). Hence using Descarte's rule of signs, we can say that there exist at most one positive root. After getting this unique positive root say D_h^*, it is easy to find Y_h^*, Z_h^*, Y_m^* using equations (3.2.7), (3.2.8) and (3.2.9). Hence the existence of unique equilibrium point is guaranteed under (3.2.15).

Stability Analysis

For non trivial equilibrium

Let, $I = I^* + i$, $Y_m = Y_m^* + y_m$, $D_m = D_m^* + d_m$, $Y_h = Y_h^* + y_h$, $Z_h = Z_h^* + z_h$, $D_h = D_h^* + d_h$

Then the linear system of (3.2.1) is given by $\frac{dI}{dt} = \frac{di}{dt}$.

Similarly,

$$\dot{i} = bd_m - (cF + \lambda + \kappa_1 + 2\lambda_1 I^*)i$$

$$\dot{y}_m = \{\beta' y_h(t-\tau) + \gamma z_h(t-\tau_1)\}(1-f_{ip})(D_m^* - Y_m^*) + \{\beta Y_h^* + \gamma Z_h^*\}(1-f_{ip})\{d_m(t-\tau_1) - y_m(t-\tau_1)\} - (\mu + \kappa_2 + \kappa_3 + \mu_1 D_m^*) y_m - \mu Y_m^* d_m$$

$$\dot{d}_m = fcFi(t-\tau) - (\mu + \kappa_2 + \kappa_3 + 2\mu_1 D_m^*) d_m$$

$$\dot{y}_h = \beta(1-f_{ip})[(D_m^* - Y_h^* - Z_h^*) * y_m(t-\tau_2) + Y_m^*\{d_h(t-\tau_2) - y_h(t-\tau_2) - z_h(t-\tau_2)\}] - \{\upsilon + \eta + d_h + \delta + (1-f)\frac{r_h}{K} D_h^*\} y_h - (1-f)\frac{r_h}{K} Y_h^* d_h$$

$$\dot{z}_h = \delta y_h - \{p_h + (1-f)\frac{r_h D_h^*}{K}\} Z_h - (1-f)\frac{r_h Z_h^*}{K} d_h$$

$$\dot{d}_h = r_h d_h - \frac{2r_h D_h^*}{K} d_h - \eta y_h \quad (3.3.1)$$

Now looking for the solution in the form of $\sim \exp(\psi t)$ to get a characteristic polynomial

$$\nabla(\psi, \tau, \tau_1, \tau_2) = [\psi^2 + \psi(cF + \lambda + \kappa_1 + 2\lambda_1 I^* + \mu + \kappa_2 + \kappa_3 + 2\mu_1 D_m^*) + (cF + \lambda + \kappa_1 + 2\lambda_1 I^*)(\mu + \kappa_2 + \kappa_3 + 2\mu_1 D_m^*) - \alpha bcF e^{-\psi\tau_1}] * [\psi^4 + \psi^3 H_3 + \psi^2 H_2 +$$

$$H_3 = m_{22} + m_{44} + m_{55} + m_{66}$$

$$H_2 = m_{22}(m_{44} + m_{55} + m_{66}) + m_{44}(m_{55} + m_{66})$$

$$+m_{55}m_{66}\{\beta(1-f_{ip})Y_m^* e^{-\psi\tau_2} - (1-f)\frac{r_h}{K}Y_h^*\}\eta$$

$$-\beta(1-f_{ip})(D_h^* - Y_h^* - Z_h^*) * \beta'(1-f_{ip})(D_m^* - Y_m^*)e^{-\psi\tau_2}$$

$$H_1 = m_{22}m_{44}m_{55} + m_{44}m_{55}m_{66} +$$

$$m_{55}m_{66}m_{22} + m_{66}m_{22}m_{44} +$$

$$(m_{22} + m_{55})\{\beta(1-f_{ip}) * Y_m^* e^{-\psi\tau_2}$$

$$-(1-f)\frac{r_h}{K}Y_h^*\}\eta - \beta(1-f_{ip})Y_m^*\eta *$$

$$(1-f)r_h Z_h^* / K e^{-\psi\tau_2} - (m_{55} + m_{66}) *$$

$$\beta(1-f)^2(D_h^* - Y_h^* - Z_h^*)\beta'(D_m^* - Y_m^*)e^{-\psi\tau_2}$$

$$H_0 = m_{22}m_{44}m_{55}m_{66} + m_{22}m_{55}\{\beta(1-f_{ip}) *$$

$$Y_m^* e^{-\psi\tau_2} - (1-f)\frac{r_h}{K}Y_h^*\}\eta - m_{22}\beta(1-f_{ip}) *$$

$$Y_m^* e^{-\psi\tau_2}\eta(1-f) * \frac{r_h}{K}Z_h^* - m_{55}m_{66}\beta\beta' *$$

$$(1-f_{ip})^2(D_h^* - Y_h^* - Z_h^*)$$

$$m_{22} = \beta'(1-f_{ip})Y_m^* e^{-\psi\tau_1} + \gamma(1-f_{ip}) *$$

$$Z_h^* e^{-\psi\tau_1} + \mu + \kappa_2 + \kappa_3 + \mu_1 D_m^*$$

$$m_{44} = \beta(1-f_{ip})Y_m^* e^{-\psi\tau_2} + \upsilon + \eta +$$

$$p_h + \delta + (1-f)\frac{r_h}{K}D_h^*$$

$$m_{55} = p_h + (1-f)\frac{r_h}{K}D_h^*,$$

$$m_{66} = \frac{r_h}{K}(2D_h^* - K)(D_m^* - Y_m^*)e^{-\psi\tau_2}$$

$$-\beta\gamma(1-f_{ip})^2(D_h^* - Z_h^*)(D_m^* - Y_m^*) *$$

$$\eta(1-f)\frac{r_h}{K}Z_h^* e^{-\psi\tau_2}(\tau_1 + \tau_2)$$

For $\tau = \tau_1 = \tau_2 = 0$, using Routh-Hurwitz criteria the non-trivial (if it exists) will be locally asymptotically stable if the following condition hold:

$$\begin{aligned}&(cF + \lambda + \kappa_1 + 2\lambda_1 L^*) *\\ &(\mu + \kappa_2 + \kappa_3 + 2\mu_1 D_m^*) > fbcF\end{aligned} \qquad (3.3.3)$$

$$\text{and} \quad H_3 > 0, \begin{vmatrix} H_3 & H_1 \\ 1 & H_2 \end{vmatrix} > 0,$$

$$\begin{vmatrix} H_3 & H_1 & 0 \\ 1 & H_2 & H_0 \\ 0 & H_3 & H_1 \end{vmatrix} > 0, \begin{vmatrix} H_3 & H_1 & 0 & 0 \\ 1 & H_2 & H_0 & 0 \\ 0 & H_3 & H_1 & 0 \\ 0 & 1 & H_2 & H_0 \end{vmatrix} > 0 \qquad (3.3.4)$$

Here the expression for H_3, H_2, H_1, H_0 are same as defined earlier with $\tau = \tau_1 = \tau_2 = 0$. It is easy to say that the condition (3.3.3) is automatically satisfied using the equation corresponding to the equilibrium of the system (3.2.1). Also the first two inequalities of (3.3.4) are satisfied, so conditions for local asymptotic stability are the third inequality and $H_0 > 0$. The result of an equilibrium analysis of the model (3.2.1) is stated in the following theorem.

Theorem

A set of necessary and sufficient conditions for $E^*(I_h^*, Y_m^*, D_m^*, Y_h^*, Z_h^*, D_h^*)$ to be asymptotically stable for $\tau \geq 0, \tau_1 \geq 0, \tau_2 \geq 0$ is the following:

(i) The real parts of all the roots of $V(\omega,0,0,0) = 0$ are negative,

(ii) For all real ω_2 and $\tau \geq 0$, $\nabla(i\omega_2, \tau, 0, 0) \neq 0$,

(iii) For all real ω_2 and $\tau_1 \geq 0$, $V(i\omega_1, 0, \tau_1, 0) \neq 0$,

(iv) For all real ω_2 and $\tau_2 \geq 0$, $V(i\omega_2, 0, 0, \tau_2) \neq 0$.

Proof: (a) The real parts of all the roots of $V(\omega,0,0,0) = 0$ are negative, when $\tau = \tau_1 = \tau_2 = 0$ then,

$$m_{22} = \beta'(1-f_{ip})Y_m^* + \gamma(1-f_{ip})Z_h^* +$$

$$(\mu + \kappa_2 + \kappa_3 + \mu_1 D_m^*), \quad m_{44} = \beta(1-f_{ip})$$

$$Y_m^* + \upsilon + \eta + p_h + \delta + (1-f)\frac{r_h}{K}D_h^*,$$

$$m_{55} = p_h + (1-f)\frac{r_h}{K}D_h, \quad m_{66} = \frac{r_h}{K}(2D_h^* - K)$$

Here we see m_{22}, m_{44}, m_{55} and m_{66} all are positive. So their multiple additions are also positive. Then the characteristic equation is

$$\nabla(\psi,0,0,0) = |\psi^2 + \psi(cF + \lambda + \kappa_1 + 2\lambda_1 L^* + \mu + 2\mu_1 D_m^* + \kappa_2 + \kappa_3) + (cF + \lambda + \kappa_1 + 2\lambda_1 L^*)(\mu + \kappa_2 + \kappa_3 + 2\mu_1 D_m^*) - fbcF] * [\psi^4 + \psi^3 H_3 + \psi^2 H_2 + H_1\psi + H_0| = 0$$

$$(cF + \lambda + \kappa_1 + 2\lambda_1 L^*) * (\mu + \kappa_2 + \kappa_3 + 2\mu_1 D_m^*) > fbcF$$

and

$$H_4 = \{(cF + \lambda + \kappa_1 + 2\lambda_1 L^*)(\mu + \kappa_2 + \kappa_3 + 2\mu_1 D_m^*) - fbcF\},$$

$$H_5 = cF + \lambda + \kappa_1 + 2\lambda_1 L^* + \mu + 2\mu_1 D_m^* + \kappa_2 + \kappa_3$$

are also positive. Then

$$\nabla(\psi,0,0,0) = \psi^6 + \psi^5(H_3 + H_5) + \psi^4(H_2 + H_5H_3 + H_4) + \psi^3(H_1 + H_2H_5 + H_4H_3) + \psi^2(H_0 + H_1H_5 + H_2H_4) + \psi(H_5H_0 + H_1H_4) + H_4H_0 = 0$$

It is a 6^{th} order polynomial of a function ψ . By using Descarte's rule of signs there is no positive root in this equation which may be negative or imaginary. i.e. some roots are negative and real. Therefore the real parts of $\nabla(\psi,0,0,0) = 0$ is negative, similarly we can say that (b), (c), and (d) is also satisfied then $E^*(L^*, Y_m^*, D_m^*, Y_h^*, Z_h^*, D_h^*)$ be asymptotically stable for $\tau \geq 0, \tau_1 \geq 0, \tau_2 \geq 0$. **(Proved)**

Condition for no stability change of the equilibrium E^* due to the delay τ

In fact in this section we are going to see the stability of two dimensional systems $(L - D_m$ System) without infection and how mosquito dynamics affects the stability of our system (3.2.1). As τ enters only in the first factor of the expression (3.3.2) then we get

$$\psi^2 + \{cF + \lambda + \kappa_1 + 2\lambda_1 L^* + \mu + \kappa_2 + \kappa_3 + 2\mu D_m^*\}\psi + (cF + \lambda + \kappa_1 + 2\lambda_1 L^*) * (\mu + \kappa_2 + \kappa_3 + 2\mu D_m^*) - fbcF e^{-\psi\tau} = 0 \quad (3.4.1)$$

$$\Rightarrow \{(\omega_1^2 - \omega_2^2) + (cF + \lambda + \kappa_1 + 2\lambda_1 L^* + \mu + \kappa_2 + \kappa_3 + 2\mu D_m^*)\omega_1 - fbcF e^{-\omega_1\tau} * \cos\omega_2\tau + (cF + \lambda + \kappa_1 + 2\lambda_1 L^*)(\mu + \kappa_2 + \kappa_3 + 2\mu D_m^*)\} + i\{2\omega_1\omega_2 + \omega_2(cF + \lambda + \kappa_1 + 2\lambda_1 L^* + \mu + \kappa_2 + \kappa_3 + 2\mu D_m^*) + fbcF e^{-\omega_1\tau}\sin\omega_2\tau\} = 0$$

$[\because$ Putting $\psi = \omega_1 + i\omega_2]$

Now separating real and imaginary part then

$$\{(\omega_1^2 - \omega_2^2) + (cF + \lambda + \kappa_1 + 2\lambda_1 L^* + \mu + \kappa_2 + \kappa_3 + 2\mu D_m^*)\omega_1 + (cF + \lambda + \kappa_1 + 2\lambda_1 L^*)(\mu + \kappa_2 + \kappa_3 + 2\mu D_m^*)\} - fbcF e^{-\omega_1\tau}\cos\omega_2\tau = 0 \quad (3.4.2)$$

$$2\omega_1\omega_2 + \omega_2(cF + \lambda + \kappa_1 + 2\lambda_1 L^* + \mu + \kappa_2 + \kappa_3 + 2\mu D_m^*) + fbcF e^{-\omega_1\tau}\sin\omega_2\tau = 0 \quad (3.4.3)$$

Now putting

$A = cF + \lambda + \kappa_1 + 2\lambda_1 L^*$ & $B = \mu + \kappa_2 + \kappa_3 + 2\mu D_m^*$; then $\omega_1 = 0$

and, $-\omega_2^2 + AB = fbcF\cos\omega_2\tau$ (3.4.4)

& $\omega_2(A + B) = -fbcF\sin\omega_2\tau$ (3.4.5)

From (3.4.4) & (3.4.5),

$$(\omega_2)^2 + (A^2 + B^2)\omega_2^2 + A^2B^2 = (fbcF)^2 \quad (3.4.6)$$

$$\therefore x=\frac{1}{2}\{-(A^2+B^2)\pm \sqrt{(A^4+B^4)-2A^2B^2+(4fbcF)^2}\}$$

$$[\text{putting } \omega_2^2 = x] \qquad (3.4.7)$$

Thus the condition for existence of real ω_2 to get positive x is

$$(4fbcF)^2-2A^2B^2>0;$$

$$\therefore fbcF>\frac{1}{\sqrt{2}}(AB) \qquad (3.4.8)$$

From (3.4.5) & (3.4.4),

$$\tau=\frac{n\pi}{\omega_2}+\frac{1}{\omega_2}\tan^{-1}\left(\frac{(A+B)\omega_2}{\omega_2^2-AB}\right) \qquad (3.4.9)$$

$$\left[\because \text{here } n\geq 0, \beta=\tan^{-1}\left(\frac{(A+B)\omega_2}{\omega_2^2-AB}\right)\right]$$

Hence the necessary and sufficient condition for no stability change is the just opposite condition of (3.4.8).

$$i.e.\ fbcF<\frac{1}{\sqrt{2}}(AB) \qquad (3.4.10)$$

Imitation

Adult female mosquitoes lay eggs in every 1-2 weeks and lay near about 300 or 10-20 eggs if they have fed or not fed respectively. Consider, an adult female mosquito lays 600 eggs per mouth averagely, larval death rate is taken as 1/16 and the death rate of adult female mosquitoes is 1/30 because they survive height 30 days. We don't know the density dependent death rate of mosquito larvae or adults,

so we take, $b=20, f=0.5, \lambda=0.0625, \lambda_1=0.0005, \mu=1/30, \mu_1=0.0002$, putting $cF=0.01, \beta=\beta'=0.000002, \gamma=0/10$. [For details Burton S.J. & Cohen, R. (1980), Nedelman, J. (1985) and Ngwa, G.A. & Shu, W.S. (1999)].

Different areas may have different epidemiological environments and it may vary. Suppose a small fraction $i.e.\ v=0.01$ which corresponds the infections period of 100 days and $\delta=0.00005$ (fraction going to reservoir class). We assume per capital birth rate is $b_h=30/365000$ $P_h=1/2190$. The disease related death rate of human is about 1.5 % implying per capita disease induced death rate $\eta=0.015/36$. The carrying capacity K is constant. To illustrate the global stability behavior of E^* and to see the effect of various parameters on the spread of the disease, the system (3.2.1) is integrated using the fourth order Runge-Kutta method. Initially we take all control parameters $\kappa_1=0$, $\kappa_2=0$, $\kappa_3=0$, $f_{rp}=0$ and $K-30000$ and delays are $\tau-20$, $\tau_1=15$, $\tau_2=15$. The equilibrium values are $L=6846.95$, $Y_m=397.91$ $D_m=1227.65$ $Y_h=6202.9$ $Z_h=4836.41$ $D_h=18954.7$

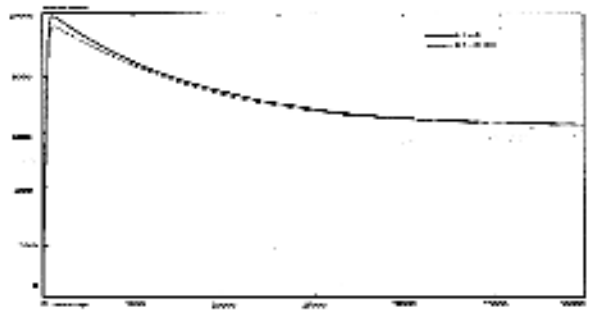


Fig. 4.1: Effect of k_1 on infected human.

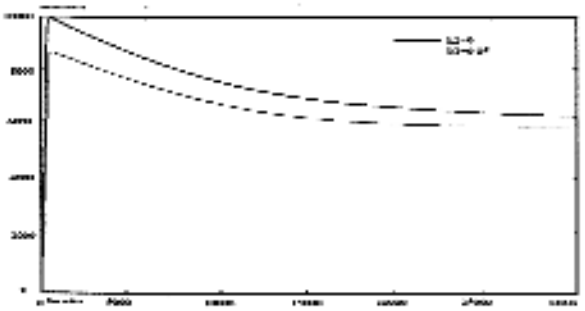


Fig. 4.2: Effect of k_2 on infected human.

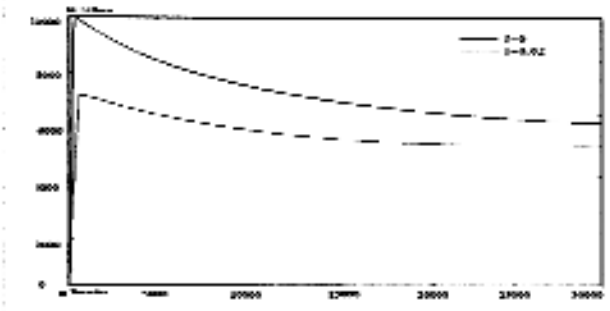


Fig. 4.3: Effect of f on infected human

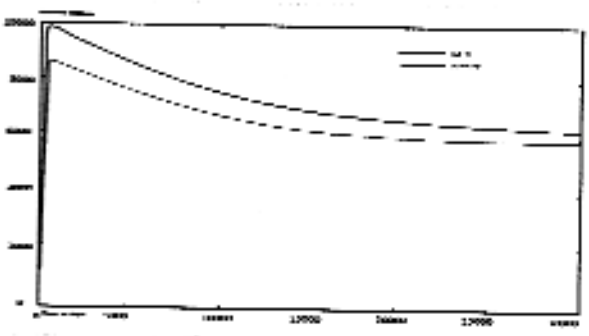


Fig. 4.4: Effect of k_3 on infected human

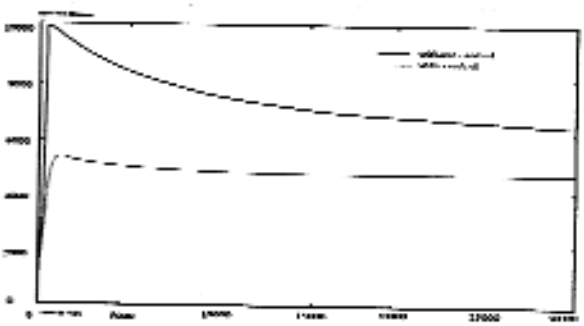


Fig. 4.5: Effect of all control measure simultaneously on infected human

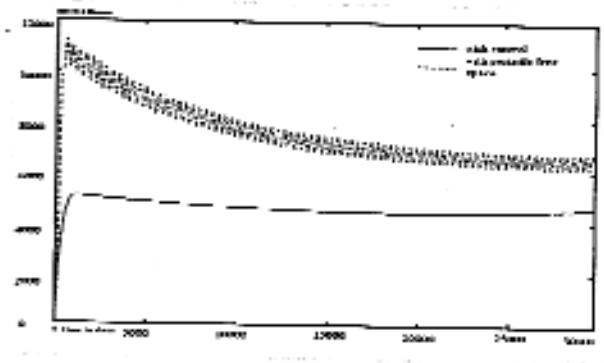


Fig. 4.6: Effect of periodic free space (F) on infected human

Imitation is performed for different initial positions. It is observed that solution curves tend to tile same point for all initial starts implying the global stability of this equilibrium. The effect of different control measures on the infected human population are shown in figures (4.1- 4.5). We also consider the case in which the free space *(F)* is periodic. For example during rainy season water for Bangladesh logging increases free space and helps mosquito breeding. It is observed that in this case disease persists with oscillation and with increased infected humans (see *fig. 4.1 - 4.6*).

Conclusion

In this paper a simple SIS model and nonlinear model for malaria transmission in Bangladesh is proposed and analyzed. The existence of non trivial equilibrium point is shown under some condition. The conditions for no stability change for the non trivial equilibrium point is obtained. Result on local asymptotic stability of boundary equilibrium is also included. Using imitation it is found that the nontrivial equilibrium point is in fact globally stable for the chosen set of parameters. The effects of different control measures for malaria control are observed and it is concluded that use of bed net is the most effective control measure for the control of malaria. Parallelism, if we can use other control measures with bed net then the number of infected human population decreases tremendously.

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