

Decay of Temperature Fluctuations in Homogeneous Turbulence before the Final Period for the Case of Multi-Point and Multi-Time in a Rotating System

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Abstract

Using Deissler's approach the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particle is studied and have considered correlation between fluctuating quantities at two and three point. Two and three point correlation equations in a rotating system is obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison to the second and third order correlations. The correlations are converted to spectral form by taking their Fourier transforms. Finally integrating the energy spectrum over all wave numbers, the energy decay law of temperature fluctuations in homogeneous turbulent before the final period for the case of multi-point and multi-time in a rotating system is obtained.

Introduction

Deissler^{1,2} developed a theory for homogeneous turbulence which was valid for times before the final period. Following Deissler's theory Loeffler and Deissler³ studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their study, they presented the theory which is valid during the period for which the quadruple correlation terms are neglected compared to the 2nd and 3rd – order correlation terms. Using Deissler's same theory Kumar and Patel⁴ studied the first-order reactions in homogeneous turbulence before the final period for the case of multi-point and single-time. The problem⁴ is extended to the case of multi-point and multi-time concentration correlation by Kumar and Patel⁵ and also the numerical result of⁵ carried out by Patel.⁶ Following

Deissler's approach Sarker and Islam studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time.⁷ M. A. Islam and M. S. A. Sarker⁸ also studied the first-order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Sarker and L. Rahman⁹ studied the decay of temperature fluctuations in MHD turbulence before the final period. Sarker and Islam¹⁰ also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. Azad and Sarker¹¹ studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle.

M. A. K. Azad and S. A. Sarker¹² also studied the decay of dusty fluid MHD turbulence before the final period in a rotating system for the case of multi-point and multi-time. S. Sultana and S. A. Sarker¹³ discussed the Decay of homogeneous dusty fluid turbulence before the final period. Further work along this same line has more recently been done by M. A. K. Azad and M. S. Alam Sarker.^{14,15}

In their approach, they considered two and three point correlations and neglecting fourth- and higher-order correlation terms compared to the second-and third-order correlation terms.

In this paper the Deisslers^{1,2} method is used and we have studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system.

Correlation and spectral equations

For an incompressible fluid with constant properties and for negligible frictional heating, the energy equation may be written as

$$\frac{\partial \tilde{T}}{\partial t} + \tilde{u}_i \frac{\partial \tilde{T}}{\partial x_i} = \frac{k}{\rho C_p} \frac{\partial^2 \tilde{T}}{\partial x_i \partial x_i} \quad \text{----- (1)}$$

where \tilde{T} and \tilde{u}_i are instantaneous values of temperature and velocity; k, thermal conductivity; ρ , fluid density; C_p heat capacity at constant pressure; x_i , space coordinate; t, time and the repeated subscripts are summed from 1 to 3.

Breaking these instantaneous values into time average and fluctuating components as

$$\tilde{T} = \langle T \rangle + T \quad \text{and} \quad \tilde{u}_i = \langle u_i \rangle + u_i$$

where T, temperature fluctuation from time average ; $\langle T \rangle$, time average value of temperature; u_i , velocity fluctuation from time average; $\langle u_i \rangle$, time average value of velocity.

From the condition of homogeneity it follows that $\frac{\partial \langle T \rangle}{\partial x_i} = 0$, and in addition the

usual assumption is made that $\langle T \rangle$ is independent of time and that $\langle u_i \rangle = 0$. Thus equation (1) becomes

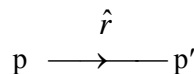
$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \left(\frac{\nu}{p_r} \right) \frac{\partial^2 T}{\partial x_i \partial x_i} \quad \text{----- (2)}$$

where $p_r = \nu / \gamma$, Prandtl number; ν , kinematic viscosity and $\gamma = \frac{k}{\rho C_p}$.

Equation (2) is assumed to hold at the arbitrary point p. For the point p' the corresponding equation can be written

$$\frac{\partial T'}{\partial t'} + u'_i \frac{\partial T'}{\partial x'_i} = \left(\frac{\nu}{p_r} \right) \frac{\partial^2 T'}{\partial x'_i \partial x'_i} \quad \text{----- (3)}$$

The points p and p' are separated by the vector \hat{r} is shown below



Multiplying equation (2) by T' , equation (3) by T and taking ensemble average, result in

$$\frac{\partial \langle TT' \rangle}{\partial t} + \frac{\partial \langle TT'u_i \rangle}{\partial x_i} = \left(\frac{\nu}{p_r} \right) \frac{\partial^2 \langle TT' \rangle}{\partial x_i \partial x_i} \quad \text{--- (4)}$$

$$\frac{\partial \langle TT' \rangle}{\partial t'} + \frac{\partial \langle TT'u'_i \rangle}{\partial x'_i} = \left(\frac{\nu}{p_r} \right) \frac{\partial^2 \langle TT' \rangle}{\partial x'_i \partial x'_i} \quad \text{--- (5)}$$

with the continuity equation,

$$\frac{\partial u_i}{\partial x} = \frac{\partial u'_i}{\partial x'_i} = 0 \quad \text{----- (6)}$$

The angular bracket has been used to denote an ensemble average.

Using the transformations

$$\begin{aligned} \frac{\partial}{\partial x_i} &= -\frac{\partial}{\partial r_i}, \quad \frac{\partial}{\partial x'_i} = \frac{\partial}{\partial r_i}, \quad \left(\frac{\partial}{\partial t} \right)_t \\ &= \left(\frac{\partial}{\partial t} \right)_{\Delta t} - \frac{\partial}{\partial \Delta t}, \quad \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t} \end{aligned}$$

into equation (4) and (5), one obtains

$$\begin{aligned} \frac{\partial \langle TT' \rangle}{\partial t} - \frac{\partial \langle u_i TT' \rangle}{\partial r_i} (-r_i - \Delta t, t + \Delta t) + \\ \frac{\partial \langle TT'u'_i \rangle}{\partial r_i} (\hat{r}, \Delta t, t) = 2 \left(\frac{\nu}{p_r} \right) \frac{\partial^2 \langle TT' \rangle}{\partial r_i \partial r_i} \quad \text{----- (7)} \end{aligned}$$

$$\begin{aligned} \frac{\partial \langle TT' \rangle}{\partial \Delta t} + \frac{\partial \langle u_i TT' \rangle}{\partial r_i} (-\hat{r}, -\Delta t, t + \Delta t) \\ = \left(\frac{\nu}{p_r} \right) \frac{\partial^2 \langle TT' \rangle}{\partial r_i \partial r_i} \quad \text{--- (8)} \end{aligned}$$

It is convenient to write this equation in spectral form by use of the following three-dimensional Fourier transforms as follows.¹⁶

$$\langle TT'(\hat{r}, \Delta t, t) \rangle = \int_{-\infty}^{\infty} \langle \tau \tau'(\hat{K}, \Delta t, t) \rangle \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad \text{----- (9)}$$

$$\langle u_i TT'(\hat{r}, \Delta t, t) \rangle = \int_{-\infty}^{\infty} \langle \phi_i \tau \tau'(\hat{K}, \Delta t, t) \rangle \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad \text{----- (10)}$$

It is more obvious by interchanging the point p and p' that

$$\begin{aligned} \langle u'_i TT'(\hat{r}, \Delta t, t) \rangle &= \langle u_i TT'(-\hat{r}, -\Delta t, t + \Delta t) \rangle \\ &= \int_{-\infty}^{\infty} \langle \phi_i \tau \tau'(-\hat{K}, -\Delta t, t + \Delta t) \rangle \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad \text{----- (11)} \end{aligned}$$

where \hat{K} is known as a wave number vector and magnitude of \hat{K} has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substitution of equations (9)-(11) into equations (7) and (8) leads to the spectral equations.

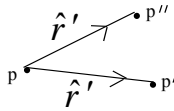
$$\begin{aligned} \frac{\partial \langle \tau \tau' \rangle}{\partial t} + 2 \left(\frac{\nu}{p_r} \right) k^2 \langle \tau \tau' \rangle = iK_i \left[\langle \phi_i \tau \tau'(\hat{K}, \Delta t, t) \rangle \right. \\ \left. - \langle \phi_i \tau \tau'(-\hat{K}, -\Delta t, t + \Delta t) \rangle \right] \quad \text{----- (12)} \end{aligned}$$

$$\begin{aligned} \frac{\partial \langle \tau \tau' \rangle}{\partial \Delta t} + 2 \left(\frac{\nu}{p_r} \right) k^2 = \\ -iK_i \langle \phi_i \tau \tau'(-\hat{K}, -\Delta t, t + \Delta t) \rangle \quad \text{----- (13)} \end{aligned}$$

In equations (12)–(13) the quantity $\tau\tau'(\hat{K})$ may be interpreted as a temperature fluctuation “energy” contribution of thermal eddies of size $1/k$. The time derivative of this “energy” as a function of the convective transfer to the wave numbers and the “dissipation” due to the action of the thermal conductivity. The term on the right hand side of equation (12) is also called transfer term while the 2nd term on the left hand side is the “dissipation” term.

Three-point, Three-time correlation and spectral equations

Considering three points p, p', p'' separated by the vectors \hat{r} and \hat{r}' , we write the Navier-Stokes equation for turbulent flow of incompressible fluid in a rotating system at the point p , energy equations at the points p' and p'' .



$$\frac{\partial u_j}{\partial t} + \frac{\partial}{\partial x_i}(u_j u_i) = -\frac{1}{\rho} \frac{\partial p}{\partial x_j} + \nu \frac{\partial^2 u_j}{\partial x_i \partial x_i} - 2 \epsilon_{mij} \Omega_m u_j \quad \text{--- (14)}$$

$$\frac{\partial T'}{\partial t} + u'_i \frac{\partial T'}{\partial x'_i} = \left(\frac{\nu}{P_r}\right) \frac{\partial^2 T'}{\partial x'_i \partial x'_i} \quad \text{----- (15)}$$

$$\text{and } \frac{\partial T''}{\partial t} + u''_i \frac{\partial T''}{\partial x''_i} = \left(\frac{\nu}{P_r}\right) \frac{\partial^2 T''}{\partial x''_i \partial x''_i} \quad \text{--- (16)}$$

Multiplying equations (14) – (16) by $T'T''$, $u_j T''$ and $u_j T'$ respectively and then taking ensemble average, we obtained

$$\frac{\partial \langle u_j T'T'' \rangle}{\partial t} + \frac{\partial \langle u_j T'T'' u_i \rangle}{\partial x_i} = -\frac{1}{\rho} \frac{\partial \langle P T'T'' \rangle}{\partial x_j} + \nu \frac{\partial^2 \langle u_j T'T'' \rangle}{\partial x_i \partial x_i} - 2 \epsilon_{mij} \Omega_m \langle u_j T'T'' \rangle \quad \text{---- (17)}$$

$$\frac{\partial \langle T'u_j T'' \rangle}{\partial t'} + \frac{\partial \langle u'_i T'u_j T'' \rangle}{\partial x'_i} = \left(\frac{\nu}{P_r}\right) \frac{\partial^2 \langle T'u_j T'' \rangle}{\partial x'_i \partial x'_i} \quad \text{----- (18)}$$

and

$$\frac{\partial \langle T''u_j T' \rangle}{\partial t''} + \frac{\partial \langle u''_i T''u_j T' \rangle}{\partial x''_i} = \left(\frac{\nu}{P_r}\right) \frac{\partial^2 \langle T''u_j T' \rangle}{\partial x''_i \partial x''_i} \quad \text{----- (19)}$$

Using the transformations

$$\begin{aligned} \frac{\partial}{\partial x_i} &= -\left(\frac{\partial}{\partial r_i} + \frac{\partial}{\partial r'_i}\right), \frac{\partial}{\partial x'_i} = \frac{\partial}{\partial r_i}, \frac{\partial}{\partial x''_i} = \frac{\partial}{\partial r'_i} \\ \left(\frac{\partial}{\partial t}\right)_{t',t''} &= \left(\frac{\partial}{\partial t}\right)_{\Delta t, \Delta t'} - \frac{\partial}{\partial \Delta t} - \frac{\partial}{\partial \Delta t'}, \frac{\partial}{\partial \Delta t'} \\ &= \frac{\partial}{\partial \Delta t}, \frac{\partial}{\partial \Delta t''} = \frac{\partial}{\partial \Delta t'} \end{aligned}$$

into equations (17)–(19), we have

$$\begin{aligned} \frac{\partial \langle u_j T'T'' \rangle}{\partial t} - \left(\frac{\partial}{\partial r_i} + \frac{\partial}{\partial r'_i}\right) \langle u_j T'T'' u_i \rangle + \frac{\partial \langle u'_i T'u_j T'' \rangle}{\partial r_i} + \frac{\partial \langle u''_i T''u_j T' \rangle}{\partial r'_i} \\ = \frac{1}{\rho} \left(\frac{\partial}{\partial r_j} + \frac{\partial}{\partial r'_j}\right) \langle P T'T'' \rangle + \nu \left(\frac{\partial}{\partial r_i} + \frac{\partial}{\partial r'_i}\right)^2 \langle u_j T'T'' \rangle + \left(\frac{\nu}{P_r}\right) \left(\frac{\partial^2}{\partial r_i \partial r_i} + \frac{\partial^2}{\partial r'_i \partial r'_i}\right) \langle u_j T'T'' \rangle - 2 \epsilon_{mij} \Omega_m \langle u_j T'T'' \rangle \quad \text{--- (20)} \end{aligned}$$

$$\frac{\partial \langle u_j T'T'' \rangle}{\partial \Delta t} + \frac{\partial \langle u'_i T'u_j T'' \rangle}{\partial r_i} = \left(\frac{\nu}{P_r}\right) \frac{\partial^2 \langle T'u_j T'' \rangle}{\partial r_i \partial r_i} \quad \text{----- (21)}$$

$$\frac{\partial \langle T' u_j T'' \rangle}{\partial \Delta t'} + \frac{\partial \langle u_i' T'' u_j T' \rangle}{\partial r_i'} = \left(\frac{\nu}{P_r} \right) \frac{\partial^2 \langle u_j T' T'' \rangle}{\partial r_i' \partial r_i'} \quad (22)$$

Using the six-dimensional Fourier transforms of the type

$$\langle u_j T' T'' \rangle(\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \beta_j \theta' \theta'' \rangle \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] d\hat{K} d\hat{K}' \quad (23)$$

$$\langle u_i u_j T' T'' \rangle(\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \beta_i \beta_j \theta' \theta'' \rangle \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] d\hat{K} d\hat{K}' \quad (24)$$

$$\langle p T' T'' \rangle(\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty} \langle \alpha \theta' \theta'' \rangle \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] d\hat{K} d\hat{K}' \quad (25)$$

Interchanging the points p' and p'' shows that

$$\langle u_j u_i' T' T'' \rangle = \langle u_j u_i T' T'' \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \beta_j \beta_i' \theta' \theta'' \rangle \exp[i(\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] d\hat{K} d\hat{K}' \quad (26)$$

By use of these facts we can write equations (20)-(22) in the form

$$\frac{\partial \langle \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial t} + \frac{\nu}{p_r} [(1 + p_r) k^2 + 2 p_r k_i k_i' + (1 + p_r) k'^2 + \frac{2 p_r}{\nu} \epsilon_{mij} \Omega_m] \langle \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = \frac{1}{\rho} i (k_j + k_j') \langle \alpha \theta' \theta'' \rangle + i (k_i + k_i') [\langle \beta_i \beta_j \theta' \theta'' \rangle - \langle \beta_i' \beta_j' \theta' \theta'' \rangle](\hat{K}, \hat{K}', \Delta t, \Delta t', t) \quad (27)$$

$$\frac{\partial \langle \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial \Delta t} + \left(\frac{\nu}{p_r} \right) k^2 \langle \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = -i k_i \langle \beta_i' \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) \quad (28)$$

$$\frac{\partial \langle \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial \Delta t'} + \left(\frac{\nu}{p_r} \right) k'^2 \langle \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = -i k_i' \langle \beta_i' \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}', \Delta t, \Delta t', t) \quad (29)$$

If the derivative with respect to x_j is taken of the momentum equation (14) for the point p , the equation multiplied by $T' T''$ and taken the ensemble average, the resulting equation is

$$\frac{\partial^2 \langle u_i u_j T' T'' \rangle}{\partial x_j \partial x_i} = -\frac{1}{\rho} \frac{\partial^2 \langle p T' T'' \rangle}{\partial x_j \partial x_i} \quad (30)$$

Writing this equation in terms of the independent variables \hat{r} and \hat{r}'

$$\left[\frac{\partial^2}{\partial r_i \partial r_i} + \frac{\partial^2}{\partial r_i \partial r_i'} + \frac{\partial^2}{\partial r_i' \partial r_j} + \frac{\partial^2}{\partial r_i' \partial r_j'} \right] \langle u_i u_j T' T'' \rangle = -\frac{1}{\rho} \left[\frac{\partial^2}{\partial r_j \partial r_j} + 2 \frac{\partial^2}{\partial r_j \partial r_j'} + \frac{\partial^2}{\partial r_j' \partial r_j'} \right] \langle p T' T'' \rangle \quad (31)$$

Taking the Fourier transforms of equation (31)

$$\langle \alpha \theta' \theta'' \rangle = -\rho \frac{(k_i k_j + k_i k_j' + k_i' k_j + k_i' k_j') \langle \beta_i \beta_j \theta' \theta'' \rangle}{k_j k_j + 2 k_j k_j' + k_j' k_j'} \quad (32)$$

Equation (32) can be used to eliminate $\langle \alpha \theta' \theta'' \rangle$ from equation (27).

Solution for times before the final period

To obtain the equation for times before the final period of decay, the three point correlations are considered and the

quadruple correlation terms decays faster than the lower-order correlation terms. If this assumption is made the equation (32) shows that the term $\langle \alpha \theta' \theta'' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (27)-(29).

$$\frac{\partial \langle \beta_j \theta \theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial t} + \frac{v}{p_r} \left[(1+p_r)k^2 + 2p_r k k' + \frac{2p_r}{v} \epsilon_{mij} \Omega_m \right] \langle \beta_j \theta \theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad \text{----- (33)}$$

$$\frac{\partial \langle \beta_j \theta' \theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial \Delta t} + \left(\frac{v}{p_r} \right) k^2 \langle \beta_j \theta' \theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad \text{---- (34)}$$

$$\frac{\partial \langle \beta_j \theta' \theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)}{\partial \Delta t'} + \left(\frac{v}{p_r} \right) k'^2 \langle \beta_j \theta' \theta'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \quad \text{--- (35)}$$

Inner multiplication of equations (33), (34) and (35) by k_j and integrating between t_0 and t we obtain

$$k_j \langle \beta_j \theta \theta'' \rangle = f_j \exp \left[-\frac{v}{p_r} \left[(1+p_r)(k^2 + k'^2) + 2p_r k k' \cos \theta + \frac{2p_r}{v} \epsilon_{mij} \Omega_m \right] (t-t_0) \right] \quad \text{----- (36)}$$

$$k_j \langle \beta_j \theta' \theta'' \rangle = g_j \exp \left(-\frac{v}{p_r} k^2 \Delta t \right) \quad \text{--- (37)}$$

and

$$k_j \langle \beta_j \theta' \theta'' \rangle = q_j \exp \left(-\frac{v}{p_r} k'^2 \Delta t' \right) \quad \text{----- (38)}$$

For these relations to be consistent, we have

$$k_j \langle \beta_j \theta' \theta'' \rangle = k_j \langle \beta_j \theta' \theta'' \rangle_0 \exp \left[-\frac{v}{p_r} \left\{ (1+p_r)(k^2 + k'^2)(t-t_0) + k^2 \Delta t + k'^2 \Delta t' + 2p_r k k' \cos \theta (t-t_0) + \frac{2p_r}{v} \epsilon_{mij} \Omega_m (t-t_0) \right\} \right] \text{---- (39)}$$

where θ is the angle between k and k' and $\langle \beta_j \theta' \theta'' \rangle_0$ is the value of $\langle \beta_j \theta' \theta'' \rangle$ at $t=t_0$, $\Delta t = \Delta t' = 0$. Letting $\hat{r}' = 0, \Delta t' = 0$ in the equation (23) and comparing the result with the equation (10) shows that

$$\langle k_j \phi_j \tau \tau' (\hat{K}, \Delta t, t) \rangle = \int_{-\infty}^{\infty} \langle k_j \beta_j \theta' \theta'' (\hat{K}, \hat{K}', \Delta t, 0, t) \rangle d\hat{K} \quad \text{---- (40)}$$

Substituting the equation (39) and (40) into the equation (12) we obtain

$$\frac{\partial \langle \tau \tau' \rangle (\hat{K}, \Delta t, t)}{\partial t} + 2 \frac{v}{p_r} k^2 \langle \tau \tau' \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} i k_j \left[\langle \beta_j \theta' \theta'' \rangle (\hat{K}, K', \Delta t, 0, t) - \langle \beta_j \theta' \theta'' \rangle (-\hat{K}, -\hat{K}', \Delta t, 0, t) \right] \exp \left[-\frac{v}{p_r} \left\{ (1+p_r)(k^2 + K'^2)(t-t_0) + k^2 \Delta t + k'^2 \Delta t' + 2p_r k k' (t-t_0) \cos \theta + \frac{2p_r}{v} \epsilon_{mij} \Omega_m (t-t_0) \right\} \right] d\hat{K}' \quad \text{(41)}$$

Now, $d\hat{K}'$ can be expressed in terms of k' and θ as $-2\pi k'^2 d(\cos \theta) d\hat{k}'$ (cf. Deissler²).

$$\frac{\partial \langle \tau \tau' \rangle (\hat{K}, \Delta t, t)}{\partial t} + 2 \frac{v}{p_r} k^2 \langle \tau \tau' \rangle (\hat{K}, \Delta t, t) = 2 \int_{-\infty}^{\infty} 2\pi i k_j \left[\langle \beta_j \theta' \theta'' \rangle (k, k') - \langle \beta_j \theta' \theta'' \rangle (-\hat{K}, \hat{K}') \right] d\hat{k}'$$

$$k'^2 \left[\int_{-1}^1 \exp \left\{ -\frac{\nu}{p_r} [(1+p_r)(k^2+k'^2)(t-t_0) + k^2 \Delta t + k'^2 \Delta t' + 2p_r k k'(t-t_0) \cos \theta + \frac{2p_r}{\nu} \epsilon_{mij} \Omega_m(t-t_0)] \right\} d(\cos \theta) \right] d\hat{k}' \quad (42)$$

The quantity $\left[\langle \beta_j \theta' \theta'' \rangle(\hat{K}, k') - \langle \beta_i \theta' \theta'' \rangle(-\hat{K}, -\hat{K}') \right]_0$ depends on the initial conditions of the turbulence.

In order to find the solution completely and following Loeffler and Deissler³ we assume that

$$ik_j \left[\langle \beta_j \theta' \theta'' \rangle(\hat{K}, \hat{K}') - \langle \beta_j \theta' \theta'' \rangle(-\hat{K}, -\hat{K}') \right]_0 = -\frac{\delta_0}{(2\pi)^2} (k^2 k'^4 - k^4 k'^2) \quad (43)$$

where δ_0 is a constant depending on the initial condition. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0 .

Substituting equation (43) into equation (42), we get

$$\begin{aligned} & \frac{\partial}{\partial t} \langle \tau \tau' \rangle(\hat{K}, \Delta t, t) 2\pi + 2 \frac{\nu}{p_r} 2\pi k^2 \langle \tau \tau' \rangle(\hat{K}, \Delta t, t) \\ & = -2\delta_0 \int_0^\infty (k^2 k'^4 - k^4 k'^2) k'^2 \\ & \times \left[\int_{-1}^1 \exp \left\{ -\frac{\nu}{p_r} [(1+p_r)(k^2+k'^2)(t-t_0) + k^2 \Delta t + k'^2 \Delta t' + 2p_r k k'(t-t_0) \cos \theta \right. \right. \end{aligned}$$

$$\left. \left. + \frac{2p_r}{\nu} \epsilon_{mij} \Omega_m(t-t_0) \right\} d(\cos \theta) \right] d\hat{k}' \quad (44)$$

Multiplying both side of equation (44) by k^2 , we get

$$\frac{\partial E}{\partial t} + 2 \frac{\nu}{p_r} k^2 E = w \quad (45)$$

where $E = 2\pi k^2 \langle \tau \tau' \rangle$, the energy spectrum function and w is given by

$$\begin{aligned} w & = -2\delta_0 \int_0^\infty (k^2 k'^4 - k^4 k'^2) k^2 k'^2 \\ & \left[\int_{-1}^1 \exp \left\{ -\frac{\nu}{p_r} [(1+p_r)(k^2+k'^2)(t-t_0) + k^2 \Delta t \right. \right. \\ & \left. \left. + 2p_r k k'(t-t_0) \cos \theta + \frac{2p_r}{\nu} \epsilon_{mij} \Omega_m(t-t_0) \right\} \right. \\ & \left. d(\cos \theta) \right] d\hat{k}' \quad (46) \end{aligned}$$

Integrating equation (44) with respect to θ , we have

$$\begin{aligned} w & = -\frac{\delta_0}{\nu(t-t_0)} \int_0^\infty (k^3 k'^5 - k^5 k'^3) \\ & \left[\exp \left\{ -\frac{\nu}{p_r} [(1+p_r)(k^2+k'^2)(t-t_0) + k^2 \Delta t \right. \right. \\ & \left. \left. - 2p_r k k'(t-t_0) + \frac{2p_r}{\nu} \epsilon_{mij} \Omega_m(t-t_0) \right\} \right] \\ & dk' + \frac{\delta_0}{\nu(t-t_0)} \int_0^\infty (k^3 k'^5 - k^5 k'^3) \\ & \left[\exp \left\{ -\frac{\nu}{p_r} [(1+p_r)(k^2+k'^2)(t-t_0) + k^2 \Delta t + \right. \right. \\ & \left. \left. 2p_r k k'(t-t_0) + \frac{2p_r}{\nu} \epsilon_{mij} \Omega_m(t-t_0) \right\} \right] dk' \quad (47) \end{aligned}$$

Again integrating equation (47) with respect to k' , we have

$$\begin{aligned}
 w = & -\frac{\delta_o \sqrt{\pi} p_r^{5/2}}{4\nu^{3/2}(t-t_o)^{3/2}(1+p_r)^{5/2}} \exp\left\{-\frac{2p_r}{\nu} \in_{mij} \Omega_m (t-t_o)\right\} \times \exp\left[\frac{-k^2 \nu(1+2p_r)}{p_r(1+p_r)}(t-t_o + \frac{1+p_r}{1+2p_r} \Delta t)\right] \\
 & \times \left[\frac{15p_r k^4}{4\nu^2(t-t_o)^2(1+p_r)} + \left\{ \frac{5p_r^2}{(1+p_r)^2} - \frac{3}{2} \right\} \frac{k^6}{\nu(t-t_o)} \right. \\
 & \left. + \left\{ \frac{p_r^3}{(1+p_r)^3} - \frac{p_r}{1+p_r} \right\} k^8 \right] \\
 & - \frac{\delta_o \sqrt{\pi} p_r^{5/2}}{4\nu^{3/2}(t-t_o + \Delta t)^{3/2}(1+p_r)^{5/2}} \exp\left\{-\frac{2p_r}{\nu} \in_{mij} \Omega_m (t-t_o + \Delta t)\right\} \times \exp\left[\frac{-k^2 \nu(1+2p_r)}{p_r(1+p_r)}(t-t_o + \frac{p_r}{1+2p_r} \Delta t)\right] \\
 & \times \left[\frac{15p_r k^4}{4\nu^2(t-t_o + \Delta t)^2(1+p_r)} + \left\{ \frac{5p_r^2}{(1+p_r)^2} - \frac{3}{2} \right\} \right. \\
 & \left. \frac{k^6}{\nu(t-t_o + \Delta t)} + \left\{ \frac{p_r^3}{(1+p_r)^3} - \frac{p_r}{1+p_r} \right\} k^8 \right] \dots (48)
 \end{aligned}$$

The series of equation (46) contains only even power of k and start with k^4 .

If we integrate equation (48) for $\Delta t=0$ over all wave numbers, we find that

$$\int_0^\infty w dk = 0 \quad \dots\dots\dots (49)$$

which is indicating that the expression for w satisfies the condition of continuity and homogeneity. Physically it was to be expected as w is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (45) can be solved to give

$$\begin{aligned}
 E = & \exp\left[-\frac{2\nu}{p_r} k^2(t-t_o + \frac{\Delta t}{2})\right] \int w \exp\left[\frac{\nu}{p_r} k^2(t-t_o + \frac{\Delta t}{2})\right] dt + \\
 & J(k) \exp\left[-\frac{2\nu}{p_r} k^2(t-t_o + \frac{\Delta t}{2})\right] \quad \dots\dots\dots (50)
 \end{aligned}$$

where $J(k) = \frac{N_o k^2}{\pi}$ is a constant of integration.

Substituting the values of w from (48) and $J(k)$ into the equation (50) and integrating with respect to t_0 we get

$$\begin{aligned}
 E = & \frac{N_o k^2}{\pi} \exp\left[-2 \frac{\nu}{p_r} k^2(t-t_o + \frac{\Delta t}{2})\right] \\
 & + \frac{\delta_o \sqrt{\pi} p_r^{5/2}}{4\nu^{3/2}(1+p_r)^{7/2}} \exp\left[-2 \in_{mij} \Omega_m (t-t_o)\right] \\
 & \times \exp\left[\frac{-k^2 \nu(1+2p_r)}{p_r(1+p_r)}(t-t_o + \frac{1+p_r}{1+2p_r} \Delta t)\right] \times \\
 & \left[\frac{3p_r k^4}{2\nu^2(t-t_o)^{5/2}} + \frac{p_r(7p_r - 6)k^6}{3\nu(1+p_r)(t-t_o)^{3/2}} \right. \\
 & - \frac{4(3p_r^2 - 2p_r + 3)k^8}{3(1+p_r)^2(t-t_o)^{1/2}} + \frac{8\sqrt{\nu}(3p_r^2 - 2p_r + 3)k^9}{3(1+p_r)^{5/2} p_r^{1/2}} F(\eta) \\
 & \left. + \frac{\delta_o \sqrt{\pi} p_r^{5/2}}{4\nu^{3/2}(1+p_r)^{7/2}} \times \exp\left[2 \in_{mij} \Omega_m (t-t_o)\right] \right. \\
 & \times \exp\left[\frac{-k^2 \nu(1+2p_r)}{p_r(1+p_r)}(t-t_o + \frac{p_r}{1+2p_r} \Delta t)\right] \\
 & \times \left[\frac{3p_r k^4}{2\nu^2(t-t_o + \Delta t)^{5/2}} \right] + \frac{p_r(7p_r - 6)k^6}{3\nu(1+p_r)(t-t_o + \Delta t)^{3/2}} \\
 & - \frac{4(3p_r^2 - 2p_r + 3)k^8}{3(1+p_r)^2(t-t_o + \Delta t)^{1/2}} \\
 & \left. + \frac{8\sqrt{\nu}(3p_r^2 - 2p_r + 3)k^9}{3(1+p_r)^{5/2} p_r^{1/2}} F(\eta) \right] \quad \dots\dots\dots (51)
 \end{aligned}$$

where $F(\eta) = \bar{e}^{\eta^2} \int_0^\eta e^{x^2} dx$

$$\eta = k \sqrt{\frac{v(t-t_o)}{p_r(1+p_r)}}$$

or $\eta = k \sqrt{\frac{v(t-t_o + \Delta t)}{p_r(1+p_r)}}$

By setting $\hat{r} = 0$ in equation (9) and use is made of the definition of E, the result is

$$\frac{\langle TT' \rangle}{2} = \frac{\langle T^2 \rangle}{2} = \int_0^\infty E dk \quad \text{----- (52)}$$

Substituting equation (51) into equation (52) and integrating with respect to k, gives

$$\begin{aligned} \frac{\langle T^2 \rangle}{2} &= \frac{N_o p_r^{3/2} (T + \frac{\Delta T}{2})^{-3/2}}{8\sqrt{2\pi} v^{3/2}} + \\ &\frac{\pi \delta_o p_r^6}{4v^6(1+p_r)(1+2p_r)^{5/2}} \exp[-2 \in_{mij} \Omega_m] \\ & \left[\frac{9}{16T^{5/2} (T + \frac{1+p_r}{1+2p_r} \Delta T)^{5/2}} + \right. \\ & \frac{9}{16(T + \Delta T)^{5/2} (T + \frac{p_r}{1+2p_r} \Delta T)^{5/2}} + \\ & \frac{5p_r(7p_r-6)}{16(1+2p_r)T^{3/2} (T + \frac{1+p_r}{1+2p_r} \Delta T)^{7/2}} \\ & \left. + \frac{5p_r(7p_r-6)}{16(1+2p_r)(T + \Delta T)^{3/2} (T + \frac{p_r}{1+2p_r} \Delta T)^{7/2}} \right] \end{aligned}$$

$$\begin{aligned} &+ \frac{35p_r(3p_r^2 - 2p_r + 3)}{8(1+2p_r)T^{1/2} (T + \frac{1+p_r}{1+2p_r} \Delta T)^{9/2}} \\ &+ \frac{35p_r(3p_r^2 - 2p_r + 3)}{8(1+2p_r)(T + \Delta T)^{1/2} (T + \frac{p_r}{1+2p_r} \Delta T)^{9/2}} \\ &+ \frac{8p_r(3p_r^2 - 2p_r + 3)(1+2p_r)^{5/2}}{3.2^{23/2}(1+p_r)^{11/2}} \sum_{n=0}^\infty \frac{1.3.5.....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n} \\ &\times \left\{ \frac{T^{(2n+1)/2}}{(T + \Delta T/2)^{(2n+1)/2}} + \frac{(T + \Delta T)^{(2n+1)/2}}{(T + \Delta T/2)^{(2n+1)/2}} \right\} \quad \text{----- (53)} \end{aligned}$$

where $T = t - t_0$.

Equation (53) is the decay law of temperature energy fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system.

Results and Discussion

In equation (53) we obtained the decay law of temperature fluctuation in homogeneous turbulence before the final period in a rotating system neglecting quadruple correlation terms in comparison with the third –order terms for the case of multi-point and multi-time. If the system is non rotating then $\Omega_m = 0$ the equation (53) becomes

$$\begin{aligned} \frac{\langle T^2 \rangle}{2} &= \frac{N_o p_r^{3/2} (T + \frac{\Delta T}{2})^{-3/2}}{8v^{3/2}\sqrt{2\pi}} + \frac{\pi \delta_o p_r^6}{4v^6(1+p_r)(1+2p_r)^{5/2}} \\ &\times \left[\frac{9}{16T^{5/2} (T + \frac{1+p_r}{1+2p_r} \Delta T)^{5/2}} \right. \end{aligned}$$

$$\begin{aligned}
 & + \frac{9}{16(T + \Delta T)^{5/2} \left(T + \frac{p_r}{1 + 2p_r} \Delta T\right)^{5/2}} + \\
 & \frac{5p_r(7p_r - 6)}{16(1 + 2p_r)T^{3/2} \left(T + \frac{1 + p_r}{1 + 2p_r} \Delta T\right)^{7/2}} \\
 & + \frac{5p_r(7p_r - 6)}{16(1 + 2p_r)(T + \Delta T)^{3/2} \left(T + \frac{p_r}{1 + 2p_r} \Delta T\right)^{7/2}} \\
 & + \frac{35p_r(3p_r^2 - 2p_r + 3)}{8(1 + 2p_r)T^{1/2} \left(T + \frac{1 + p_r}{1 + 2p_r} \Delta T\right)^{9/2}} \\
 & + \frac{35p_r(3p_r^2 - 2p_r + 3)}{8(1 + 2p_r)(T + \Delta T)^{1/2} \left(T + \frac{p_r}{1 + 2p_r} \Delta T\right)^{9/2}} \\
 & + \frac{8p_r(3p_r^2 - 2p_r + 3)(1 + 2p_r)^{5/2}}{3 \cdot 2^{23/2} (1 + p_r)^{1/2}} \sum_{n=0}^{\infty} \frac{1.3.5 \dots (2n+9)}{n!(2n+1)2^{2n}(1 + p_r)^n} \\
 & \times \left\{ \frac{T^{(2n+1)/2}}{(T + \Delta T / 2)^{(2n+1)/2}} + \frac{(T + \Delta T)^{(2n+1)/2}}{(T + \Delta T / 2)^{(2n+1)/2}} \right\} \\
 & \text{----- (54)}
 \end{aligned}$$

which is obtained earlier by Sarker and Islam.¹⁰

If we put $\Delta T = 0$, we can easily find out

$$\begin{aligned}
 \frac{\langle T^2 \rangle}{2} & = \frac{N_o p_r^{3/2} T^{-3/2}}{8\sqrt{2\pi}v^{3/2}} + \frac{\pi\delta_o p_r^6 T^{-5}}{4v^6(1 + p_r)(1 + 2p_r)^{5/2}} \times \\
 & \left[\frac{9}{16} + \frac{5}{16} \cdot \frac{p_r(7p_r - 6)}{1 + 2p_r} + \frac{35}{8} \cdot \frac{p_r(3p_r^2 - 2p_r + 3)}{(1 + 2p_r)^2} + \dots \right] \\
 & = AT^{-3/2} + BT^{-5} = A(t - t_0)^{-3/2} + B(t - t_0)^{-5} \text{ - (55)}
 \end{aligned}$$

where $A = \frac{N_o p_r^{3/2}}{8\sqrt{2\pi}v^{3/2}}$ and

$$\begin{aligned}
 B & = \frac{\pi\delta_o p_r^6}{2v^6(1 + p_r)(1 + 2p_r)^{5/2}} \times \left[\frac{9}{16} + \frac{5}{16} \cdot \frac{p_r(7p_r - 6)}{1 + 2p_r} \right. \\
 & \left. + \frac{35}{8} \cdot \frac{p_r(3p_r^2 - 2p_r + 3)}{(1 + 2p_r)^2} + \dots \right]
 \end{aligned}$$

which is obtained earlier by Loeffler and Deissler.³

This study shows that the effect of rotation in homogeneous turbulence the temperature energy fluctuations decays more rapidly than the energy for non-rotating fluid for times before the final period.

If higher order correlation equation are considered in the analysis it appears that more terms of higher power of time would be added to the equation (53). For large times, the second term in the equation becomes negligible leaving the -3/2 power decay law for the final period.

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