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Comparison of Several Transmission Loss Allocation Procedures

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Abstract

Transmission-loss is an inherent nature of power system. Determination of transmission losses for the purposes of billing in various interconnected trans-actions is an important issue to be solved exactly. Loss allocation is a procedure for subdividing the system transmission losses into fractions, the cost of which becomes the responsibility of network users. This paper focuses on transmission loss allocation procedures and provides a detailed comparison of some alternative algorithms: viz. 1) Incremental loss allocation, 2) A proportional allocation, 3) Preliminary loss allocation and 4) A direct methodology for loss allocation. A case study based on a 6-bus model power system has been provided. Finally conclusions and recommendations have been stated.

Key words: Transmission loss allocation, DC optimal power flow, Deregulated power market, Loss coefficient matrix, Non-volatile procedure.

Introduction

Transmission losses form a significant component of the amount of power that has to be generated in order to meet the power demand. As an example, in a power network with a demand of 10,000 MW and 7% transmission losses, the implication is that the generation must be capable of supplying 10700 MW, an extra 700 MW, fully a large power plant that must be built and operated. Clearly, someone must pay for both the capital investment and the fuel needed to generate the 700 MW of lost power. In the traditional utility, this cost is bundled into the rates together with other ancillary services and charged in some prorata fashion. With competition, this practice still persists but, more and more, there will be a need to allocate losses to transactions in a more systematic manner, particularly one that will account for the network location of the buyer and seller as well as the non-linear interaction among simultaneous transactions (Ilic, *et. al.* 1998). For example, transactions where the seller and buyer are electrically close may not generate much in the way of losses. Similarly, some transactions may actually reduce overall system losses while others can have an opposite effect (Ilic, *et. al.* 1998). Methods that can systematically identify such differences in behavior are therefore required.

This research work concerns about the investigations regarding the transmission loss allocation of power systems in the deregulated power industry. The growth in the volume of power produced and consumed in every country; increasing efficiency of power plants and distribution systems; and demand for distributed power generation, especially the demand for guaranteed power supplies, have changed the power industry. Recently the monopolistic electric utility industry has entered an era of freewheeling competition and deregulation, allowing consumers to buy electricity from any company offering it. The increasing prominence of ideas such as conservation, energy efficiency, and free markets helped propel the power system toward open competition. For the sake of competition, the ultimate goal is the reduction of consumer prices. There are certain operating cost associated with power losses cannot be unbundled because of quadratic functions i.e. non-linear nature of power flows. The loss allocation approaches developed so far used to allocate losses to generators and/or consumers based on average loss factors. The loss allocation factor developed so far can be categorized into incremental, circuitbased, proportional sharing and miscellaneous approaches for loss allocation including bilateral transactions. Loss allocation does not affect generation levels or power flows in transmission lines, however, it does modify the distribution of revenues and payments at the network buses among suppliers and consumers. The total loss assigned to a transaction may differ significantly depending on allocation methodology adopted. Therefore, an acceptable procedure is the crying need to con-

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quer the loss allocation problem. There is no unique or ideal procedure existed for loss allocation, but they should have most of the desirable properties stated below.

a. To be consistent with the result of a power flow;
b. To depend on the amount of energy either produced or consumed;
c. To depend on the relative location in the transmis sion network;
d. To avoid volatility;
e. To provide appropriate economic marginal signals;
f. To be easy to understand;
g. To be simple to implement.

The issue of how to consider the power loss to the network users has not been established properly. So, to solve the loss allocation problem, this work demonstrates loss allocation methodologies based on DC Optimal Power Flow (DC-OPF).

II. Basics of the procedures

In this study, four procedures have been studied and discussed in detail.

1. A successive approximation methodology based on Incremental Transmission Loss (ITL) of node is called Incremental Loss Allocation (ILA) procedure (Chiba, *et. al.* 2003). In this procedure, starting from the minimum load level, system total load is incremented by a small amount; then total loss, power output and power output change of every generator are estimated by DC-OPF. Incremented loss is calculated and adjusted to the total loss computed by DC-OPF. This adjusted loss is the allocated loss to a generator. After every iteration (except the first iteration), incremented loss is added to the previous allocated loss and then adjusted to the total loss as before. Economic Load Dispatch (ELD) mode of operation of the generators has been applied in this process.

2. A directly ITL based loss allocation procedure called Proportional Allocation (PA) procedure (Kabir, 2007) has also been studied. In this procedure, the positive power injection in a generator bus is multiplied by the ITL of that bus and then adjusted to the total loss calculated by DC-OPF.

3. A Preliminary Loss Allocation (PLA) (Kabir, *et. al.* 2005) formula has been studied considering that the generators are running under ELD approach. Using this formula the preliminary losses to the generators are calculated at first and then correction factors are calculated from them. According to the correction factors the preliminary losses are adjusted to the total loss computed by using DC-OPF.

4. A New DC-OPF method (NDC) (Kabir, 2007; Matsumoto, *et. al.* 2005), has been studied which is much faster and accurate than the DC methods developed so far. The main feature of the procedure using NDC method is that the real impact of every line flow on the transmission system has been considered extensively and properly. This procedure allocates losses to loads at first and then to generators. The numerical analysis using this procedure proves that the system losses are shown to be separable among the buses naturally. No special approximations are required in the derivation of loss coefficient matrices that are useful tools in the loss allocation equations (one for loss allocation to loads and the other for loss allocation to generators). It is extremely simple to formulate and to implement. This procedure also yields allocation levels generally consistent with the power flow computations. Comparing several features of the procedures studied in this work, it is worth noting that the Direct Methodology for Loss Allocation (DMLA) procedure by using the NDC method has got the highest priority regarding performance and quality in every sphere of calculation.

Thus, it can be concluded that the proposed loss allocation strategy based on the NDC method is robust and provides a simple and effective way to solve the loss allocation problem, that is, the main barrier in the deregulated power market has apparently been subjugated.

III. Mathematical overview

The mathematical overview of the transmission loss allocation procedures considered in this comparative study has been described in the four subsections below.

1) Incremental loss allocation (Chiba, *et. al.* 2003)

Starting from the minimum load level, system total load is incremented by a small amount; then total loss, power output and power output change ΔP_k are estimated by DC-OPF. At every iteration, incremented loss ΔL_k to generator k is calculated as

$$\Delta L_k = \Delta P_k \times \frac{\partial L}{\partial P_k} \qquad (1)$$

Where, $[\partial L/\partial P_k]$ is the incremental transmission loss (ITL) of the generator bus. Therefore, the new loss is obtained as follows:

$$L_k^{New} = L_k^{Old} + \Delta L_k \qquad (2)$$

Where, L_k^{Old} is the allocated loss estimated at the previous iteration. (1) and (2) are used at each incremented load level. This process is repeated until the desired load level is reached. So, finally, adjusted loss becomes the allocated loss to generator *k* at the desired load level.

2) A Proportional allocation (Kabir, 2007):

We know, if bus voltages are assumed to be constant i.e. 1 p.u. in every node, it can be shown that the partial derivative of total loss with respect to phase angle θ of node voltage is

$$\frac{\partial L}{\partial \theta_i} = -2\sum_{j=1}^{n} G_{ij} Sin(\theta_i - \theta_j) \qquad (3)$$

Where G_{ij} is the real part of transfer admittance matrix. θ_i and θ_j are phase angles of voltages at bus *i* and bus *j* respectively in a system of *n* buses, $i = 1,2,..,n$ and $[\partial L/\partial \theta_i]$ is an *n* dimensional row vector [11]. We know there is a close relationship between node power injection, node voltage angle and susceptance matrix *B* of the network. Therefore, ITL (Willis, *et. al.*) has been calculated as the following (4). Where $\hat{B}$ is the submatrix of an (nxn) susceptance matrix *B*.

$$\frac{\partial L}{\partial P_i} = \frac{\partial L}{\partial \theta_i} \times \hat{B}^{-1} \qquad (4)$$

Using generated power P_k, in bus *i* and by (4) we can calculate the Preliminary Loss (L_{pk}) allocated to generator *k* as

$$L_{pk} = P_k \times \frac{\partial L}{\partial P_i} \qquad (5)$$

Where $[\partial L/\partial P_i]$ is the sensitivity of the system losses with respect to injection at bus *i*. These are the wellknown ITLs. The sum of preliminary losses, (ΣL_{pk}), is not equal to the total loss *L* calculated by DC-OPF. Assuming that the loss allocation can be done according to the proportion of preliminary losses. So, the loss allocation rate R_k (for generator *k*.) has been calculated as

$$R_k = \frac{L_{pk}}{\sum L_{pk}} \qquad (6)$$

Therefore, at a specific load level, the final loss allocation to generator *k* becomes

$$L_k = R_k \ x\ L \qquad (7)$$

Now, the total of allocated losses is equal to the total loss calculated by DC-OPF. i.e. $\Sigma L_k = L$.

3) Preliminary loss allocation (Kabir, *et al.* 2005)

In this procedure, the power outputs of the generators are realized under the ELD mode of operation. At the same time; total loss *L* and angle of every node voltage have been calculated by DC-OPF. Hence, in every load level, ITLs are determined considering the influence of power outputs that have met the total load and loss. Therefore, at a specific load level, ITL (for generator k) can be expressed by (8). Where c_k can be considered constant for a small change in P_k.

$$\frac{dL_{pk}}{dP_k} = a_k P_k^{\ 2} + b_k P_k + c_k \qquad (8)$$

Here, we call L_{pk} as the preliminary loss to generator *k* in a system of *n* buses. The coefficients in (8) calculated by using polynomial curve fitting have individual set of numerical values for each generator. The solution of (8) is as follows:

$$L_{pk} = \tfrac{1}{3} a_k P_k^3 + \tfrac{1}{2} b_k P_k^2 + c_k P_k + d_k \qquad (9)$$

The constant for integration 'd_k' is calculated by putting allocated loss and power output in (9) at minimum load level only. For this loss allocation at minimum load level, the preliminary loss (L_{pk}) allocated to generator *k* is calculated by (5) (where P_k is the lower limit of the generator) and adjusted by (10). In other load levels (except the minimum level) the preliminary losses are calculated by using (9). Assuming that the total loss mismatch, ($L- \Sigma L_{pk}$), can be adjusted according to the proportion of preliminary losses in (9). The correction factor (C_k) has been calculated as (6) using the preliminary losses calculated by (9). Therefore, at a specific load level, the final loss allocation to generator *k* becomes as follows:

$$L_k = L_{pk} + C_k \left(L - \sum L_{pk} \right) \qquad (10)$$

4) Direct methodology for loss allocation (Kabir, 2007; Matsumoto, *et. al.* 2005):

From the known power injections at the generator (PV) buses a matrix *f* has been developed that establishes a prompt relationship between the power injections at PV buses to the injections at the load demand (PQ) buses as shown below.

$$(P) = \begin{pmatrix} f_{11} & \cdots & f_{1D} \\ \vdots & f_{ij} & \vdots \\ f_{G1} & \cdots & f_{GD} \end{pmatrix} (d) \qquad (11)$$

Where *P* consists of net power injections in the PV buses, *d* consists of gross power injections (includ-ing losses) at the PQ buses. At first, taking penalty factor=1 for all the generating units, the power out-puts of the generators are determined by ELD mode of operation. In other cases the net power injections in the PV buses are calculated by (11). *As a result, this procedure neither depends on Incremental Transmission Loss (ITL) of buses nor on arbitrary slack bus selection. Hence, it is a non-volatile procedure.* The elements f_{11}, ... , f_{G1} *are calculated as follows*

$$f_{i1} = \frac{\text{Net power injection at PV bus i}}{\text{Sum of net power injections at PV buses}} \qquad (12)$$

Expressing the *f* as (13).

$$(F) = \begin{pmatrix} f_{11} & \cdots & f_{1D} \\ \vdots & f_{ij} & \vdots \\ f_{G1} & \cdots & f_{GD} \\ -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{pmatrix} \qquad (13)$$

The power injection matrix ***I*** at all buses can be expressed as follows:

$$(I) = (F)(d) = (B)(\delta) \qquad (14)$$

Where ***F*** is an (*nxD*) matrix, ***B*** is an (*nxn*) suscep-tance matrix, δ is an (*nx1*) column vector of node voltage phase angles. Here, *n*=total bus number, *D*= number of PQ buses. Therefore, from (14) the matrix δ can be expressed as

$$(\delta) = (B^{-1})(F)(d) \qquad (15)$$

In DC load flow method (Exposito, *et.al.* 2000; Wood, 1996; Yamashiro, 1977; Conejo *et.al.* 2003; Elgerd, 1970; MATLAB, 2005; Schweppe, 1998; Bialek, *et.al.* 1996), a power flow matrix P_F for the whole system can be written as follows.

$$\begin{aligned} (P_F) &= (b)(A)(\delta) \\ &= (b)(A)(B^{-1})(F)(d) \\ &= (K)(d) \end{aligned} \qquad (16)$$

Where ***b*** is an (*mx m*) diagonal matrix that consists of transmission line susceptances as diagonal elements. ***A*** is an (*mx n*) matrix that contains -1 for starting nodes and -1 for ending nodes and rest of the elements is zero, ***K*** is an (*mxD*) matrix, *m*= number of transmission lines. Therefore, power flow in line *l* can be written as

$$P_{Fl} = (K_l)(d) \qquad (17)$$

Where, $\boldsymbol{K_l}$ is the l^{th} row of the matrix ***K***. In DC method, power flow in a line is assumed to be current. Therefore, power loss in the line *l* is computed as

$$\begin{aligned} L_l &= (P_{Fl})^2 r_l = (K_l d)^2 r_l \\ &= (d^T)(K_l^T)(K_l)(d) r_l \\ &= (d^T)(S^l)(d) = \sum_{i=1}^{D} \sum_{q=1}^{D} d_i S_{iq}^l d_q \\ &= \sum_{i=1}^{D} d_i^2 S_{ii}^l + \sum_{i=1}^{D} \sum_{\substack{q=1 \\ q \neq i}}^{D} d_i S_{iq}^l d_q \end{aligned} \qquad (18)$$

Where r_l is the resistance of the transmission line *l*. The coefficient $(S^l) = (K_l^T)(K_l) r_l$ is a square matrix developed to allocate the loss of line *l* to the PQ buses. Because of this loss coefficient matrix, the procedure accounts for the geographical location of the PQ buses in the network.

Assuming that the loss caused by d_i and d_q is equally allo cated to each node of *i* and *q* (Exposito, *et.al.* 2000). From

(18) the allocated loss L_{Dli} to PQ bus i for transmission line l can be written as follows:

$$L_{Dli} = d_i^2 S_{ii}^l + \sum_{\substack{q=1\\ q\neq i}}^{D} d_i S_{iq}^l d_q \qquad (19)$$

Similarly, loss allocations considering the impact of all other line flows in the network can easily be computed. Therefore, total loss allocated to PQ bus i is as

$$L_{Di} = \sum_{l=1}^{m} L_{Dli} \qquad (20)$$

The sum of the allocated losses to PQ buses is exactly equals to the sum of the losses in transmission lines as calculated by using (18). Adding the allocated losses to the respective loads, the elements in ***d*** are refreshed and used in (11). In the proposed method, the (11) through (20) are repeated until the network converges. It is worth noting that the proposed method starts from the no loss condition.

Finally, to allocate the transmission losses to the PV buses, the following mathematical formulation has been presented. Now, (21) shows the relationship between the power injections at PQ buses to the injections at the PV buses.

$$(d) = \begin{pmatrix} c_{11} & \cdots & c_{1G} \\ \vdots & c_{ij} & \vdots \\ c_{D1} & \cdots & c_{DG} \end{pmatrix} (P) \qquad (21)$$

The elements $c_{11}, \ldots, c_{D1}$ are calculated from the elements in ***d*** that is exactly as follows:

$$c_{i1} = -\left(\frac{\text{Load} + \text{allocated loss to load bus i}}{\text{Total load} + \text{Total loss}} \right) \qquad (22)$$

Expressing the c as (13).

$$(C) = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \\ c_{11} & \cdots & c_{1G} \\ \vdots & c_{ij} & \vdots \\ c_{D1} & \cdots & c_{DG} \end{pmatrix} \qquad (23)$$

The power injection matrix ***I*** at all buses can be expressed as follows:

$$(I) = (C)(P) = (B)(\delta) \qquad (24)$$

Therefore, the power flow matrix $\boldsymbol{P_F}$ in (16) for the whole system can be expressed as follows:

$$\begin{aligned} (P_F) &= (b)(A)(B^{-1})(C)(P) \\ &= (H)(P) \end{aligned} \qquad (25)$$

Where, ***H*** is an (mxG) matrix, G= number of PV buses. Therefore, power flow in line l can be written as

$$P_{Fl} = (H_l)(P) \qquad (26)$$

Where, $\boldsymbol{H_l}$ is the l^{th} row of the matrix ***H.*** Therefore, power loss in the line ***l*** is computed as

$$\begin{aligned} L_l &= (P_{Fl})^2 r_l = (H_l P)^2 r_l \\ &= (P^T)(H_l^T)(H_l)(P) r_l \\ &= (P^T)(R^l)(P) = \sum_{i=1}^{G} \sum_{q=1}^{G} P_i R_{iq}^l P_q \\ &= \sum_{i=1}^{G} P_i^2 R_{ii}^l + \sum_{i=1}^{G} \sum_{\substack{q=1\\ q\neq i}}^{G} P_i R_{iq}^l P_q \end{aligned} \qquad (27)$$

The coefficient $(R^l) = (H_l^T)(H_l) r_l$ is a square matrix developed to allocate the loss of line l to the PV buses. Because of this loss coefficient matrix, the procedure accounts for the geographical location of the PV buses in the network

Assuming that the loss caused by P_i and P_q is equally allocated to each node of i and q. From (27) the allocated loss L_{Gli} to PV bus i for transmission line l can be written as follows:

$$L_{Gli} = P_i^2 R_{ii}^l + \sum_{\substack{q=1\\ q\neq i}}^{G} P_i R_{iq}^l P_q \qquad (28)$$

Similarly, loss allocations considering the impact of all other line flows in the network can easily be computed. Therefore, total loss allocated to PV bus i is as

$$L_{Gi} = \sum_{l=1}^{m} L_{Gli} \qquad (29)$$

The sum of the allocated losses to PV buses is exactly equals to the sum of the losses in transmission lines as calculated by using (27). It may be mentioned that the total loss calculated by using (27) is the same as that of calculated by using (18). Also note that sum of the allocated losses to the PQ buses is exactly equals to the sum of the allocated losses to the PV buses.

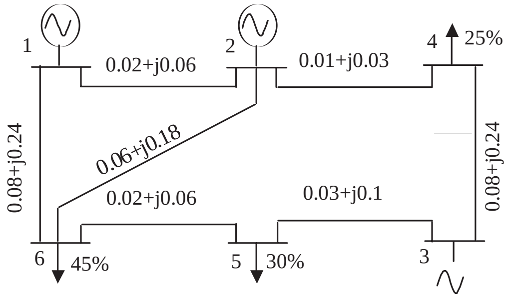

Fig. 1. Model power system

Results and Discussion

The 6-bus model power system Fig.1 has been used to compare the four transmission loss allocation procedures considered in this paper. Power flow and total loss data have been computed by DC-OPF for the four loss allocation algorithms. Data calculated at 400 MW load level have been analyzed for easy comprehension. The power flow calculations in every procedure have been done by using simulation programs in MATLAB. Execution time (in second) of each algorithm has been calculated and recorded in Table IV.

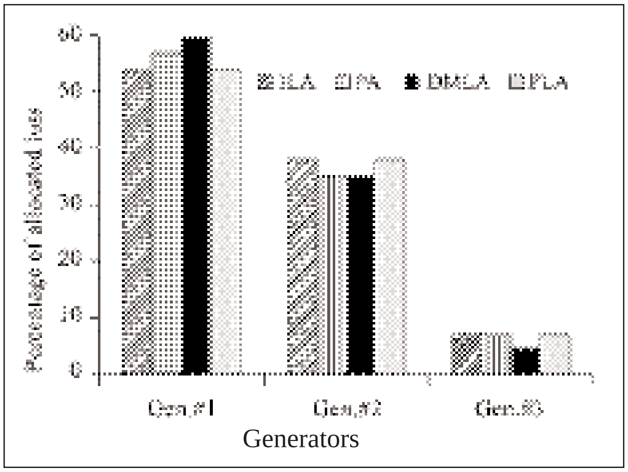

Fig. 2. Percentage of allocated losses to generators.

The total loss calculated by (18) and/or (27) is little bit higher (but closer to the loss calculated by Newton-Raphson method [11]) than those of other procedures, because (18) and/or (27) calculates the losses more accurately. In other procedures, the power losses in the transmission lines are added to the corresponding loads and hence, the node voltage angles are computed. This process is slightly inaccurate, because loss allocations need to be adjusted to the total loss calculated by DC-OPF. The DMLA procedure also takes care for negative loss allocation Table III. Negative allocation provides monetary incentives to those generators "well" positioned in the network. Alternatively, generators or loads "poorly" positioned receive proportionately higher loss allocations (Conejo, *et. al.* 2001). So, the allocated losses in ILA, PA and PLA procedures are different from those of DMLA procedure.

Table I. Losses to generators at 400 MW load level

Units	ILA	PA	DMLA	PLA
1	12.767	13.544	14.2261	12.849
2	9.1605	8.397	8.4531	9.0932
2	1.7821	1.7678	1.2197	1.7671
Total loss	23.709	23.7093	23.8989	23.7093

Table II. Percentage of allocated losses to generators.

Units	ILA	PA	DMLA	PLA
1	53.8481	57.1253	59.5262	54.1939
2	38.6367	35.4165	35.3702	38.3529
2	7.5165	7.4561	5.1036	7.4532
Total	100	100	100	100

The extra-ordinary feature of DMLA procedure is that the allocated losses to generators for every transaction can be computed properly and effectively because of transaction based calculations for loss. This prominent feature is proved by the example set of data as shown in Table III at load level 400 MW.

Table III. Allocation done by DMLA method

Line #	From bus	To bus	Unit 1	Unit 2	Unit 3
1	1	2	2.2285	-0.5985	-0.0424
2	1	6	5.9375	2.8481	0.2019
3	2	4	0.9635	0.8434	-0.0833
4	2	6	3.461	3.7493	0.2657
5	4	3	0.6357	0.6532	-0.5792
6	3	5	0.8387	0.8617	1.5936
7	6	5	0.1613	0.0958	-0.1366
Total loss			14.2261	8.4531	1.2197

Table IV. Execution time and percentage of them.

Units	ILA	PA	DMLA	PLA
Execution time (s)	1.30	0.11	0.02	2.203
Percentage of time	59%	5%	0.9%	100%

The percentages have been calculated on the basis of the execution time of the slowest one (the PLA procedure). It is proved that the DMLA procedure is the fastest one (around 1000 times faster than PLA, 50 times faster than PA and 590 times faster than ILA procedures), because it does not need any external and complex function to manipulate the bus voltage angles for power flow calculations. Analyzing several simulation results, it is worth noting that:

1. ILA and PLA are directly based on the computation of ITLs. These have been used to allocate losses to generators.
2. ILA suffers from the lack of integrity because ITL variations cannot influence the loss allocation if power output of a generator remains unchanged, that is, a generator output may remain fixed to its upper or lower limits.
3. PA procedure is simple to understand and implement. It includes the impact of network parameters by the computation of ITLs. As a result, two identical demands located respectively near generating buses and far away from these buses are not equally treated, which is important for fair loss allocation.
4. PLA is initially time consuming procedure. Calculation of coefficients sensitively depends on the number of over-determined equations that should be as many as possible for more accuracy
5. The DMLA procedure calculates allocation results significantly different from those produced by other algorithms. In this procedure losses are first allocated to demands and then to generators.

Conclusion

From the analysis, the following conclusions can be drawn:

1. ILA procedure suffers from the lack of integrity.
2. After calculating the coefficients from the overdetermined equations, the allocation trend of the PLA is almost similar to the PA procedure. But PLA procedure is better than the PA procedure, because the preliminary loss formula in PLA procedure can calculate the loss allocation at the vicinity of the loss should be finally allocated to a generator.
3. The DMLA procedure considers the network efficiently. The impact of every transaction is considered in computation of loss coefficient for loss allocation to generators or demands. So this procedure is reasonably acceptable than the other procedures considered.

Final recommendations are as follows:

The ILA procedure is not advisable because it is unfair to the generators which output remains fixed to its lower or upper limits. If the slack bus is unique and volatility is acceptable the PA and PLA procedures are recommendable. Above all the DMLA procedure is the most recommended one among the procedures considered in this paper. Because the DMLA procedure meets all the points cited in the introductory section.

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