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Magnetohydrodynamic Three Dimensional Flow and Heat Transfer Past a Vertical Porous Plate Through a Porous Medium with Periodic Suction

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Abstract

This paper analyzes the effect of magnetic field and the permeability of the medium on the three dimensional flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by an infinite vertical porous plate in presence of periodic suction and a transverse magnetic field. The governing equations for the velocity and temperature of the flow field are solved employing perturbation technique and the effects of the pertinent parameters such as magnetic parameter (M), suction parameter (α), permeability parameter (K_p), Reynolds number (R_e) and Prandtl number (P_r) on the velocity field, temperature field, skin friction and the rate of heat transfer are discussed with the help of figures and tables. It is observed that both magnetic parameter and the permeability parameter have accelerating effect on the velocity of the flow field. The effect of growing Prandtl number/suction parameter/ Reynolds number is to enhance the temperature of the flow field at all points while a growing magnetic parameter has retarding effect on the temperature field. The magnetic parameter increases the x-component of skin friction and reduces the magnitude of z-component of skin friction at the wall while the permeability parameter shows the reverse effect on both the components of skin friction. The rate of heat transfer at the wall grows as we increase the magnetic parameter or suction parameter or Prandtl number in the flow field and the effect reverses with the increase of the permeability parameter.

Key words: MHD, Three dimensional flow, Heat transfer, Vertical plate, Porous medium, Periodic suction

Introduction

In recent years the problem of MHD flow and heat transfer has attracted the attention of a number of researchers because of its possible applications to several geophysical and astrophysical studies. In view of these applications, a series of investigations were made to study the flow past a vertical wall. Chauhan and Sahal (2005) analyzed the flow and heat transfer over a naturally permeable bed of very small permeability with a variable suction. Das *et al.* (2006) numerically studied the effect of mass transfer on unsteady flow past an accelerated vertical porous plate with suction. In a separate paper, Das and his associates (2006) investigated the free convective mass transfer flow of a viscous incompressible fluid past an infinite vertical porous plate through a porous medium in presence of source/sink with constant suction and heat flux. The effect of variable suction on free convection on a vertical plate in a porous medium has been presented by Govindarajulu and Thangaraj (1992). The unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction was investigated by Kim (2000) and its steady part past a vertical moving plate by Sharma and Pareek (2002). The unsteady free convection flow with suction on an accelerating porous plate has been studied by Makinde *et al.* (2003).

The effect of free convection on steady MHD flow past a vertical porous plate was discussed by Soundalgekar (1974), Raptis and Singh (1983) and Raptis (1986) under different physical situations. Sarangi and Jose (2005) discussed the unsteady free convective MHD flow and mass transfer past a vertical porous plate with variable temperature. Sharma and Gupta (2004) analyzed the steady three dimensional flow and heat transfer along an infinite hot vertical porous surface in presence of heat sources, uniform free stream and periodic suction. Singh and Sharma (2002) discussed the three dimensional free convective flow and heat transfer through a porous medium with periodic permeability.

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Soundalgekar and Haldavnekar (1973) analyzed the MHD free convective flow in a vertical channel.Vershney and Singh (2005) investigated the three dimensional free convective flow with heat and mass transfer through a porous medium with periodic permeability.

The study reported herein analyzes the effect of magnetic field and the permeability of the medium on the three dimensional flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by an infinite vertical porous plate in presence of a transverse magnetic field and periodic suction. The governing equations for the velocity and temperature of the flow field are solved employing perturbation technique and the effects of the pertinent parameters such as magnetic parameter (M), suction parameter (α), permeability parameter (K_p), Reynolds number (R_e) and Prandtl number (P_r) on the velocity field, temperature field, the skin friction and the rate of heat transfer are discussed with the help of figures and tables.

Formulation of the problem

Consider the three dimensional flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by a vertical infinite porous plate in presence of a uniform transverse magnetic field B_0. A coordinate system is chosen with the plate lying vertically on x^*-z^* plane such that x^*-axis is taken along the plate in the direction of flow and y^*-axis is taken normal to the plane of the plate and directed into the fluid which is flowing with the free stream velocity U. The plate is assumed to be at constant temperature T_w. The plate is subjected to a transverse sinusoidal suction velocity of the form:

$$v^*(z^*) = - V\,(1+ \varepsilon \cos\pi z^* / l), \tag{1}$$

where ε (<<1) is a very small positive constant quantity, l is taken equal to the half wavelength of the suction velocity. The negative sign in the above equation indicates that the suction is towards the plate. Due to this kind of injection velocity the flow remains three dimensional. All the physical quantities involved are independent of x^* for this fully developed laminar flow. Denoting the velocity components u^*, v^*, w^* in x^*, y^*, z^* directions, respectively, and the temperature by T^*, the problem is governed by the following equations:

Continuity equation:

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \tag{2}$$

Momentum equation

$$v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) - \frac{\nu}{K^*}u^* - \frac{\sigma B_0^2}{\rho}u^*, \tag{3}$$

$$v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) - \frac{\nu}{K^*}v^*, \tag{4}$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \nu\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\nu}{K^*}w^* - \frac{\sigma B_0^2}{\rho}w^*, \tag{5}$$

Energy equation:

$$\rho C_p\left(v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*}\right) = k\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) + \mu\phi^*, \tag{6}$$

where

$$\phi^* = 2\left\{\left(\frac{\partial v^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial z^*}\right)^2\right\} + \left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*} + \frac{\partial v^*}{\partial z^*}\right)^2 + \left(\frac{\partial u^*}{\partial z^*}\right)^2\right\}, \tag{7}$$

ρ is the density, σ is the electrical conductivity, $p^*$is the pressure, K^* is the permeability of the porous medium, ν is the coefficient of kinematic viscosity and α is the thermal diffusivity. The initial and the boundary conditions of the problem are

$u^* = 0$, $v^* = -V\,(1+ \varepsilon \cos\pi z^* / l)$, $w^* = 0$, $T^* = T_w^*$ at $y^* = 0$,

$u^*=U,\ v^*=V,\ w^*=0,\ p^*=p_\infty^*$ as $y^* \to \infty$. (8)

Introducing the following non-dimensional quantities

$$y=\frac{y^*}{l},\ z=\frac{z^*}{l},\ u=\frac{u^*}{U},\ v=\frac{v^*}{U},$$

$$w=\frac{w^*}{U},\ p=\frac{p^*}{\rho U^2},\ \theta=\frac{T^*-T_\infty^*}{T_w^*-T_\infty^*}, \quad (9)$$

equations (2) - (6) reduce to the following forms:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \quad (10)$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{R_e}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\left(M+\frac{1}{K_p}\right)u \quad (11)$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{R_e}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{v}{K_p}, \quad (12)$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{R_e}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\left(M+\frac{1}{K_p}\right)w \quad (13)$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{R_eP_r}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)+\frac{E_c}{R_e}\phi, \quad (14)$$

where

$$\phi=2\left\{\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}+\left\{\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right\}, \quad (15)$$

$R_e=\frac{Ul}{\nu}$ (Reynolds number), $M=\frac{\sigma B_0^2}{U}\frac{\nu l}{\mu}$ (Magnetic parameter), $P_r=\frac{\mu C_p}{k}$ (Prandtl number), $E_c=\frac{U^2}{C_p\left(T_w^*-T_\infty^*\right)}$ (Eckert number), $K_p=\frac{K^*U}{\nu l}$ (Permeability parameter), $\alpha=\frac{V}{U}$ (Suction parameter). (16)

The corresponding boundary conditions now reduce to the following form:

$u=0,\quad v=1+\varepsilon\cos\pi z,\quad w=0,\quad \theta=1$ at $y=0$,

$u=1,\quad v=1,\ p=p_\infty,\quad w=0,\quad \theta=0$ as $y\to\infty$. (17)

Materials and Methods

In order to solve the problem, we assume the solutions of the following form because the amplitude ε (<<1) of the permeability variation is very small:

$u(y,z)=u_0(y)+\varepsilon\ u_1(y,z)+\ldots\ldots$ (18)
$v(y,z)=v_0(y)+\varepsilon\ v_1(y,z)+\ldots\ldots$ (19)
$w(y,z)=w_0(y)+\varepsilon\ w_1(y,z)+\ldots\ldots$ (20)
$p(y,z)=p_0(y)+\varepsilon\ p_1(y,z)+\ldots\ldots$ (21)
$\theta(y,z)=\theta_0(y)+\varepsilon\ \theta_1(y,z)+\ldots\ldots$ (22)

When $\varepsilon=0$, the problem reduces to the two dimensional free convective MHD flow through a porous medium with constant permeability which is governed by the following equations:

$$\frac{dv_0}{dy}=0, \quad (23)$$

$$\frac{d^2u_0}{dy^2}+\alpha R_e\frac{du_0}{dy}-R_e\left(M+\frac{1}{K_p}\right)u_0=0, \quad (24)$$

$$\frac{d^2\theta_0}{dy^2}+\alpha R_eP_r\frac{d\theta_0}{dy}=-2E_cP_ru_0'^2. \quad (25)$$

The corresponding boundary conditions become

$u_0=0,\quad v_0=-\alpha,\ w_0=0,\quad \theta_0=1$ at $y=0$,

$u_0=1,\ p=p_\infty,\ v_0=1,\ w_0=0,\ \theta_0=0$ as $y\to\infty$. (26)

The solutions for $u_0(y)$ and $T_0(y)$ under boundary conditions (26) for this two dimensional problem are

$$u_0(y) = 1 - e^{-my}, \quad (27)$$

$$\theta_0(y) = e^{\alpha P_r R_e y} + E_1\left(e^{-\alpha P_r R_e y} - e^{-2my}\right), \quad (28)$$

with $v_0 = -\alpha$, $w_0 = 0$, $p_0 =$ constant, (29)

where

$$m = \frac{1}{2}\left[\alpha R_e + \sqrt{\alpha^2 R_e^2 + 4R_e\left(M + \frac{1}{K_p}\right)}\right]$$

and $E_1 = \dfrac{mE_c P_r}{2(2m - \alpha R_e P_r)}$.

When $\varepsilon \neq 0$, substituting equations (18)-(22) into equations (10) - (14) and comparing the coefficients of like powers of ε, neglecting those of ε^2, we get the following first order equations with the help of equation (29):

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \quad (30)$$

$$v_1\frac{\partial u_0}{\partial y} - \alpha\frac{\partial u_1}{\partial y} = \frac{1}{R_e}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \left(M + \frac{1}{K_p}\right)u_1, \quad (31)$$

$$-\alpha\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{R_e}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{v_1}{K_p}, \quad (32)$$

$$-\alpha\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{R_e}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \left(M + \frac{1}{K_p}\right)w_1, \quad (33)$$

$$v_1\frac{\partial \theta_0}{\partial y} - \alpha\frac{\partial \theta_1}{\partial y} = \frac{1}{R_e P_r}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) + \frac{2E_c}{R_e}\cdot\frac{\partial u_0}{\partial y}\cdot\frac{\partial u_1}{\partial y} \quad (34)$$

The corresponding boundary conditions are

$u_1 = 0$, $v_1 = -\alpha\cos\pi z$, $w_1 = 0$, $\theta_1 = 0$ at $y = 0$,
$u_1 = 0$, $v_1 = 0$, $p_1 = 0$, $w_1 = 0$, $\theta_1 = 0$ as $y \to \infty$. (35)

Equations (30) - (34) are the linear partial differential equations which describe the MHD three-dimensional flow through a porous medium. For solution, we shall first consider three equations (30), (32) and (33) being independent of the main flow component u_1 and the temperature field T_1. We assume v_1, w_1 and p_1 of the following form:

$$v_1(y,z) = v_{11}(y)\cos\pi z, \quad (36)$$

$$w_1(y,z) = -\frac{1}{\pi}v'_{11}(y)\sin\pi z, \quad (37)$$

$$p_1(y,z) = p_{11}(y)\cos\pi z, \quad (38)$$

where the prime in $v'_{11}(y)$ denotes the differentiation with respect to y. Expressions for $v_1(y, z)$ and $w_1(y, z)$ have been chosen so that the equation of continuity (30) is satisfied. Substituting these expressions (36)-(38) into (32) and (33) and solving under corresponding transformed boundary conditions, we get the solutions of v_1, w_1 and p_1 as:

$$v_1 = \frac{\alpha}{A_1 - A_2}\left(A_2 e^{-A_1 y} - A_1 e^{-A_2 y}\right)\cos\pi z \quad (39)$$

$$w_1 = \frac{\alpha A_1 A_2}{\pi(A_1 - A_2)}\left(e^{-A_1 y} - e^{-A_2 y}\right)\sin\pi z \quad (40)$$

where

$$A_1 = \frac{1}{2}\left[m + \sqrt{m^2 + 4\pi^2}\right], A_2 = \frac{1}{2}\left[n + \sqrt{n^2 + 4\pi^2}\right],$$

$$n = \frac{1}{2}\left[\alpha R_e - \sqrt{\alpha^2 R_e^2 + 4R_e\left(M + \frac{1}{K_p}\right)}\right].$$

In order to solve equations (31) and (34), we assume

$$u_1(y,z) = u_{11}(y)\cos\pi z, \quad (41)$$

$$\theta_1(y,z) = \theta_{11}(y)\cos\pi z, \quad (42)$$

Substituting equations (41) and (42) in equations (31) and (34), we get

$$u''_{11} + \alpha u'_{11} - \left(\pi^2 + MR_e + \frac{R_e}{K_p}\right)u_{11} = R_e v_{11} u'_0 \quad (43)$$

$$\theta''_{11} + \alpha P_r\theta'_{11} - \pi^2\theta_{11} = R_e P_r\theta'_0 v_{11} - 2E_c P_r u'_0 u'_{11} \quad (44)$$

The corresponding boundary conditions are

$u_{11} = 0,\ \theta_{11} = 0$ at $y = 0$,

$u_{11}=0,\quad \theta_{11} = 0$ as $y \to \infty$. (45)

Solving equations (43) and (44) under boundary condition (45) and using equations (18), (22), (25) and (26), we get

$$u = 1 - e^{-my} + A_0 e^{-my} \left[A_3 e^{(m-A_1)y} - A_4 e^{-(m+A_2)y} - e^{-A_5 y}\right]\cos \pi z \quad (46)$$

$$\theta = e^{\alpha P_r R_e y} + E_1\left(e^{-\alpha P_r R_e y} - e^{-2my}\right) + A\left[A_7 e^{(\alpha P_r R_e - A_1)y} - A_8 e^{(\alpha P_r R_e - A_2)y} - A_9 e^{-(\alpha P_r R_e + A_1)y} + A_{10} e^{-(\alpha P_r R_e + A_2)y}\right] + \varepsilon B \left[A_{11} e^{-(A_1+2m)y} - A_{12} e^{(2m-A_2)y} - A_{13} e^{-(m+A_5)y}\right] - \varepsilon A_{14} e^{-A_6 y} \quad (47)$$

where

$$A = \frac{\varepsilon\alpha^2 R_e^2 P_r^2}{A_1 - A_2},\ A_0 = \frac{\varepsilon\alpha R_e m}{A_1 - A_2},\ B = \frac{2m\alpha R_e P_r}{A_1 - A_2},$$

$$A_3 = \frac{A_2}{(A_1 + m)^2 - \alpha R_e (A_1 + m) - \left(\pi^2 + MR_e + \frac{R_e}{K_p}\right)},$$

$$A_4 = \frac{A_1}{(A_2 + m)^2 + \alpha R_e (A_2 + m) - \left(\pi^2 + MR_e + \frac{R_e}{K_p}\right)},$$

$$A_5 = \frac{1}{2}\left[\alpha R_e + \sqrt{\alpha^2 R_e^2 + 4\left(\pi^2 + MR_e + \frac{R_e}{K_p}\right)}\right],$$

$$A_6 = \frac{1}{2}\left[\alpha R_e P_r + \sqrt{\alpha^2 R_e^2 P_r^2 + 4\pi^2}\right],$$

$$A_7 = \frac{A_2}{(A_1 - \alpha P_r)^2 - \alpha P_r (A_1 - \alpha P_r) - \pi^2},$$

$$A_8 = \frac{A_1}{(A_2 - \alpha P_r)^2 - \alpha P_r (A_2 - \alpha P_r) - \pi^2},$$

$$A_9 = \frac{E_1 A_2}{(A_1 + \alpha P_r)^2 - \alpha P_r (A_1 + \alpha P_r) - \pi^2},$$

$$A_{10} = \frac{A_1 E_1}{(A_2 + \alpha P_r)^2 - \alpha P_r (A_2 + \alpha P_r) - \pi^2},$$

$$A_{11} = \frac{A_{15}}{(A_1 + 2m)^2 - \alpha P_r (A_1 + 2m) - \pi^2},$$

$$A_{12} = \frac{A_{16}}{(A_2 + 2m)^2 - \alpha P_r (A_2 + 2m) - \pi^2},$$

$$A_{13} = \frac{mE_c A_5}{(A_5 + m)^2 - \alpha P_r (A_5 + m) - \pi^2},$$

$$A_{14} = \frac{\alpha^2 R_e^2 P_r^2}{A_1 - A_2}(A_7 - A_8 - A_9 + A_{10}) - B(A_{11} - A_{12} - A_{13}),$$

$$A_{15} = E_1 A_2 + mE_c A_3 (A_1 + m),$$

$$A_{16} = E_1 A_1 + mE_c A_4 (A_2 + m).$$

Skin friction

The x- and z-components of skin friction at the wall are given by

$$\tau_x = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{du_1}{dy}\right)_{y=0}$$

and $$\tau_z = \varepsilon\left(\frac{dw_1}{dy}\right)_{y=0}. \quad (48)$$

Using equations (46) and (40), the x- and z-components of skin friction at the wall become

$$\tau_x = m + A_0(-A_1 A_3 + A_2 A_4 + A_5 + m)\cos \pi z, \quad (49)$$

$$\tau_z = \frac{\varepsilon\alpha A_1 A_2}{\pi(A_1 - A_2)}(A_2 - A_1)\sin \pi z. \quad (50)$$

Rate of Heat Transfer

The rate of heat transfer i.e. heat flux at the wall in terms of Nusselt number (N_u) is given by

$$N_u = \left(\frac{d\theta_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{d\theta_1}{dy}\right)_{y=0} \quad (51)$$

Using equation (47), the heat flux at the wall becomes

$$N_u = \alpha R_e P_r + E_1(-\alpha R_e P_r + 2m) + A[A_7(\alpha R_e P_r - A_1) - A_8(\alpha R_e P_r - A_2) + A_9(\alpha R_e P_r + A_1) - A_{10}(\alpha R_e P_r + A_2)] + \varepsilon B[-A_{11}(2m + A_1) - A_{12}(2m - A_2) + A_{13}(m + A_5)] + \varepsilon A_6 A_{14}$$

Results and Discussion

The problem of magnetohydrodynamic three dimensional flow and heat transfer past a vertical porous plate through a porous medium with periodic suction is considered. The governing equations for the velocity and temperature of the flow field are solved employing perturbation technique and the effects of the pertinent parameters such as magnetic parameter (M), suction parameter (α), permeability parameter (K_p), Reynolds number (R_e) and Prandtl number (P_r). on the velocity and temperature of the flow field and also on the skin friction and the rate of heat transfer have been discussed with the help of figures (1)-(8) and tables (1)-(4), respectively.

Velocity field

The velocity of the flow field is found to change substantially with the variation of flow parameters such as magnetic parameter (M), permeability parameter (K_p), suction parameter (α) and Reynolds number (R_e). These variations are show in figures (1)-(4).

In figure (1), we present the velocity profiles against the non-dimensional distance with the variation of the magnetic parameter (M). A growing magnetic parameter is found to accelerate the velocity of the flow field at all points. This variation in the velocity of the flow field is due to the magnetic pull of the Lorenz force acting on the flow field.

Figure (2) depicts the effect of permeability of the medium on the velocity of the flow field. Keeping other parameters of the flow field constant, the permeability parameter (K_p) is varied in steps and its effect on the velocity field is studied.

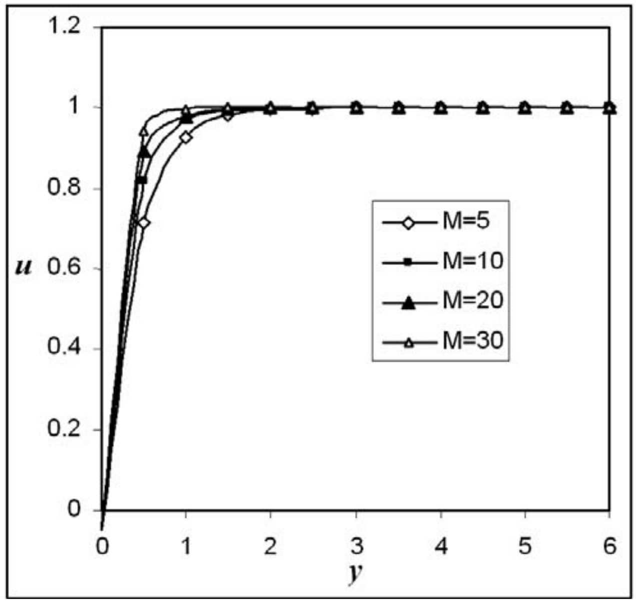

Fig. 2: Velocity profiles against *y* for different values of K_p with $R_e = 1$, $M = 1$, $P_r = 0.71$, $\alpha = 0.2$, $E_c = 0.01$, $\varepsilon = 0.2$ and $z = 0$

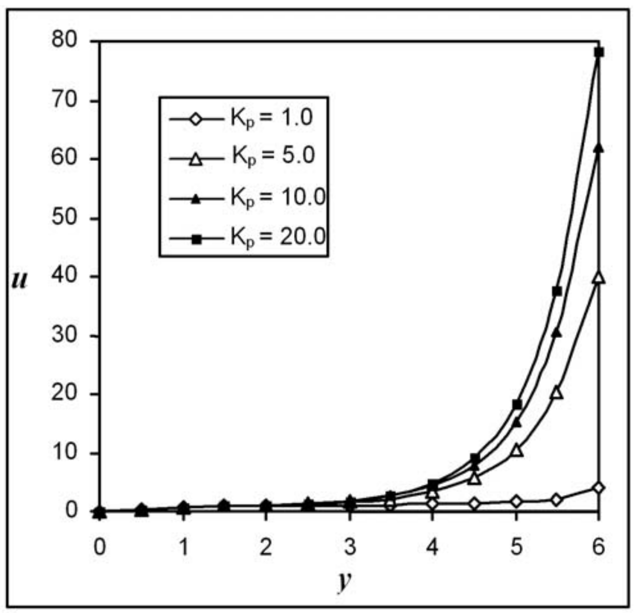

Fig. 2: Velocity profiles against *y* for different values of K_p with $R_e = 1$, $M = 1$, $P_r = 0.71$, $\alpha = 0.2$, $E_c = 0.01$, $\varepsilon = 0.2$ and $z = 0$

It is observed that a growing permeability parameter has an accelerating effect on the velocity of the flow field.

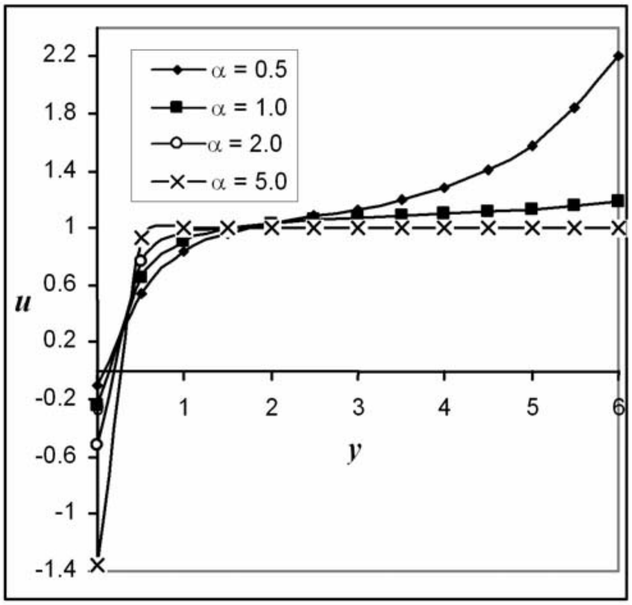

Fig. 3: Velocity profiles against *y* for different values of α with $R_e = 1$, $M = 1$, $P_r = 0.71$, $K_p = 1$, $E_c = 0.01$, ε = 0.2 and $z = 0$

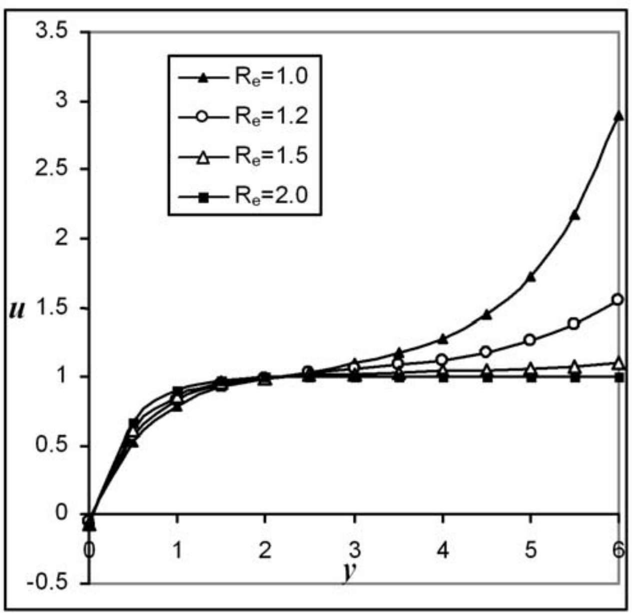

Fig. 2: Velocity profiles against *y* for different values of K_p with $R_e = 1$, $M = 1$, $P_r = 0.71$, α = 0.2, $E_c = 0.01$, ε = 0.2 and $z = 0$

In figure (3), we analyze the effect of suction parameter (α) on the velocity of the flow field. The suction parameter shows a dual behaviour. It increases the magnitude of the velocity upto a certain distance y=1.5 near the plate and thereafter the flow behaviour reverses.

The effect of Reynolds number (R_e) on the velocity of the flow field is shown in figure (4).It is observed that R_e increases the velocity near the plate upto y=1.5 and thereafter, it retards the velocity. The behaviour of R_e is similar to the suction parameter in this respect.

Temperature field

The temperature of the flow field varies vastly with the variation of the flow parameters. The major flow parameters affecting this field are Prandtl number (P_r), suction parameter (α), Reynolds number (R_e) and the magnetic parameter

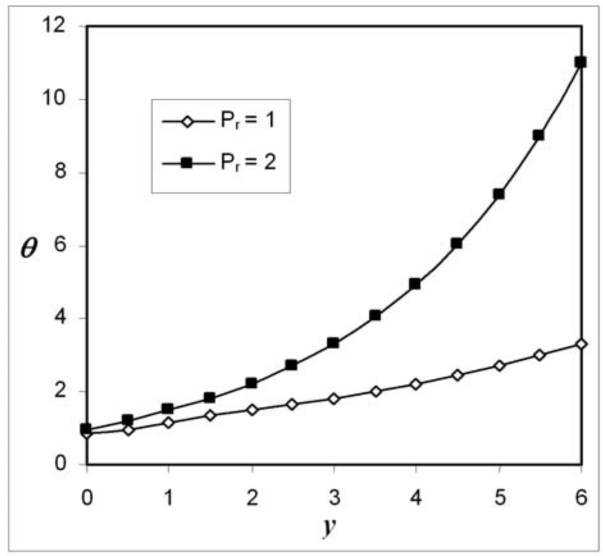

Fig. 2: Velocity profiles against *y* for different values of K_p with $R_e = 1$, $M = 1$, $P_r = 0.71$, α = 0.2, $E_c = 0.01$, ε = 0.2 and $z = 0$

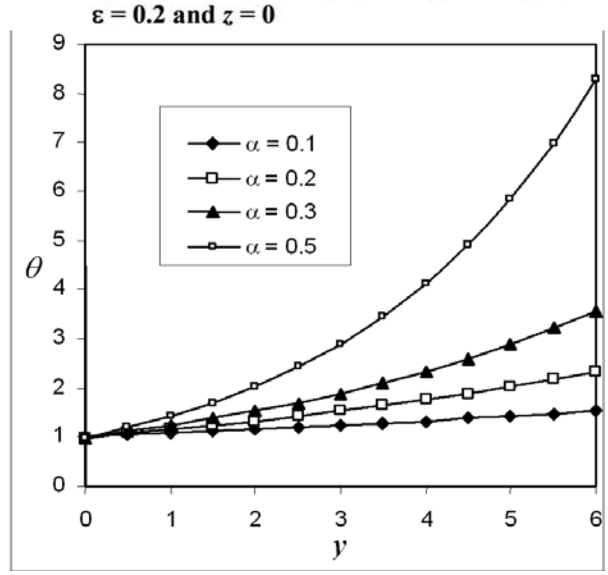

Fig. 6: Temperature profiles against *y* for different values of α with $R_e = 1$, $K_p = 1$, $M = 1$, $P_r = 0.71$, $E_c = 0.01$ and ε = 0.2

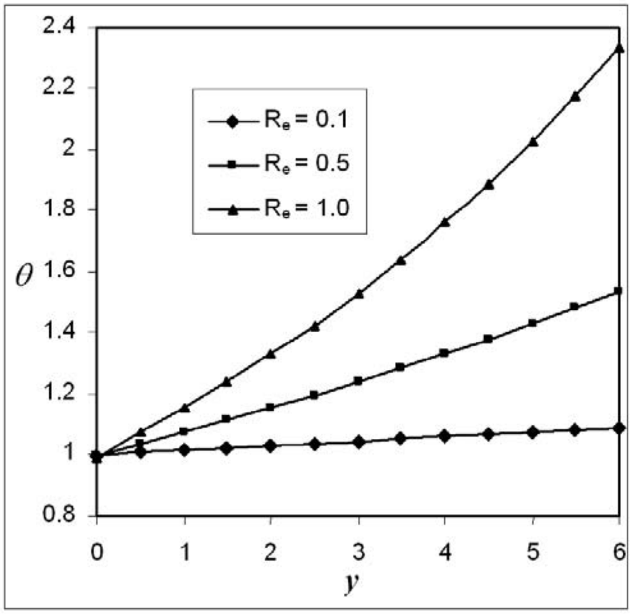

Fig. 3: Velocity profiles against y for different values o α with $R_e = 1$, $M = 1$, $P_r = 0.71$, $K_p = 1$, $E_c = 0.01$, = 0.2 and $z = 0$

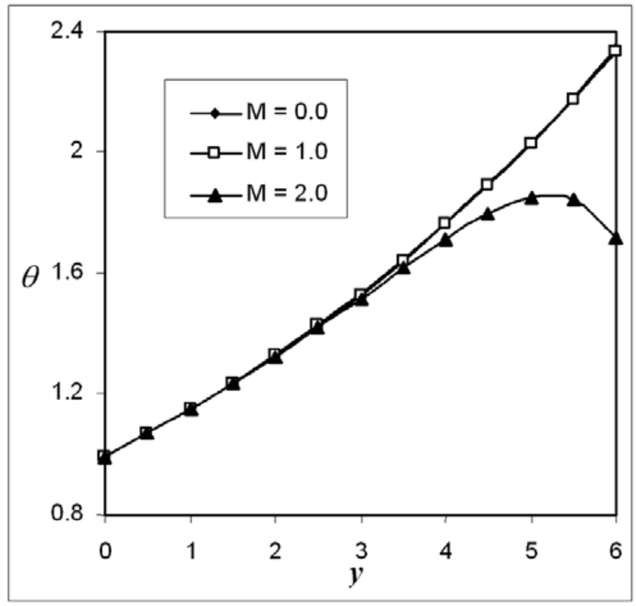

Fig. 3: Velocity profiles against y for different values of α with $R_e = 1$, $M = 1$, $P_r = 0.71$, $K_p = 1$, $E_c = 0.01$, ε = 0.2 and $z = 0$

(M). The effects of these parameters on the temperature of the flow field are analyzed graphically with the help of figures (5)-(8).

Figures (5), (6) and (7) depict the effect of Prandtl number (Pr), suction parameter (α) and the Reynolds number (R_e) respectively, on the temperature of the flow field. The effect of growing Prandtl number or suction parameter or Reynolds number is to enhance the temperature of the flow field at all points. On careful observation of figure (8), it is observed that the magnetic parameter reduces the temperature of the flow field at all points.

Skin friction

The x- and z-components of skin friction at the wall for different values of α , M and K_p are entered in Tables I and II respectively. It is observed that the magnetic parameter increases the x-component of skin friction and reduces the magnitude of z-component of skin friction at the wall while the permeability parameter shows the reverse effect on both the components of skin friction at the wall. The effect of suction parameter is to enhance the magnitude of both the components of skin friction at the wall.

Rate of heat transfer

The rate of heat transfer i.e. the heat flux at the wall for different values of α, P_r, M and K_P are entered in Tables III, IV and V respectively. The rate of heat transfer i.e. the heat flux at the wall grows as we increase the magnetic parameter or suction parameter or Prandtl number in the flow field

Table I: Variation in the value of x- and z-component of skin friction (τ_x, τ_z) against α for different values of M with $R_e = 1$, $K_p = 1$, $P_r = 0.71$, 0.2, $E_c = 0.01$, $\varepsilon = 0.2$ and $z = 0$ (=1/2 for τ_z)

α	M =0		M =1.0		M =5.0		M =10.0	
	τ_x	τ_z	τ_x	τ_z	τ_x	τ_z	τ_x	τ_z
0.0	1.00000	0.0000	1.41421	0.00000	2.44949	0.0000	3.31663	0.0000
0.1	1.13681	-0.0638	1.56217	-0.0637	2.62907	-0.0636	3.52547	-0.0632
0.3	1.44731	-0.1975	1.88975	-0.1974	3.01318	-0.1969	3.96512	-0.1965
0.5	1.81087	-0.3397	2.26202	-0.3393	3.43172	-.3382	4.43482	-0.3368
1.0	2.96388	-0.7345	3.39983	-0.7331	4.63633	-0.7281	5.74603	-0.7226

Table II: Variation in the value of x- and z-component of skin friction (τ_x, τ_z) against α for different values of K_p with R_e = 1, M = 1, P_r= 0.71, E_c = 0.01, α = 0.2 ε =0.2 and z = 0 (=1/2 for τ_z)

α	K_p =0.1		K_p =0.5		K_p =1.0		K_p =20.0	
	τ_x	τ_z	τ_x	τ_z	τ_x	τ_z	τ_x	τ_z
0.0	3.31662	0.0000	1.73205	0.0000	1.41421	-0.0000	1.02469	0.0000
0.1	3.52547	-0.0636	1.88921	-0.0637	1.56216	-0.0639	1.16213	-0.0643
0.3	3.96512	-0.1965	2.23251	-0.1972	1.88975	-0.1973	1.47352	-0.1975
0.5	4.43482	-0.3368	2.61631	-0.3390	2.26201	-0.3393	1.83732	-0.3396
1.0	5.74603	-0.7226	3.76335	-0.7317	3.39983	-0.7331	2.98822	-0.7344

Table III: Variation in the value of rate of heat transfer (N_u) against α for different values of M with R_e=1, K_p = 1, P_r = 0.71, α = 0.2, E_c = 0.01 and ε = 0.2

α	N_u			
	M =0	M =1.0	M =5.0	M =10.0
0.0	0.003550	0.005020	0.008696	0.011774
0.1	0.091124	0.092275	0.096668	0.101162
0.3	0.272650	0.272680	0.277564	0.284493
0.5	0.463426	0.464281	0.465496	0.474290
1.0	0.948500	0.957327	0.969674	0.980480

Table IV: Variation in the value of rate of heat transfer (N_u) against α for different values of K_p with R_e=1, M = 1, P_r = 0.71, α = 0.2, E_c = 0.01 and ε = 0.2

α	N_u			
	K_p =0.2	K_p =0.5	K_p =1.0	K_p =20.0
0.0	0.008696	0.006149	0.005020	0.003638
0.1	0.096668	0.093453	0.092275	0.091176
0.3	0.277564	0.273548	0.272608	0.272595
0.5	0.465496	0.461471	0.461281	0.461199
1.0	0.969674	0.969285	0.967327	0.965392

Table V: Variation in the value of rate of heat transfer (N_u) against α for different values of P_r with R_e=1, M = 1, P_r = 0.71, α = 0.2, E_c = 0.01 and ε = 0.2

α	N_u			
	P_r =0.71	P_r =1.0	P_r =2.0	P_r =7.0
0.0	0.005020	0.007071	0.014142	0.049497
0.1	0.092275	0.130467	0.264591	1.012477
0.3	0.272608	0.388372	0.812274	4.140528
0.5	0.461281	0.662648	1.441254	7.500890

and the effect reverses with the increase of permeability parameter in the flow field.

Conclusion

The above analysis brings out the following results of physical interest on the velocity, temperature, skin friction and the rate of heat transfer at the wall in the flow field.

1. A growing magnetic parameter/ permeability parameter has an accelerating effect on the velocity of the flow field at all points.

2. The effect of increasing suction parameter / Reynolds number is to enhance the magnitude of velocity of the flow field near the plate upto a certain distance (y=1.5) and thereafter the flow behaviour reverses.

3. The effect of growing Prandtl number/suction parameter/Reynolds number is to enhance the temperature of the flow field at all points while a growing magnetic parameter has a retarding effect on the temperature of the flow field.

4. The magnetic parameter increases the x-component of skin friction and reduces the magnitude of z-component of skin friction at the wall while the permeability parameter shows the reverse effect on both the components of skin friction at the wall. The effect of suction parameter is to enhance the magnitude of both the components of skin friction at the wall.

5. The rate of heat transfer i.e. the heat flux at the wall grows as we increase the magnetic parameter or suction parameter or Prandtl number in the flow field and the effect reverses with the increase of the permeability parameter in the flow field.

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