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JORDAN HIGHER DERIVATIONS IN PRIME Γ-RINGS

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Abstract

The objective of this paper is to study Jordan higher derivations in prime Γ-rings. We introduce a higher derivation and a Jordan higher derivation in Γ-rings. For a 2-torsion free prime Γ-ring M which satisfies the condition $a\alpha b\beta c = a\beta b\alpha c$ for all $a,b,c \in M$ and $\alpha,\beta \in \Gamma$, we prove that every Jordan higher derivation $D = (d_i)_{i \in N_0}$ of M is a higher derivation of M .

Keywords: Higher derivation, Jordan higher derivation, prime Γ-ring.

Introduction

We begin with the general definition of a Γ-ring. The notion of a Γ-ring was introduced by Nobusawa (1964) and generalized by Barnes (1966) as defined below. Let M and Γ be additive abelian groups. If there is a mapping $M \times \Gamma \times M \to M$ such that the conditions

- $(x+y)\alpha z = x\alpha z + y\alpha z, x(\alpha+\beta)y = x\alpha y + x\beta y, x\alpha(y+z) = x\alpha y + x\alpha z;$
- $(x\alpha y)\beta z = x\alpha(y\beta z)$

are satisfied for all $x, y, z \in M, \alpha, \beta \in \Gamma$, then M is called a Γ*-ring*. This concept is more general than that of a ring. From the definition it is clear that every ring is a Γ-ring but the converse is not necessarily true. A Γ-ring M is *2-torsion free* if $2a = 0$ implies $a = 0$ for all $a \in M$; M is called a *prime* Γ*-ring* if for all $a,b \in M, a\Gamma M\Gamma b = 0$ implies $a = 0$ or $b = 0$.

The concepts of derivation and Jordan derivation of a Γ-ring have been introduced by Sapanci and Nakajima (1997). For classical ring theory, Herstien (1957) proved a well known result that every Jordan derivation of a 2-torsion free prime ring is a derivation. Bresar (1988) proved this result for semiprime rings. Sapanci and Nakajima (1997) proved the same result for completely prime Γ-rings. Haetinger (2002) worked on higher derivations on prime rings and extended this result to Lie ideals in a prime ring. In this article, we introduce a higher derivation and a Jordan higher derivation in Γ-rings. We extend the result of Cortes and Haetinger (2005) concerning Jordan higher derivations in prime Γ-rings. We prove that every Jordan higher derivation of a 2-torsion free prime Γ-ring satisfying the condition $a\alpha b\beta c = a\beta b\alpha c$ for all $a,b,c \in M$ and $\alpha,\beta \in \Gamma$, is a higher derivation of M .

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Throughout the article, we asume the condition $a\alpha b\beta c = a\beta b\alpha c$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$ and refer it to by (*).

Jordan Derivations in a Prime Γ-ring

The notions of derivation and Jordan derivation of Γ-rings have been introduced by Sapanci and Nakajima (1997) as follows.

Definition 1. *For a Γ-ring M, if $d : M \to M$ is an additive mapping such that $d(a\alpha b) = d(a)\alpha b + a\alpha d(b)$ holds for all $a, b \in M$ and $\alpha \in \Gamma$, then d is called a derivation of M; d is called a Jordan derivation of M if $d(a\alpha a) = d(a)\alpha a + a\alpha d(a)$ holds for all $a \in M$ and $\alpha \in \Gamma$.*

First, we show that every Jordan derivation of a 2-torsion free prime Γ-ring is a derivation. For this purpose we prove the following Lemmas.

Lemma 1. *Let M be a Γ-ring, and let d be a Jordan derivation of M. Then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, the following statements hold:*

(i) $d(a\alpha b + b\alpha a) = d(a)\alpha b + d(b)\alpha a + a\alpha d(b) + b\alpha d(a)$

(ii) $d(a\alpha b\beta a + a\beta b\alpha a) = d(a)\alpha b\beta a + d(a)\beta b\alpha a + a\alpha d(b)\beta a$
$+ a\beta d(b)\alpha a + a\alpha b\beta d(a) + a\beta b\beta d(a).$

In particular, if M is 2-torsion free and satisfies the condition (*), then

(iii) $d(a\alpha b\beta a) = d(a)\alpha b\beta a + a\alpha d(b)\beta a + a\alpha b\beta d(a)$

(iv) $d(a\alpha b\beta c + c\alpha b\beta a) = d(a)\alpha b\beta c + d(c)\alpha b\beta a + a\alpha d(b)\beta c$
$+ c\alpha d(b)\beta a + a\alpha b\beta d(c) + c\alpha b\beta d(a).$

Proof. Compute $d((a+b)\alpha(a+b))$ and cancel the like terms from both sides to obtain (i). Then replace $a\beta b + b\beta a$ for b in (i) to get (ii). Using the condition (*), and since M is 2-torsion free, (iii) follows from (ii). Finally, (iv) is obtained by replacing $a + c$ for a in (iii).

Definition 2. *Let d be a Jordan derivation of a Γ-ring M. Then for all $a, b \in M$ and $\alpha \in \Gamma$, we define $\phi_\alpha(a,b) = d(a\alpha b) - d(a)\alpha b - a\alpha d(b)$. Thus*

$\phi_\alpha(b,a) = d(b\alpha a) - d(b)\alpha a - b\alpha d(a)$.***Lemma 2.*** *Let d be a Jordan derivation of a Γ-ring M. Then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, the following statements hold:*

(i) $\phi_\alpha(a,b)+\phi_\alpha(b,a)=0$; (ii) $\phi_\alpha(a+b,c)=\phi_\alpha(a,c)+\phi_\alpha(b,c)$
(iii) $\phi_\alpha(a,b+c)=\phi_\alpha(a,b)+\phi_\alpha(a,c)$; (iv) $\phi_{\alpha+\beta}(a,b)=\phi_\beta(a,b)+\phi_\beta(a,b)$.

Proof. Obvious.

Remmark 1. *d is a derivation of a Γ-ring M if and only if $\phi_\alpha(a,b)=0$ for all $a,b\in M$ and $\alpha\in\Gamma$.*

Lemma 3. *Let M be a 2-torsion free Γ-ring satisfying the condition(*), and let d be a Jordan derivation of M. Then $\phi_\alpha(a,b)\beta m\gamma[a,b]_\alpha+[a,b]_\alpha\beta m\gamma\phi_\alpha(a,b)=0$ for all $a,b,m\in M$ and $\alpha,\beta,\gamma\in\Gamma$.*

Proof. For any $a,b,m\in M$ and $\alpha,\beta,\gamma\in\Gamma$, by using Lemma 1(iv), we have

$$\begin{aligned}d(a\alpha b\beta m\gamma b\alpha a+b\alpha a\beta m\gamma a\alpha b)&=d((a\alpha b)\beta m\gamma b\alpha a+(b\alpha a)\beta m\gamma(a\alpha b))\\&=d(a\alpha b)\beta m\gamma b\alpha a+a\alpha b\beta d(m)\gamma b\alpha a+a\alpha b\beta m\gamma d(b\alpha a),\\&\quad+d(b\alpha a)\beta m\gamma a\alpha b+b\alpha a\beta d(m)\gamma a\alpha b+b\alpha a\beta m\gamma d(a\alpha b)\end{aligned}$$

On the other hand, by using Lemma 1 (iii)

$$\begin{aligned}d(a\alpha(b\beta m\gamma b)\alpha a+b\alpha(a\beta m\gamma a)\alpha b)&=d(a\alpha(b\beta m\gamma b)\alpha a)+d(b\alpha(a\beta m\gamma a)\alpha b)\\&=d(a)\alpha b\beta m\gamma b\alpha a+a\alpha d(b\beta m\gamma b)\alpha a+a\alpha b\beta m\gamma b\alpha d(a)\\&\quad+d(b)\alpha a\beta m\gamma a\alpha b+b\alpha d(a\beta m\gamma a)\alpha b+b\alpha a\beta m\gamma a\alpha d(b)\\&=d(a)\alpha b\beta m\gamma b\alpha a+a\alpha d(b)\beta m\gamma b\alpha a+a\alpha b\beta d(m)\gamma b\alpha a\\&\quad+a\alpha b\beta m\gamma d(b)\alpha a+a\alpha b\beta m\gamma b\alpha d(a)+d(b)\alpha a\beta m\gamma a\alpha b+b\alpha d(a)\\&\quad\beta m\gamma a\alpha b+b\alpha a\beta d(m)\gamma a\alpha b+b\alpha a\beta m\gamma d(a)\alpha b+b\alpha a\beta m\gamma a\alpha d(b).\end{aligned}$$

Comparing the two relations and using the Definition 2, we obtain

$$\phi_\alpha(a,b)\beta m\gamma b\alpha a+\phi_\alpha(b,a)\beta m\gamma a\alpha b+a\alpha b\beta m\gamma\phi_\alpha(b,a)+b\alpha a\beta m\gamma\phi_\alpha(a,b)=0.$$

This implies that

$$\phi_\alpha(a,b)\beta m\gamma[a,b]_\alpha+[a,b]_\alpha\beta m\gamma\phi_\alpha(a,b)=0,\forall a,b,m\in M \text{ and } \alpha,\beta,\gamma\in\Gamma$$

Lemma 4. *Let M be a 2-torsion free prime Γ-ring and let $a,b\in M$.*
If $a\alpha m\beta b+b\alpha m\beta a=0$ for all $m\in M,\alpha,\beta\in\Gamma$, then $a=0$ or $b=0$.

Proof. Replacing m by $s\delta a\mu t$ in $a\alpha m\beta b + b\alpha m\beta a = 0$, we have $a\alpha s\delta a\mu t\beta b + b\alpha s\delta a\mu t\beta a = 0$.

Now $b\alpha s\delta a = -a\alpha s\delta b$ and $a\mu t\beta b = -b\mu t\beta a$. Substituting these we get

$$-a\alpha s\delta b\mu t\beta a - a\alpha s\delta b\mu t\beta a = 0.$$

$$\Rightarrow 2a\alpha s\delta b\mu t\beta a = 0.$$

As M is 2-torsion free, so $a\alpha s\delta b\mu t\beta a = 0$.

Therefore, $(a\alpha s\delta b)\Gamma M\Gamma a = 0$. As M is prime, so $a\alpha s\delta b = 0$ or $a = 0$.

Suppose $a\alpha s\delta b = 0$. Again applying the primeness of M, we have $a = 0$ or $b = 0$.

Theorem 1. *Let M be a 2-torsion free prime Γ-ring satisfying the condition (*), and let d be a Jordan derivation of M. Then d is a derivation of M.*

Proof. By Lemma 3 and Lemma 4, and M being prime, we have

$$\phi_\alpha(a,b) = 0 \text{ or } [a,b]_\alpha = 0.$$

If $[a,b]_\alpha = 0$ for all $a,b \in M, \alpha \in \Gamma$, then $a\alpha b = b\alpha a$. Using this in Lemma 1(i), we have $2d(a\alpha b) = 2d(a)\alpha b + 2a\alpha d(b)$. Since M is 2-torsion free, we obtain d is a derivation of M.

If $\phi_\alpha(a,b) = 0$, then d is also a derivation of M.

Jordan Higher Derivations in Prime Γ-Rings

We introduce higher derivation and Jordan higher derivation of Γ-rings in the following way.

Definition 3. *Let $D = (d_i)_{i\in N_0}$ be a family of additive mappings of a $\Gamma -$ ring M such that $d_0 = id_M$, where id_M is an identity mapping on M and $N_0 = N \cup \{0\}$. Then D is a higher derivation of M if for each $n \in N_0$ and $i, j \in N_0$,*

$$d_n(a\alpha b) = \sum_{i+j=n} d_i(a)\alpha d_j(b), \text{ holds for all } a,b \in M; \alpha \in \Gamma,$$

D is a *Jordan higher derivation* of M if

$$d_n(a\alpha a) = \sum_{i+j=n} d_i(a)\alpha d_j(a), \text{ holds for all } a \in M; \alpha \in \Gamma.$$

Example 1. *Let* R *be an associative ring with 1. Let us consider* $M = M_{1,2}(R)$ *and* $\Gamma = \left\{ \begin{pmatrix} n.1 \\ 0 \\ \end{pmatrix} : n \in Z \right\}$*, then* M *is a* Γ*-ring. Let* $f_n : R \to R$ *be a higher derivation for each* $n \in N_0$*. For* $n \in N_0$*, we define additive mappings* $d_n : M \to M$ *by* $d_n((a,b)) = (f_n(a), f_n(b))$*. Then an easy verifications leads to us that* d_n *is a higher derivation of* M*. Let* $P = \{(a,a) : a \in R\}$*, then* P *is a* Γ*-ring contained in* M*. In fact,* P *is a sub* Γ*-ring. Define* $d_n((a,a)) = (f_n(a), f_n(a))$*, then* d_n *is a Jordan higher derivation of* P*.*

Lemma 5. *Assume that* $D = (d_i)_{i \in N}$ *is a Jordan higher derivation of* M*. Then for all* $a,b,c \in M; \alpha, \beta \in \Gamma$ *and* $n \in N$*,*

(i) $d_n(a\alpha b + b\alpha a) = \sum_{i+j=n}[d_i(a)\alpha d_j(b) + d_i(b)\alpha d_j(a)]$;

(ii) $d_n(a\alpha b\beta a) = \sum_{i+j+k=n}[d_i(a)\alpha d_j(b)\beta d_k(a)]$;

(iii) $d_n(a\alpha b\beta c + c\alpha b\beta a) = \sum_{i+j+k=n}[d_i(a)\alpha d_j(b)\beta d_k(c) + d_i(c)\alpha d_j(b)\beta d_k(a)]$.

Proof. The proofs of (i) and (ii) are similar to the proofs of Lemma 1(i) and Lemma 1(iii). Replacing a by $a + c$ in (ii) and using (ii), we obtain

$$W = d_n((a+c)\alpha b\beta(a+c)) = \sum_{i+j+k=n} d_i(a+c)\alpha d_j(b)\beta d_k(a+c)$$

$$= \sum_{i+j+k=n} (d_i(a) + d_i(c))\alpha d_j(b)\beta(d_k(a) + d_k(c)) = \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(a)$$

$$+ \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(c) + \sum_{i+j+k=n} d_i(c)\alpha d_j(b)\beta d_k(a) + \sum_{i+j+k=n} d_i(c)\alpha d_j(b)\beta d_k(c).$$

Also, we have

$$W = d_n(a\alpha b\beta a + a\alpha b\beta c + c\alpha b\beta a + c\alpha b\beta c)$$

$$= d_n(a\alpha b\beta a) + d_n(c\alpha b\beta c) + d_n(a\alpha b\beta c + c\alpha b\beta a)$$

$$= \sum_{i+j+k=n} d_i(a)\alpha d_j(b)\beta d_k(a) + \sum_{i+j+k=n} d_i(c)\alpha d_j(b)\beta d_k(c) + d_n(a\alpha b\beta c + c\alpha b\beta a).$$

By comparing the two expressions for W, we obtain (iii).

Definition 4 *For any Jordan higher derivation* $D=(d_i)_{i\in N}$ *of* M *, we define*

$$\phi_n^\alpha(a,b)=d_n(a\alpha b)-\sum_{i+j=n}d_i(a)\alpha d_j(b) \text{ for all } a,b\in M;\alpha\in\Gamma \text{ and } n\in N.$$

Remmark 2. D *is a higher derivation of* M *if and only if* $\phi_n^\alpha(a,b)=0$ *holds for all* $a,b\in M;\alpha\in\Gamma$ *and* $n\in N$ *.*

Lemma 6. *For every* $a,b,c\in M;\alpha,\beta\in\Gamma$ *and* $n\in N$ *,*

(i) $\phi_n^\alpha(a,b)+\phi_n^\alpha(b,a)=0;$ (ii) $\phi_n^\alpha(a+b,c)=\phi_n^\alpha(a,c)+\phi_n^\alpha(b,c)$

(iii) $\phi_n^\alpha(a,b+c)=\phi_n^\alpha(a,b)+\phi_n^\alpha(a,c);$ (iv) $\phi_n^{\alpha+\beta}(a,b)=\phi_n^\alpha(a,b)+\phi_n^\beta(a,b).$

Proof. (i) By Definition 4 and using Lemma 5(i), we obtain

$$\begin{aligned}\phi_n^\alpha(a,b)+\phi_n^\alpha(b,a) &= d_n(a\alpha b)-\sum_{i+j=n}d_i(a)\alpha d_j(b)+d_n(b\alpha a)-\sum_{i+j=n}d_i(b)\alpha d_j(a)\\ &= d_n(a\alpha b+b\alpha a)-\sum_{i+j=n}d_i(a)\alpha d_j(b)-\sum_{i+j=n}d_i(b)\alpha d_j(a)\\ &= \sum_{i+j=n}d_i(a)\alpha d_j(b)+\sum_{i+j=n}d_i(b)\alpha d_j(a)-\sum_{i+j=n}d_i(a)\alpha d_j(b)\\ &\quad -\sum_{i+j=n}d_i(b)\alpha d_j(a)=0.\end{aligned}$$

(ii) By Definition 4, we get

$$\begin{aligned}\phi_n^\alpha(a+b,c) &= d_n((a+b)\alpha c)-\sum_{i+j=n}d_i(a+b)\alpha d_j(c)\\ &= d_n(a\alpha c+b\alpha c)-\sum_{i+j=n}d_i(a)\alpha d_j(c)-\sum_{i+j=n}d_i(b)\alpha d_j(c)\\ &= d_n(a\alpha c)-\sum_{i+j=n}d_i(a)\alpha d_j(c)+d_n(b\alpha c)-\sum_{i+j=n}d_i(b)\alpha d_j(c)\\ &= \phi_n^\alpha(a,c)+\phi_n^\alpha(b,c).\end{aligned}$$

(iii)-(iv): The proofs are straight forward.

Lemma 7. *Suppose* $D=(d_i)_{i\in N}$ *is a Jordan higher derivation of a* Γ *-ring* M *. Let* $n\in N$ *and assume that* $a,b\in M;\alpha,\beta,\gamma\in\Gamma$ *. If* $\phi_m^\alpha(a,b)=0$ *, for every* $m<n$ *, then*

$$\phi_n^\alpha(a,b)\beta w\gamma[a,b]_\alpha+[a,b]_\alpha\beta w\gamma\phi_n^\alpha(a,b)=0 \text{, for every } w\in M.$$

Proof. We consider $G = d_n(a\alpha b\beta w\gamma b\alpha a + b\alpha a\beta w\gamma a\alpha b)$. First, we compute

$$G = d_n(a\alpha(b\beta w\gamma b)\alpha a) + d_n(b\alpha(a\beta w\gamma a)\alpha b).$$

Using Lemma 5(ii), we have on one hand

$$G = \sum_{i+p+l=n} d_i(a)\alpha d_p(b\beta w\gamma b)\alpha d_l(a) + \sum_{i+p+l=n} d_i(b)\alpha d_p(a\beta w\gamma a)\alpha d_l(b)$$

$$= \sum_{i+j+k+h+l=n} d_i(a)\alpha d_j(b)\beta d_k(w)\gamma d_h(b)\alpha d_l(a) + \sum_{i+j+k+h+l=n} d_i(b)\alpha d_j(a)\beta d_k(w)\gamma d_h(a)\alpha d_l(b).$$

On the other hand

$$G = d_n((a\alpha b)\beta w\gamma(b\alpha a) + (b\alpha a)\beta w\gamma(a\alpha b)).$$

Using Lemma 5(iii), we obtain

$$G = \sum_{r+s+t=n} (d_r(a\alpha b)\beta d_s(w)\gamma d_t(b\alpha a) + d_r(b\alpha a)\beta d_s(w)\gamma d_t(a\alpha b))$$

$$= \sum_{r+s+t=n} d_r(a\alpha b)\beta d_s(w)\gamma d_t(b\alpha a) + \sum_{r+s+t=n} d_r(b\alpha a)\beta d_s(w)\gamma d_t(a\alpha b).$$

Comparing the two expressions for G, we obtain

$$\sum_{i+j+k+h+l=n} d_i(a)\alpha d_j(b)\beta d_k(w)\gamma d_h(b)\alpha d_l(a) - \sum_{r+s+t=n} d_r(a\alpha b)\beta d_s(w)\gamma d_t(b\alpha a)$$
$$+ \sum_{i+j+k+h+l=n} d_i(b)\alpha d_j(a)\beta d_k(w)\gamma d_h(a)\alpha d_l(b) - \sum_{r+s+t=n} d_r(b\alpha a)\beta d_s(w)\gamma d_t(a\alpha b) = 0. \quad (1)$$

By the inductive assumption we can put $d_r(x\alpha y)$ for $\sum_{i+j=r} d_i(x)\alpha d_j(y)$, when $r < n$.

Therefore,

$$\sum_{i+j+k+h+l=n} d_i(a)\alpha d_j(b)\beta d_k(w)\gamma d_h(b)\alpha d_l(a) - \sum_{r+s+t=n} d_r(a\alpha b)\beta d_s(w)\gamma d_t(b\alpha a)$$
$$= (\sum_{i+j=n} d_i(a)\alpha d_j(b))\beta w\gamma b\alpha a + a\alpha b\beta w\gamma(\sum_{h+l=n} d_h(b)\alpha d_l(a))$$
$$+ \sum_{i+j+k+h+l=n}^{i+j<n,h+l<n} d_i(a)\alpha d_j(b)\beta d_k(w)\gamma d_h(b)\alpha d_l(a) - d_n((a\alpha b)\beta w\gamma(b\alpha a)$$
$$-(a\alpha b)\beta w\gamma d_n(b\alpha a) - \sum_{r+s+t=n}^{i+j=r<n,\,p+q=t<n} d_i(a)\alpha d_j(b)\beta d_s(w)\gamma d_p(b)\alpha d_q(a)$$

$$= -(d_n((a\alpha b) - \sum_{i+j=n} d_i(a)\alpha d_j(b))\beta(w\gamma b\alpha a) - (a\alpha b\beta w)\gamma(d_n(b\alpha a) - \sum_{h+l=n} d_h(b)\alpha d_l(a))$$

$$= -(\phi_n^\alpha(a,b)\beta w\gamma b\alpha a + a\alpha b\beta w\gamma\phi_n^\alpha(b,a)). \tag{2}$$

Similarly,

$$\sum_{i+j+k+h+l=n} d_i(b)\alpha d_j(a)\beta d_k(w)\gamma d_h(a)\alpha d_l(b) - \sum_{r+s+t=n} d_r(b\alpha a)\beta d_s(w)\gamma d_t(a\alpha b)$$

$$= -(\phi_n^\alpha(b,a)\beta w\gamma a\alpha b + b\alpha a\beta w\gamma\phi_n^\alpha(a,b)). \tag{3}$$

Hence, by using (2) and (3) in (1), we get

$$\phi_n^\alpha(a,b)\beta w\gamma b\alpha a + a\alpha b\beta w\gamma\phi_n^\alpha(b,a) + \phi_n^\alpha(b,a)\beta w\gamma a\alpha b + b\alpha a\beta w\gamma\phi_n^\alpha(a,b) = 0.$$

By Lemma 6(i), we have

$$\phi_n^\alpha(a,b)\beta w\gamma b\alpha a - a\alpha b\beta w\gamma\phi_n^\alpha(a,b) - \phi_n^\alpha(a,b)\beta w\gamma a\alpha b + b\alpha a\beta w\gamma\phi_n^\alpha(a,b) = 0.$$

This implies,

$$\phi_n^\alpha(a,b)\beta w\gamma[a,b]_\alpha + [a,b]_\alpha\beta w\gamma\phi_n^\alpha(a,b) = 0, \forall\ w \in M.$$

Here, we extend the result of Cortes and Haetinger (2005) concerning Jordan higher derivations in prime Γ-rings.

Theorem 2. *Let M be a 2-torsion free prime Γ-ring satisfying the condition (*). Then every Jordan higher derivation of M is a higher derivation of M.*

Proof. By definition, we have

$$\phi_0^\alpha(a,b) = 0, \quad \text{for all } a,b \in M, \alpha \in \Gamma.$$

Also, by Theorem 1,

$$\phi_1^\alpha(a,b) = 0, \quad \text{for all } a,b \in M, \alpha \in \Gamma.$$

Now, we proceed by induction. Suppose that, $\phi_m^\alpha(a,b) = 0.$

This implies, $d_m(a\alpha b) = \sum_{i+j=m} d_i(a)\alpha d_j(b)$ for all $a,b \in M; \alpha \in \Gamma$ and $m < n.$

Taking $a,b \in M$, by Lemma 7, we get

$$\phi_n^\alpha(a,b)\beta w\gamma[a,b]_\alpha + [a,b]_\alpha\beta w\gamma\phi_n^\alpha(a,b) = 0, \forall\ w \in M, \alpha, \beta, \gamma \in \Gamma.$$

Since M is prime, so by Lemma 4 $\phi_n^\alpha(a,b) = 0$, or $[a,b]_\alpha = 0$. Using the similar arguments as used in the proof of Theorem 1, we obtain that every Jordan higher derivation of M is a higher derivation of M.

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