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Decay of Temperature Fluctuations in MHD Turbulence before the Final Period in a Rotating System

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Abstract

Using Deissler's approach the Decay of temperature fluctuations in MHD turbulence before the final period in a rotating system is studied and have considered correlations between fluctuating quantities at two and three point. In this case two and three-point correlation equations in a rotating system is obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison to the second and third order correlations. The correlation equations are converted to special form by taking their Fourier-transforms. Finally integrating the energy spectrum over all wave numbers the energy decay law of temperature fluctuations in MHD turbulence before the final period in a rotating system is obtained.

Key words : Deissler's method, MHD turbulence, Rotating system, Temperature fluctuation.

Introduction

Deissler (1958 and 1960) developed a theory for homogeneous turbulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler (1961) studied the decay of temperature fluctuations in homogeneous turbulence before the final period. Following Deissler's approach Sarker and Islam (2001) also studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time. Sarker and L. Rahman (1998) studied the decay of temperature fluctuations in MHD turbulence before the final period. Islam and Sarker (2001) studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Kumar and Patel (1975) also studied on fist-order reactant in homogeneous turbulence before the final period of decay for the case of multipoint and multi-time. Sarker and Islam (2001) studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Further work along this same line for the case of multi-point and single time had been done by Sarker and Kishore (1991).

In their approach they considered two and three-point correlations after neglecting higher order correlation terms compared to the second-and third-order correlation terms.

Kishore and Dixit (1979), Kishore and Singh (1984) discussed the effect of coriolis force on acceleration covariance in ordinary and MHD turbulence. Shimomura and Yoshizawa (1986), Shimomura (1986 and 1989) also discussed the statistical analysis of turbulent viscosity, turbulent scalar flux and turbulent shear flows respectively in a rotating system by two-scale direct interaction approach. Sarker and Islam (2001) studied the decay of dusty fluid turbulence before the final period in rotating system. Azad, Sarker and Mondol (2006) studied the decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system and also Azad and Sarker (2003) decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle has more recently been done by Azad and Sarker (2003).

By analyzing the above theories we have studied the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system using two-and three-point correlation equations neglecting fourth order correlation terms compared to the second-and third-order correlation terms. Finally, the energy decay law of temperature fluctuations in MHD turbulence before the final period in a rotating system is obtained.

Basic Equations

The equation of motion and continuity for viscous, incompressible MHD turbulent flow in a rotating system are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}\left(u_i u_k - h_i h_k\right) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\in_{mki}\Omega_m u_i, \quad \text{-- (1)}$$

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$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k)=\frac{\nu}{P_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k}, \quad \text{----- (2)}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_k}=-\frac{k}{m_s}(v_i - u_i) \quad \text{--------- (3)}$$

with

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial v_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}=0 \quad \text{--------- (4)}$$

and the equation of energy for an incompressible fluid with constant properties and for negligible frictional heating.

$$\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}=\left(\frac{\nu}{p_r}\right)\frac{\partial^2 T}{\partial x_i \partial x_i}. \quad \text{---------(5)}$$

The subscripts can take on the values 1, 2 or 3.

Here,

u_i, turbulent velocity component; hi, magnetic field fluctuation component,

$W(\hat{x},t)=\frac{p}{\rho}+\frac{1}{2}\langle h^2\rangle+\frac{1}{2}\left|\hat{\Omega}\times\hat{x}\right|^2$, total MHD pressure inclusive of potential and centrifugal force,

$p(\hat{x},t)$ = hydro-dynamic pressure,

ρ = fluid density,

$p_M=\frac{\nu}{\lambda}$, magnetic Prandtl number,

$p_r=\frac{\nu}{\gamma}$, Prandtl number,

ν = kinematic viscosity,

$\gamma=\frac{K}{\rho c_p}$, thermal diffusivit y,

$\lambda=(4\pi\mu\sigma)^{-1}$, magnetic diffusivity,

c_p = heat capacity at constant pressure ,

Ω_m = constant angular velocity components,

$\in_{mki}$ = alternating tensor,

$m_s=\frac{4}{3}\pi R_s^3\rho_s$, mass of single spherical dust particle of radius R_s ,

ρ_s = constant density of the material in dust particle,

x_k = Space co-ordinate, the subscripts can take on the values 1, 2 or 3.

Two-point Correlation and Spectral Equations

The induction equation of a magnetic field at the point p is

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\frac{\nu}{p_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k} \quad \text{--- (6)}$$

and the energy equation at the point p' is

$$\frac{\partial T_j'}{\partial t}+u_k'\frac{\partial T_j'}{\partial x_k'}=\frac{\nu}{p_r}\frac{\partial^2 T_j'}{\partial x_k' \partial x_k'}. \quad \text{---------- (7)}$$

The points p and P' are separated by the vector $\hat{r}$ is shown bellow

$p \longrightarrow p'$

Multiplying equation (6) by T_j' and (7) by h_i, adding and taking ensemble average, we get

$$\frac{\partial\langle h_i T_j'\rangle}{\partial t}+u_k\frac{\partial\langle h_i T_j'\rangle}{\partial x_k}+u_k'\frac{\partial\langle h_i T_j'\rangle}{\partial x_k'}-h_k\frac{\partial\langle u_i T_j'\rangle}{\partial x_k'}$$
$$=\nu\left[\frac{1}{P_M}\frac{\partial^2\langle h_i T_j'\rangle}{\partial x_k \partial x_k}+\frac{1}{P_r}\frac{\partial^2\langle h_i T_j'\rangle}{\partial x_k' \partial x_k'}\right]. \quad \text{----------(8)}$$

Angular bracket 〈..................〉 is used to denote an ensemble average.

The continuity equation is

$$\frac{\partial u_k}{\partial x_k}=\frac{\partial u_k'}{\partial x_k'}=0 \quad \text{--------- (9)}$$

Substituting equation (4.4) in to equation (4.3) yields

$$\frac{\partial\langle h_i T_j'\rangle}{\partial t}+\frac{\partial\langle u_k h_i T_j'\rangle}{\partial x_k}+\frac{\partial\langle u_k' h_i T_j'\rangle}{\partial x_k'}-\frac{\partial\langle u_i h_k T_j'\rangle}{\partial x_k}$$
$$=\nu\left[\frac{1}{p_M}\frac{\partial^2\langle h_i T_j'\rangle}{\partial x_k dx_k}+\frac{1}{p_r}\frac{\partial^2\langle h_i T_j'\rangle}{\partial x_k' \partial x_k'}\right]. \quad \text{---------(10)}$$

Using the transformations

$$\frac{\partial}{\partial r_k}=-\frac{\partial}{\partial x_k}=\frac{\partial}{\partial x_k'}$$

and the Chandrasekhar relations (1951a)

$$\langle u_k h_i T_j' \rangle = -\langle u_k' h_i T_j' \rangle \ ,$$

in to equation (10) one obtains

$$\frac{\partial}{\partial t}\langle h_i T_j' \rangle + 2\frac{\partial}{\partial r_k}\langle u_k' h_i T_j' \rangle + \frac{\partial \langle u_i h_k T_j' \rangle}{\partial r_k}$$

$$= \nu \left[\frac{\partial^2 \langle h_i T_j' \rangle}{\partial r_k \partial r_k} \left(\frac{1}{P_M} + \frac{1}{p_r} \right) \right]. \qquad \text{------------(11)}$$

Now we write this equation in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms.

$$\langle h_i T_j'(\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \psi_i \tau_j'(\hat{K}) \rangle \exp\left[\hat{i}(\hat{K}.\hat{r}) \right] d\hat{K} \ , \text{--(12)}$$

$$\langle u_i h_k T_j'(r) \rangle = \int_{-\infty}^{\infty} \langle \phi_i \psi_k \tau_j'(\hat{K}) \rangle \exp\left[\hat{i}(\hat{K}.\hat{r}) \right] d\hat{K} \ , \text{---(13)}$$

$$\langle u_k' h_i T_j'(r) \rangle = \langle u_k h_i T_j'(-\hat{r}) \rangle$$

$$= \int_{-\infty}^{\infty} \langle \phi_k \psi_i \tau_j'(-\hat{k}) \rangle \exp\left[\hat{i}(\hat{k}.\hat{r}) \right] d\hat{K} \qquad \text{----(14)}$$

Equation (14) is obtained by interchanging the subscripts i and j and then the points p and p'.

Substituting of equation (12) to (14) in to equation (11) leads to the spectral equation

$$\frac{\partial \langle \psi_i \tau_j \rangle}{\partial t} + iK_k \left[2\langle \phi_k \psi_i \tau_j'(-\hat{K}) + \langle \phi_i \psi_k \tau_j'(\hat{K}) \rangle \right]$$

$$= -\nu \left[\left(\frac{1}{p_M} + \frac{1}{p_r} \right) k^2 \langle \psi_i \tau_j'(\hat{K}) \rangle \right]. \qquad (15)$$

The tensor equation (15) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial \langle \psi_i \tau_i'(\hat{K}) \rangle}{\partial t} + iK_k \left[2\langle \phi_k \psi_i \tau'(-\hat{K}) \rangle + \langle \phi_i \psi_k \tau_i'(\hat{K}) \rangle \right]$$

$$= -\nu \left[\left(\frac{1}{P_M} + \frac{1}{P_r} \right) k^2 \langle \psi_i \tau_i'(\hat{k}) \rangle \right] . \qquad \text{----(16)}$$

Three-point Correlation and Spectral Equations

Similar Procedure can be used to find the three-point correlation equation. For this purpose, considering three points p, p', p'' separated by the vectors $\hat{r}$ and $\hat{r}'$ we take the momentum equation of MHD turbulence at the point P, the induction equation at the point P' and the energy equation at P'' as

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} - h_k \frac{\partial h_i}{\partial x_k} = -\frac{\partial w}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k} - 2\in_{mki} \Omega_m u_i \ , \quad \text{---(17)}$$

$$\frac{\partial h_i'}{\partial t} + u_k' \frac{\partial h_i'}{\partial x_k} - h_k' \frac{\partial u_i'}{\partial x_k'} = \frac{\nu}{P_M} \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} \quad \text{--(18)}$$

and

$$\frac{\partial T_j''}{\partial t} + u_k'' \frac{\partial T_j''}{\partial x_k''} = \left(\frac{\nu}{P_r} \right) \frac{\partial^2 T_j''}{\partial x_k'' \partial x_k''} \ , \quad \text{---- (19)}$$

where

$W(\hat{x},t) = \dfrac{P}{\rho} + \dfrac{1}{2}\langle h^2 \rangle + \dfrac{1}{2}\left| \hat{\Omega} \times \hat{x} \right|^2$, total MHD pressure inclusive of potential and centrifugal force $P(\hat{x},t)$, hydrodynamic pressure; Ωm, constant angular velocity components; $\in_{mki}$, alternating tensor.

Multiplying equation (17) by $h_i' T_j''$, (18) by $u_i T_j''$ and (19) by $u_i h_i'$, adding and taking ensemble average, one obtains

$$\frac{\partial \langle u_i h_i' T_j'' \rangle}{\partial t} + \frac{\partial \langle u_i u_k h_i' T_j \rangle}{\partial x_k} - \frac{\partial \langle h_i h_k h_i' T_j'' \rangle}{\partial x_k''}$$

$$+ \frac{\partial \langle u_i u_k' h_i' T_j'' \rangle}{\partial x_k'} - \frac{\partial \langle u_i u_i' h_k' T_j'' \rangle}{\partial x_k'} + \frac{\partial \langle u_i h_i' u_k'' T_j'' \rangle}{\partial x_k''}$$

$$= -\frac{\partial \langle w h_i' T_j'' \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle u_i h_i' T_j'' \rangle}{\partial x_k \partial x_k} +$$

$$\left[\frac{1}{P_M} \frac{\partial^2 \langle u_i h_i' T_j'' \rangle}{\partial x_k' \partial x_k'} + \frac{1}{P_r} \frac{\partial^2 \langle u_i h_i' T_j'' \rangle}{\partial x_k'' \partial x_k''} \right]. \qquad \text{--(20)}$$

$$-2 \in_{mki} \Omega_m \langle u_i h_i' T_j'' \rangle$$

Using the transformations

$$\frac{\partial}{\partial x_k} = -\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'} \right), \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'}$$

into equation (20)

$$\frac{\partial\langle u_i h_i' T_j''\rangle}{\partial t} - \nu\left[(1+\frac{1}{p_M})\frac{\partial^2\langle u_i h_i' T_j''\rangle}{\partial r_k \partial r_k} + (1+\frac{1}{p_r})\frac{\partial^2\langle u_i h_i' T_j''\rangle}{\partial r_k' \partial r_k'} + 2\frac{\partial^2\langle u_i h_i' T_j''\rangle}{\partial r_k \partial r_k'}\right]$$

$$= \frac{\partial\langle u_i u_k h_i' T_j''\rangle}{\partial r_k} + \frac{\partial\langle u_i u_k h_i' T_j''\rangle}{\partial r_k'} - \frac{\partial\langle h_i h_k h_i' T_j''\rangle}{\partial r_k} - \frac{\partial\langle h_i h_k h_i' T_j''\rangle}{\partial r_k'} - \frac{\partial\langle u_i u_k' h_i' T_j''\rangle}{\partial r_k}$$

$$+ \frac{\partial\langle u_i u_i' h_k' T_j''\rangle}{\partial r_k} - \frac{\partial\langle u_i u_k'' h_i' T_j''\rangle}{\partial r_k'} + \frac{\partial\langle w h_i' T_j''\rangle}{\partial r_i} + \frac{\partial\langle w h_i' T_j''\rangle}{\partial r_i'} - 2\in_{mki}\Omega_m\langle u_i h_i' T_j''\rangle \quad \text{------------ (21)}$$

In order to write the equation (10) to spectral form, we can define the following six dimensional Fourier transforms:

$$\langle u_i h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad \text{----- (22)}$$

$$\langle u_i u_k h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_k \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad \text{----- (23)}$$

$$\langle h_i h_k h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \beta_i \beta_k \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad \text{----- (24)}$$

$$\langle u_i u_k' h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_k'(\hat{k}) \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad \text{----- (25)}$$

$$\langle u_i u_i'(\hat{r}) h_k'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \phi_i \phi_i' \beta_k'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad \text{----- (26)}$$

$$\langle w h_i'(\hat{r}) T_j''(\hat{r}')\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle \gamma \beta_i'(\hat{k})\theta_j''(\hat{k}')\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\, d\hat{k}' \quad \text{----- (27)}$$

Interchanging the points p' and p'' along with the subscripts i and j,

$\langle u_i u_k'' h_i' T_j''\rangle = \langle u_i u_k' h_i' T_j''\rangle$.

By use this fact we can write equation (21) in the form

$$\frac{\partial\langle \phi_i \beta_i' \theta_j''\rangle}{\partial t} + \nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2k_k k_k' + \frac{2\in_{mki}\Omega_m}{\nu}\right]\langle \phi_i \beta_i' \theta_j''\rangle$$

$$= i(k_k + k_k')\langle \phi_i \phi_k \beta_i' \theta_j''\rangle - i(k_k + k_k')\langle \beta_i \beta_k \beta_i' \theta_j''\rangle - i(k_k + k_k')\langle \phi_i \phi_k' \beta_i' \theta_j''\rangle$$

$$+ ik_k\langle \phi_i \phi_i' \beta_k' \theta_j''\rangle + i(k_i + k_i')\langle \gamma \beta_i' \theta_j''\rangle . \quad \text{--------- (28)}$$

The tensor equation (28) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\langle\partial\langle \phi_i \beta_i' \theta_i''\rangle}{\partial t} + \nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2k_k k_k' + 2\frac{\in_{mki}\Omega_m}{\nu}\right]\langle \phi_i \beta_i' \theta_i''\rangle$$

$$= i(k_k + k_k')\langle \phi_i \phi_k \beta_i' \theta_i''\rangle - i(k_k + k_k')\langle \beta_i \beta_k \beta_i' \theta_i''\rangle - i(k_k + k_k')\langle \phi_i \phi_k \beta_i' \theta_i''\rangle$$

$$+ ik_k\langle \phi_i \phi_i' \beta_k' \theta_i''\rangle + i(k_i + k_i')\langle \gamma \beta_i' \theta_i''\rangle . \quad \text{------- (29)}$$

If the derivative with respect to x_i is taken of the momentum equation (17) for the point p, the equation multiplied through by h_i T_j" and taken time average, the resulting equation

$$-\frac{\partial^2 \langle w h_i' T_j'' \rangle}{\partial x_i \partial x_i} = \frac{\partial^2}{\partial x_i \partial x_k} \left(\langle u_i u_k h_i' T_j'' \rangle - \langle h_i h_k h_i' T_j'' \rangle \right) \quad \text{------(30)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_i \partial r_i} + 2\frac{\partial^2}{\partial r_i \partial r_i'} + \frac{\partial^2}{\partial r_i' \partial r_i'} \right] \langle w h_i' T_j'' \rangle$$
$$= \left[\frac{\partial^2}{\partial r_i \partial r_k} + \frac{\partial^2}{\partial r_i' \partial r_k} + \frac{\partial^2}{\partial r_i \partial r_k'} + \frac{\partial^2}{\partial r_i' \partial r_k'} \right]$$
$$\times \left(\langle u_i u_k h_i' T_j'' \rangle - \langle h_i h_k h_i' T_j'' \rangle \right). \quad \text{------- (31)}$$

Now taking the Fourier transforms of equation (31), we get

$$-\langle \gamma \beta_i' \theta_j'' \rangle = \frac{\left(k_i k_k + k_i' k_k + k_i k_k' + k_i' k_k' \right)\left(\langle \phi_i \phi_k \beta_i' \theta_j'' \rangle - \langle \beta_i \beta_k \beta_i' \theta_j'' \rangle \right)}{k_i k_i + 2k_i' k_i + k_i' k_i'}. \quad \text{------- (32)}$$

Equation (32) can be used t o eliminate $\langle \gamma \beta_i' \theta_j'' \rangle$ from equation (28).

Solution for times before the final period

It is known that equation for final period of decay is obtained by considering the two-point correlations after neglecting the 3rd order correlation terms. To study the decay for times before the final period, the three point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. Equation (32) shows that term $\langle \gamma \beta_i' \theta_j'' \rangle$ associated with the pressure fluctuations

should also be neglected. Thus neglecting all the terms on the right hand side of equation (29)

$$\frac{\partial \langle \phi_i \beta_i' \theta_i'' \rangle}{\partial t} + \nu \left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2k_k k_k' + \frac{2 \in_{mki} \Omega_m}{\nu} \right] \langle \phi_i \beta_i' \theta_i'' \rangle = 0 \quad \text{------(33)}$$

Integrating the equation (33) between t_o and t with inner multiplication by k_k and gives

$$k_k \langle \phi_i \beta_i' \theta_i'' \rangle = k_k \left[\phi_i \beta_i' \theta_i'' \right]_0 \exp\left[-\nu \left\{ (1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2kk'\cos\theta + \frac{2 \in_{mki} \Omega_m}{\nu} \right\}(t-t_0) \right] \quad \text{---(34)}$$

where θ is the angle between k and k' and $\langle \phi_i \beta_i' \theta_i'' \rangle_0$ is the value of $\langle \phi_i \beta_i' \theta_i'' \rangle$ at t= t_o.

Now by letting $r' = 0$ in equation (22) and comparing with equations (13) and (14), we get

$$\langle \phi_i \psi_k \tau_i'(\hat{k}) \rangle = \int_{-\infty}^{\infty} \langle \phi_i \beta_i' \theta_i'' \rangle d\hat{k}', \quad \text{-------- (35)}$$

$$\langle \phi_i \psi_i \tau_i'(-\hat{k}) \rangle = \int_{-\infty}^{\infty} \phi_k \beta_i'(-\hat{k}) \theta_i''(-\hat{k}') d\hat{k}'. \quad \text{------ (36)}$$

Substituting equation (34) to (36) in equation (16)

$$\frac{\partial \langle \psi_i \tau_i'(\hat{k}) \rangle}{\partial t} + \nu (\frac{1}{p_M} + \frac{1}{p_r})k^2 \langle \psi_i \tau_i'(\hat{k}) \rangle =$$
$$-\int_{-\infty}^{\infty} ik_k \left[\langle \phi_i \beta_i' \theta_i'' \rangle + 2\langle \phi_k \beta_i'(-\hat{k}) \theta_i''(-\hat{k}') \rangle \right]_0$$
$$\exp\left[-\nu (t-t_o) \left\{ (1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2kk'\cos\theta + 2\frac{\in_{mki} \Omega_m}{\nu} \right\} \right] d\hat{k}' \quad \text{-------------(37)}$$

Now, $d\hat{k}'$ can be expressed in terms of k' and θ as $-2\pi k'^2 d(\cos\theta) dk'$ (cf. Deissler [1])

Henc $d\hat{k}' = -2\pi k'^2 d(\cos\theta) d\hat{k}'$. --------- (38)

Putting equation (38) in equation (37) yields

$$\frac{\partial \langle \psi_i \tau_i'(\hat{k}) \rangle}{\partial t} + \nu (\frac{1}{p_M} + \frac{1}{p_r})k^2 \langle \psi_i \tau_i'(\hat{k}) \rangle =$$
$$-\int_0^{\infty} 2\pi i k_k \left[\langle \phi_i \beta_i' \theta_i'' \rangle + 2\langle \phi_k \beta_i'(-\hat{k}) \theta_i''(-\hat{k}') \rangle \right]_0 k'^2 \times$$

$$\left[\int_{-1}^{1}\exp\left\{-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+ 2kk'\cos\theta+2\frac{\in_{mki}\Omega_m}{\nu}]\right\}\right]d(\cos\theta)\,d\hat{k}' \qquad \text{------(39)}$$

In order to find the solution completely and following Loeffler and Deissler (1961) we assume that

$$ik_k\left[\langle\phi_i\beta_i'\theta_i''\rangle+2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0 = \frac{\beta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right), \qquad \text{------- (40)}$$

where β_o is a constant depending on the initial conditions. Substituting equation (40) into equation (39) and completing the integration with respect to cosθ, one obtains

$$\frac{\partial(2\pi\langle\psi_i\tau_i'(\hat{k})\rangle)}{\partial t}+\nu(\frac{1}{p_M}+\frac{1}{p_r})k^2(2\pi\langle\psi_i\tau_i'(\hat{k})\rangle)= -\frac{\beta_0}{2\nu(t-t_o)}\int_0^\infty\left(k^3k'^5-k^5k'^3\right)\times$$

$$\left[\exp\left\{-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2-2kk'+ \frac{2\in_{mki}\Omega_m}{\nu}]\right\}-\exp\left\{-\nu(t-t_o)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2 +2kk'+2\frac{\in_{mki}\Omega_m}{\nu}]\right\}\right]dk' \qquad \text{-------(41)}$$

Multiplying both sides of equation (41) by k^2, we get

$$\frac{\partial Q}{\partial t}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2Q=F, \qquad \text{------(42)}$$

where, $Q=2\pi k^2\langle\psi_i\tau_i'(\hat{k})\rangle$, ------(43)

Q is the Magnetic energy Spectrum function.

and

$$F=-\frac{\beta_0}{2\nu(t-t_0)}\int_0^\infty(k^3k'^5-k^5k'^3)\times$$

$$\left[\exp\left\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2- 2kk'+\frac{2\in_{mki}\Omega_m}{\nu}]\right\}\right.$$

$$\left.-\exp\left\{-\nu(t-t_0)[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2 +2kk'+\frac{2\in_{mki}\Omega_m}{\nu}]\right\}\right]. \qquad \text{----(44)}$$

Integrating equation (44) with respect to k', we have

$$F=-\frac{\beta_0\sqrt{\pi}P_r^{5/2}}{2\nu^{3/2}(t-t_o)(1+p_r)^{5/2}}\exp\left[-2\frac{\in_{mki}\Omega_m}{\nu}(t-t_o)\right]\times \exp\left[-\nu(t-t_o)(1+\frac{1}{p_M}-\frac{p_r}{1+p_r})k^2\right]$$

$$\left[\frac{15p_rk^4}{4\nu^2(t-t_o)^2(1+p_r)}+\left\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\right\}\frac{k^6}{\nu(t-t_o)} +\left\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{(1+p_r)}\right\}k^8\right] \qquad \text{------(45)}$$

The series of equation (45) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that if we integrate equation (44) over all wave numbers, we find that

$$\int_0^\infty Fdk=0 \qquad \text{-----(46)}$$

which is indicating that the expression for F satisfies the condition of continuity and homogeneity.

The linear equation (42) can be solved to give

$$Q = \exp\left[-\nu k^2 (\frac{1}{p_M} + \frac{1}{p_r})(t - t_o)\right] \int F$$

$$\exp\left[\nu k^2 (\frac{1}{p_M} + \frac{1}{p_r})(t - t_o)\right] dt$$

$$+ J(k) \exp\left[-\nu k^2 (\frac{1}{p_M} + \frac{1}{p_r})(t - t_o)\right] , \qquad \text{-----(47)}$$

where J(K)= $\frac{N_0 k^2}{\pi}$ is a constant of integration. Substituting the values of F from equation (45) in to equation (47) and integrating with respect to t, we get

$$Q(\hat{k}, t) = \frac{N_0 k^2}{\pi} \exp\left[-\nu k^2 (\frac{1}{p_M} + \frac{1}{p_r})(t - t_o)\right]$$

$$+ \frac{\beta_0 \sqrt{\pi}\, p_r^{\,3/2}}{2\nu^{3/2}(1+p_r)^{7/2}} \times \exp[-\{2 \in_{mki} \Omega_m (t - t_o)\}]$$

$$\exp\left[-\nu k^2 (t - t_o)\left\{\frac{1 + p_r + p_M}{p_M(1+p_r)}\right\}\right]$$

$$\left[\frac{3 p_r k^4}{2\nu^2 (t - t_o)^{5/2}} + \frac{p_r (7 p_r - 6) k^6}{3\nu (1 + p_r)(t - t_o)^{3/2}}\right.$$

$$\left. - \frac{4(3 p_r^{\,2} - 2 p_r + 3) k^8}{3(1 + p_r)^2 (t - t_o)^{1/2}} + \frac{8\sqrt{\nu}\,(3 p_r^{\,2} - 2 p_r + 3) k^9}{3(1 + p_r)^{5/2} \sqrt{p_r}} N(\omega)\right] \qquad \text{-------(48)}$$

where $N(\omega) = e^{-\omega^2} \int_o^{\omega} e^{x^2} dx$

and $\omega = k \sqrt{\frac{\lambda (t - t_o)}{p_r (1 + p_r)}}$.

The function $N_{(\omega)}$ has been calculated numerically and tabulated in Sarker and Islam (2001).

By setting $\hat{r} = 0,\ \ j = i,\ \ d\hat{K} = -2\pi k^2 d(\cos\theta) dk$ and $Q = 2\pi k^2 \left\langle \psi_i \psi_i' (\hat{K}) \right\rangle$ in equation (12), we get the expression for temperature energy decay as

$$\frac{\langle T^2 \rangle}{2} = \frac{T_i T_i'}{2} = \int_0^{\infty} Q(\hat{k}) d\hat{k} . \qquad \text{--------- (49}$$

Substituting equations (48) in to (49) and after integration, we get

$$\frac{\langle T^2 \rangle}{2} = \frac{N_o p_r^{\,3/2} p_M^{\,3/2} (t - t_o)^{-3/2}}{4\sqrt{\pi}\nu^{3/2} (p_r + p_M)^{3/2}} + \exp\left[-2 \in_{mki} \Omega_m\right]$$

$$\times \frac{\beta_0 \pi p_r^{\,7/2} p_M^{\,5/2} (t - t_o)^{-5}}{2\nu^6 (1 + p_r)(1 + p_r + p_M)^{5/2}} \times$$

$$\left\{\frac{9}{16} + \frac{5 p_M (7 p_r - 6)}{16(1 + p_r + p_M)} - \frac{35 p_M^2 (3 p_M^2 - 2 p_r + 3)}{8 p_r (1 + p_r + p_M)^2} + \right.$$

$$\left. \frac{8 p_M^3 (3 p_r^2 - 2 p_r + 3)}{3.2^6 . p_r^2 (1 + p_r + p_M)^3} \sum_{n=0}^{\infty} \frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n} \right\}$$

or

$$\frac{\langle T^2 \rangle}{2} = \frac{N_0 P_r^{3/2} p_M^{3/2} (t - t_o)^{-3/2}}{4\sqrt{\pi}\nu^{3/2} (p_r + p_M)^{3/2}} +$$

$$\beta_0 z \nu^{-6} (t - t_o)^{-5} \times \exp\left[-2 \in_{mki} \Omega_m\right], \qquad \text{------(50)}$$

where

$$Z = \frac{\pi p_r^{\,7/2} p_M^{5/2}}{2(1 + p_r)(1 + p_r + p_M)^{5/2}} \times$$

$$\left[\frac{9}{16} + \frac{5 P_M (7 P_r - 6)}{16(1 + P_r + p_M)} - \frac{35 p_M^2 (3 p_r^2 - 2 p_r + 3)}{8 p_r (1 + p_r + p_M)^2} + \right.$$

$$\left. \frac{8 p_M^3 (3 p_r^2 - 2 p_r + 3)}{3.2^6 p_r^2 (1 + p_r + p_M)^3} \sum_{n=0}^{\infty} \frac{1.3.5....(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n} \right].$$

Thus the energy decay law for temperature field fluctuations of MHD turbulence in a rotating system before the final period may be written as

$$\langle T^2 \rangle = X (t - t_o)^{-3/2} + \exp$$

$$\left[-\{2 \in_{mki} \Omega_m\}\right] Y (t - t_o)^{-5} , \qquad \text{------(51)}$$

where

$$X = \frac{N_0 P_r^{3/2} p_M^{3/2}}{2\sqrt{\pi}\nu^{3/2} (p_r + p_M)^{3/2}} \text{ and } Y = 2\beta_0 Z \nu^{-6} .$$

$\langle T^2 \rangle$ is the total "energy" (the mean square of the temperature fluctuations) t is the time, x and t_o are constants determined by the initial conditions. The constant Y depends on both initial conditions and the fluid Prandtl number.

Results and Discussion

In equation (51) we obtained the decay law of temperature fluctuations in MHD turbulence before the final period in a rotating system considering three-point correlation equation after neglecting quadruple correlation terms. If the system is non-rotating, then Ω_m=0 the equation (50) becomes.

$$\langle T^2 \rangle = X(t-t_o)^{-3/2} + \Upsilon(t-t_o)^{-5} \text{-------- (52)}$$

which was obtained earlier by Sarker and Rahman (1998)

In the absence of a magnetic field, magnetic Prandtl number coincides with the Prandtl number (i.e. P_r = PM) and the system is non rotating the equation (50) becomes

$$\frac{\langle T^2 \rangle}{2} = \frac{N_0 p_r^{\ 3/2}}{8\sqrt{2\pi}\nu^{3/2}(t-t_o)^{3/2}} + \frac{\beta_0 Z}{\nu^6 (t-t_o)^5} \quad \text{----(53)}$$

which was obtained earlier by Loeffler and Deissler (1961).

We conclude that due to the effect of rotation of fluid in the flow field, the turbulent energy decays more rapidly than the energy for non-rotating fluid. The 1st term of the right hand side of equation (51) corresponds to the temperature energy for two-point correlation and second term represents temperature energy for three-point correlation. For large times the last term in the equation (51) becomes negligible, leaving the -3/2power decay law for the final period. If we considering the higher order correlation terms in the analysis, it appears that more terms in higher power of time would be added to the equation (51).

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