The Chittagong Univ. J. Sci. 40 : 59-74, 2018

Stochastic Inventory Model with Reworks

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Abstract

In most of the inventory models, a single stock is considered from where items are served for the customers. In this paper, two stocks are considered. One is for fresh items and another is for returned items. The model is more appropriate where warranties are provided for a fixed time duration. Moreover, inventory which are kept to the stock may not have finite shelf-life is also considered here such as milk, meat, vegetables, radioactive materials, volatile liquids. In this current model, we considered the inventory is decayed in a constant rate θ. It is assumed that inventory level for both the fresh and returned items are pre-determined. When inventory level reaches at s, a replenishment takes place with parameter γ. The demands that arrive for fresh items and returned items follow Poisson process with parameter λ & δ respectively. Service will be provided with Poisson process for returned items with parameter μ. The joint probability distribution for inventory level of returned items and for fresh items is obtained in the steady state analysis. Some systems characteristics of the model are derived and the results are illustrated with the help of numerical examples.

***Keywords**: Inventory*; *Warranties*; *Reworks; Shelf-life*; *Replenishment*; *Poisson process*; *Joint probability distribution*.

অধিকাংশ ভান্ডার ব্যবস্থাপনা মডেলে একটি ভান্ডার বিবেচনা করা হয় যেখান থেকে পণ্যসমূহ ক্রেতাকে পরিবেশন করা হয়। এখানে আমরা দু'টি ভান্ডার বিবেচনা করেছি। একটি ভান্ডার থেকে নতুন পণ্য বিতরণ করা হবে এবং অপর ভান্ডারটি ফেরৎ পণ্যের জন্য। যে সকল পণ্যে নির্দিষ্ট সময়ের জন্য বিক্রয় পরবর্তী সেবার নিশ্চয়তা প্রদান করা হয়, সে সকল পণ্যের ক্ষেত্রে এই মডেলটি বেশি প্রযোজ্য। অধিকন্তু, ভান্ডারে রক্ষিত সকল পণ্য যেমন দুধ, মাংশ, শাক-সব্জি, তেজস্ক্রিয় পণ্য, উদ্বায়ী তরল, ইত্যাদি, যাদের স্থায়িত্বকাল নির্ধারিত নয়, সে সকল পণ্যের ক্ষেত্রেও মডেলটি বিবেচনা করা হয়েছে। এ মডেলে আমরা ভান্ডার θ ধ্রুব হারে ক্ষয়িষ্ণু বিবেচনা করেছি। এটি ধরে নেয়া হয়েছে যে, নতুন পণ্য ও ফেরৎ পণ্য উভয় ভান্ডারের মজুদ পণ্যের পরিমাণ পূর্ব নির্ধারিত। যখন পণ্যের সীমা s পর্যায়ে পৌঁছাবে, তখন γ পরামিতিতে পণ্যের পুনঃপূরণ ঘটবে। আবির্ভূত নতুন পণ্য ও ফেরৎ পণ্যের চাহিদা সমূহ যথাক্রমে λ ও δ পরামিতিতে পৈঁসু প্রক্রিয়া (Poisson process) অনুসরণ করে। ফেরৎ পণ্যের জন্য পৈঁসু প্রক্রিয়ায় μ পরামিতিতে সেবা প্রদান করা হবে। যৌথ সম্ভাবনা বিন্যাসে ফেরৎ পণ্য সমূহ ও নতুন পণ্য সমূহের ভান্ডার মাত্রা সাম্যাবস্থা বিশ্লেষণে প্রাপ্ত হয়েছে। এ মডেলটির কিছু পদ্ধতি বৈশিষ্ট্য বের করা হয়েছে এবং ফলাফল সমূহ সাংখ্যিক উদাহরণের সাহায্যে ব্যাখ্যা করা হয়েছে।

1. Introduction

Most products lose their market value (outdate) over time. Some products lose value faster than others; these are known as perishable products. Traditionally, perishables outdate due to their chemical structure. Examples of such perishable products are fresh produce, blood products, dairy products, meat, drugs, vitamins etc. Today many products that are not perishable in the traditional sense (i.e. products that do not decay chemically over time) can still be considered as perishable. Such products, mostly high-tech products, outdate because of changes in market conditions. Personal computers, computer components (such as micro-processors, memory, data storage units), cellular phones, digital cameras, digital music players, personal digital assistants (PDAs) are examples of high-tech products that rapidly lose market value. The life cycles of such products are getting shorter every year due to technological advances.

S. Kalpakam and G. Arivarignan (1988) [1] considered a continuous review of perishable inventory with instantaneous replenishment at single location**.** Also a complete review was provided by Benita M. Beamon (1998) [2]. However, there has been increasing attention placed on performance, design. G. Arivarignan, C. Elango and N. Arumugam (2002) [3] had studied continuous review Perishable inventory at service facility**.** Mohammad Ekramol Islam et al. (2007) [4] had studied perishable inventory system with different rates of production and random switching time. Ajanta Roy (2008) [5] developed a deterministic inventory model when the deterioration rate is time proportional and demand rate is a function of selling price and holding cost is time dependent. Biswaranjan Mandal (2010) [6] developed An EOQ inventory model for Weibull distributed deterioration items under ramp type demand and shortages where he considered that the inventory level depleted not only by demand but also by deterioration. The Weibull distribution, which is capable of representing constant, increasing and decreasing rate of deterioration, is used to represent the distribution of time to deterioration. S. Sing et al. (2011) [7] developed an inventory model for decaying items with selling price depend demand in inflationary environment. S. K. Ghosh et al. (2012) [8] studied an optimal inventory replenishment policy for a deteriorating time-quadratic demand and time-dependent partial backlogging which depends on the length of the waiting time for the next replenishment over a finite time horizon and variable replenishment cycle. Zeinab Sazvar et al. (2013) [9] developed an inventory model for a main class of deteriorating items, under stochastic lead time assumption and a non-linear holding cost is considered. Vinod Kumar Mishra et al. (2013) [10] considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deteriorating is time proportional and the model considered here allows shortages and the demand is partially backlogged. Sanjai Sharma et al. (2015) [11] considered the production rate as demand dependent, which is more realistic in our general life. The time dependent rate of deterioration is taken into consideration and demand rate is price dependent. S. Mandal et al. (2015) [12] developed an optimal production inventory problem in a specific time period.

Returned policy is one of the most important challenges in the customer driven business world. By returned policy we understand a contract between the manufacturer and forward positions in the supply chain (retailers, suppliers, customers), concerning the procedure of accepting back products after having sold them, either used or in an as-good-as-new state. Customer returns of as-good-as-new products have increased dramatically in the recent years. Growth in mail-order and transactions over the Internet has increased the volume of product returns as customers are unable to see and touch the items they decide to buy, so they are more likely to return them. Several studies by Y. Arar (2008) [13] and B. C. Saskatchewan (2008) [14] draw attention to possible causes for high number of returns: in 2007, Americans returned between 11% - 20% of electronic items, which adds up to the staggering amount of $13.8 billion, out of which just 5% were actually broken. The rest failed to meet the customers' expectations. The way management handles return items plays an important role in the company's strategy to success, whatever the reason is optimization cannot be derived, accept the defective items for shake of organization branding image and loss of valued customers. They have to include the return items into their godown for reworks and then return to the customers. Manufacturing processes are sometime imperfect; for that reason, their output may contain defective items. These defective items can be reworked, scrapped, subjected to other corrective processes or sold at reduced prices, but the result is increased extensive costs in every case. In this paper we tried to reduce the total cost by developing a mathematical model on inventory system where service warranties are provided.

2. Mathematical Model

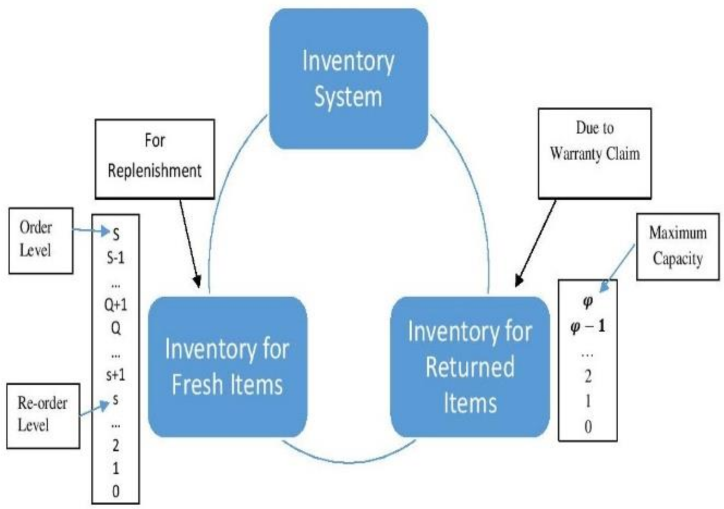


Fig-1. Stochastic Perishable Inventory Model with Reworks.

2.1 Notations

i. S→Maximum inventory level for fresh items.

ii. φ →Maximum inventory level for returned items.

iii. λ → Arrival rate of demands for fresh items.

iv. δ →Arrival rate of demands for returned items.

v. γ →Replanishment rate for fresh items.

vi. μ →Service rate for returned items.

vii. θ →Decay rate for fresh items.

viii. $I(t)$ →Inventory level at time t for fresh items.

ix. x(t) → Inventory level at time t for retuned items.

x. $E = E_1 \times E_2$ →The state space of the process.where, E_1={0, 1, 2, … , S}; E_2={0, 1, 2, … , φ }; and

xi. $e_{\varphi+1} = (1, 1,1, \dots ,1)'$; an ($\varphi$ +1)-Components column vector of 1's.

2.2 Assumptions

i. Initially the inventory level for fresh items is S and for return items is φ.

ii. Arrival rate of demands follow poisson process with parameter λ for fresh items and $\delta$for return items.

iii. Lead-time is exponentially distributed with parameter γ for fresh items.

iv. If the inventory level for returned items is at φthen home service will be provided for return items.

v. Service will be provided for the return items with exponential parameter μ.

vi. Fresh items will decay at a constant rate with parameter θ.

2.3 Model Analysis

In our model, we fixed maximum inventory level for fresh items at S and for return items at φ. The inter-arrival time between two sucessive demands are assumed to be exponentially distributed with parameter λ for fresh items and δ for return items. Each demand is for exactly one unit for each items. When inventory level reduced to s, an order for replenishment is placed. Lead-time is exponentially distributed with parameter γ. When inventory level for the return items reached at φ home service will be provided for return items.

Now, the infinitesimal generator of the two dimensional markov process $\{I(t), x(t), t \geq 0\}$ can be defined $\tilde{A} = \big(a(i,j,k,l)\big); (i,j),(k,l) \in E$

Hence, we get

$$\tilde{A}(i,j,k,l) = \begin{cases} (\lambda+\theta) & : \quad i = 1,2,3,\dots S; \quad k = i-1, \quad j = 0,1,2,\dots,\varphi, \quad l = j \\ -(\lambda+\theta+\delta+\mu) & : \quad i = s+1, s+2, \dots S; \quad k = i, \quad j = ,1,2,\dots,\varphi-1, \quad l = j \\ -(\lambda+\theta+\delta) & : \quad i = s+1, s+2, \dots S; \quad k = i, \quad j = 0, \quad l = j \\ -(\lambda+\theta+\mu) & : \quad i = s+1, s+2, \dots S; \quad k = i, \quad j = \varphi, \quad l = j \\ -(\gamma+\lambda+\theta+\delta) & : \quad i = 1,2,\dots s; \quad k = i, \quad j = 0, \quad l = j \\ -\mu & : \quad i = 0; \quad k = i, \quad j = 1,2,\dots,\varphi, \quad l = j \\ \delta & : \quad i = 0,1,2,\dots S; \quad k = i, \quad j = 0,1,2,\dots,\varphi-1, \quad l = j+1 \\ \mu & : \quad i = 0,1,2,\dots S; \quad k = i, \quad j = 1,2,\dots,\varphi, \quad l = j-1 \\ \gamma & : \quad i = 0,1,2,\dots s; \quad k = i+Q, \quad j = 0,1,2,\dots,\varphi, \quad l = j \end{cases}$$

Now, the infinitesimal generator $\tilde{A}$ can be conveniently express as a partition matrix

$\tilde{A} = (A_{ik})$, whereA_{ik} is a $(\varphi+1) \times (\varphi+1)$ sub-matrix which is given by

$$A_{ik} = \begin{cases} A_1 & if \quad k = i-1, i = s+1, s+2, \dots, S \\ A_2 & if \quad k = i, i = s+1, s+2, \dots, S \\ A_3 & if \quad k = i, i = 1,2,\dots,s \\ A_4 & if \quad k = i, i = 0 \\ A_5 & if \quad k = i-1, i = 1,2,\dots,s \\ A_6 & if \quad k = i+Q, i = 0,1,2,\dots,s \\ 0 & Otherwise \end{cases}$$

With

$A_1 = (a_{ij})_{(\varphi+1)\times(\varphi+1)} = diag((\lambda+\theta) \dots \dots (\lambda+\theta))$

$$where\ (i,j) \to (i-1,j)\ \forall\ i = (s+1),(s+2),\dots,S;\ j = j$$

$A_2 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$

$$= \begin{cases} (i,j) \to (i,j)\ \ is\ -(\lambda+\mu+\theta) & \forall \quad i = (s+1),\dots,S;\ j = \varphi \\ (i,j) \to (i,j)\ \ is\ -(\lambda+\mu+\theta+\delta) & \forall \quad i = (s+1),\dots,S;\ j = 1,2,\dots,\varphi-1 \\ (i,j) \to (i,j)\ \ is\ -(\lambda+\theta+\delta) & \forall \quad i = (s+1),\dots,S;\ j = 0 \\ (i,j) \to (i,j-1)\ \ is\ \mu & \forall \quad i = (s+1),\dots,S;\ j = 1,2,\dots,\varphi \\ (i,j) \to (i,j+1)\ \ is\ \delta & \forall \quad i = (s+1),\dots,S;\ j = 0,1,2,\dots,\varphi-1 \end{cases}$$

$A_3 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$

$$= \begin{cases} (i,j) \to (i,j)\ \ is\ -(\lambda+\mu+\theta+\gamma) & \forall \quad i = 1,2,\dots,s;\ j = \varphi \\ (i,j) \to (i,j)\ \ is\ -(\lambda+\mu+\theta+\delta+\gamma) & \forall \quad i = 1,2,\dots,s;\ j = 1,2,\dots,\varphi-1 \\ (i,j) \to (i,j)\ \ is\ -(\lambda+\theta+\delta+\gamma) & \forall \quad i = 1,2,\dots,s;\ j = 0 \\ (i,j) \to (i,j-1)\ \ is\ \ \mu & \forall \quad i = 1,2,\dots,s;\ j = 1,2,\dots,\varphi \\ (i,j) \to (i,j+1)\ \ is\ \ \delta & \forall \quad i = 1,2,\dots,s;\ j = 0,1,2,\dots,\varphi-1 \end{cases}$$

$A_4 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$

$$= \begin{cases} (i,j) \to (i,j)\ \ is\ -(\mu+\gamma) & \forall \quad i = 0;\ j = \varphi \\ (i,j) \to (i,j)\ \ is\ -(\mu+\delta+\gamma) & \forall \quad i = 0;\ j = 1,2,\dots,\varphi-1 \\ (i,j) \to (i,j)\ \ is\ -(\delta+\gamma) & \forall \quad i = 0;\ j = 0 \\ (i,j) \to (i,j-1)\ \ is\ \mu & \forall \quad i = 0;\ j = 1,2,\dots,\varphi \\ (i,j) \to (i,j+1)\ \ is\ \delta & \forall \quad i = 0;\ j = 0,1,2,\dots,\varphi-1 \end{cases}$$

$$A_5 = (a_{ij})_{(\varphi+1)\times(\varphi+1)} = diag((\lambda+\theta) \dots \dots (\lambda+\theta))\ where\ (i,j) \to (i-1,j) for\ all\ i = 1,2,\dots,s;\ j = j$$

$$A_6 = (a_{ij})_{(\varphi+1)\times(\varphi+1)} = diag(\gamma\gamma \dots \dots \gamma)\ where\ (i,j) \to (i+Q,j) for\ all\ i = 0,1,2,\dots,s;\ j = j$$

So, we can write the partioned matrx as follows:

$$\tilde{A}=\begin{cases} A_1 & for\ i=i-1; i=(s+1),(s+2),\dots,S \\ A_2 & for\ i=i; i=(s+1),(s+2),\dots,S \\ A_3 & for\ i=i;\ \ i=1,2,\dots,s \\ A_4 & for\ i=i;\ \ i=0 \\ A_5 & for\ i=i-1;\ \ i=1,2,\dots,s \\ A_6 & for\ i=i+Q;\ \ i=0,1,\dots,s \end{cases}$$

2.4 Steady State Analysis

It can be seen from the structure of matrix $\tilde{A}$ that the state space E is irreducible.Let the limiting distribution be denoted by $\pi^{(i,j)}$:

$$\pi^{(i,j)} = \underset{t\to\infty}{Lt} \Pr[I(t),N(t)=(i,j)],(i,j)\in E.$$

Let $\pi = (\pi^{(\varphi)},\pi^{(\varphi-1)},\pi^{(\varphi-2)},\dots\dots,\pi^{(2)},\pi^{(1)},\pi^{(0)})$ with $\pi^{(k)} = \left(\pi^{(k,\varphi)},\pi^{(k,\varphi-1)},\pi^{(k,\varphi-2)},\dots\dots,\pi^{(k,2)},\pi^{(k,1)},\pi^{(k,0)}\right)$,

$$\forall\ k=0,1,2,\dots\dots,S.$$

The limiting distribution exists, satisfies the following equations:

$$\pi\tilde{A}=0\ \dots \qquad (1)$$

$$and \quad \sum_{i=0}^{s}\sum_{j=0}^{\varphi}\pi^{(i,j)}=1\ \dots \qquad (2)$$

The equation (1) of the above yields the sets of equations:

$\pi^{(1)}A_5+\pi^{(0)}A_4=0$

$\pi^{(i+1)}A_5+\pi^{(i)}A_4=0 \qquad : i=0$

$\pi^{(i+1)}A_5+\pi^{(i)}A_3=0 \qquad : i=1,2,\dots\dots,s-1$

$\pi^{(i+1)}A_1+\pi^{(i)}A_3=0 \qquad : i=s$

$\pi^{(i+1)}A_1+\pi^{(i)}A_2=0 \qquad : i=s+1,s+2,\dots\dots,Q-1$

$\pi^{(i+1)}A_1+\pi^{(i)}A_2+\pi^{(i-Q)}A_6=0 \quad : i=Q,Q+1,\dots\dots,S-1$

$\pi^{(S)}A_2+\pi^{(s)}A_6=0$

The solution of the above equations (except the last one) can be conveniently express as:

$\pi^{(i)} = \pi^{(0)}\beta_i$; i=0,1,… …,S.

$$\text{Where } \beta_i = \begin{cases} I & i = 0 \\ -A_5A_4^{-1} & i = 1 \\ (-I)^{i-1}\beta_i(A_5A_4^{-1})^{i-1} & i = 1,2,\dots,s-1 \\ (-I)^{i-1}\beta_i(A_5A_4^{-1})^{i-1}(A_1A_3^{-1}) & i = s \\ (-I)^{i-1}\beta_i(A_5A_4^{-1})^{i-1}(A_1A_3^{-1})(A_2A_1^{-1})^{i-1} & i = s+1, s+2,\dots\dots,Q \\ -\beta_{i-1}(A_2A_1^{-1}) - (A_4A_1^{-1})\beta_{i+Q-1} & i = Q+1, Q+2,\dots\dots,S \end{cases}$$

To compute $\pi^{(0)}$, we can use the following equations:

$\pi^{(S)}A_2 + \pi^{(s)}A_6 = 0$ and $\sum \pi^{(k)}e_{K+1} = 1$Which yeilds respectively

$$\pi^{(0)}(\beta_S A_2 + \beta_s A_6) = 0 \quad \text{and } \pi^{(0)}(I + \sum \beta_i)e_{K+1} = 1$$

2.5 System Characteristics

(a) Mean inventory level:

(i) Mean inventory level for fresh items in the steady state, then we have

$L_1=\sum_{i=1}^{S} i \sum_{j=0}^{\varphi} \pi^{(i,j)}$

(ii) Mean inventory level for return items in the steady state, then we have

$L_2=\sum_{j=1}^{\varphi} j \sum_{i=0}^{S} \pi^{(i,j)}$

b) Re-order rate for fresh items in the steady state, then we have

$R=\lambda \sum_{j=0}^{\varphi} \pi^{(s+1,\ j)} + \theta \sum_{j=0}^{\varphi} \pi^{(s+1,\ j)}$

c) Average customer lost to the system, $CL=\lambda \sum_{j=0}^{S_2} \pi^{(0,j)}$

d) The service Rate for return items in the steady state, $S_R= \mu \sum_{j=1}^{\varphi} \sum_{i=0}^{S} \pi^{(i,j)}$

e) Expected total cost of the system, $E(TC)= c_1*L_1+c_2*L_2+c_3*R+c_4*CL+c_5*S_R$;

where, c_1= Holding cost per unit for fresh items,

c_2= Holding cost per unit for return items,

c_3= Replenishement cost per order,

c_4= Cost of customer lost for per unit,

c_5 = Service cost per unit for Reworks Item.

3. Result Analysis

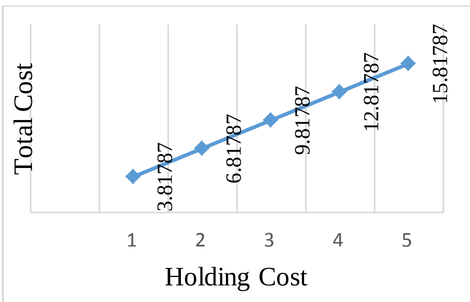
3.1 Numerical Illustrations

Putting, S=5, s=2, φ=3, Q=3, $\lambda = 0.25$, δ=0.15, $\mu = 0.20$, $\gamma = 0.30$, $\theta = 0.05$, $c_1 = 1.5$, $c_2 = 0.7$, $c_3 = 0.2$ and $c_4 = 0.1$ and c_5 = 0.15 We get

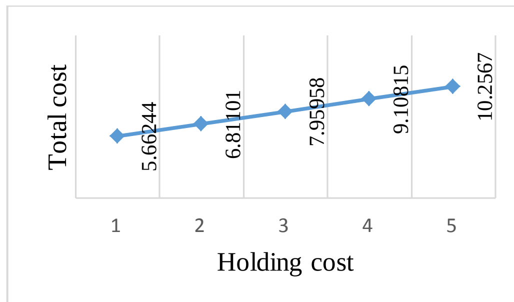
Table-1. Expected results of the characteristics of the model.

Mean inventory level for fresh items	Mean inventory level for return items	Re-order rate for fresh items	Average customer lost	Aaverage service rate	Expected total cost
3.0000000	1.1485700	0.0923075	0.0230769	0.1846150	5.352460

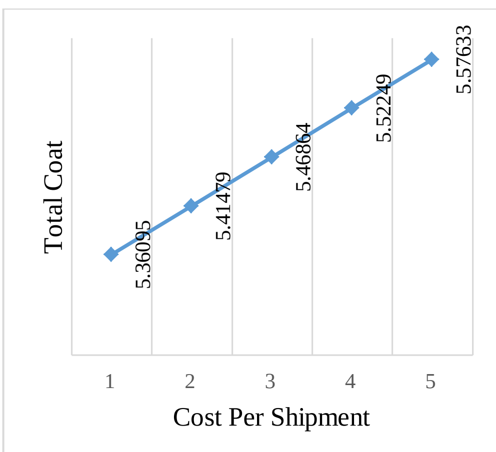
3.2 Graphical Presentation of the System



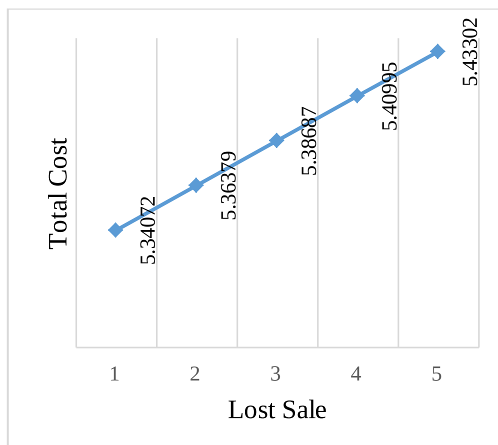
Graph-1: Total cost vs Holding cost per unit for fresh items.



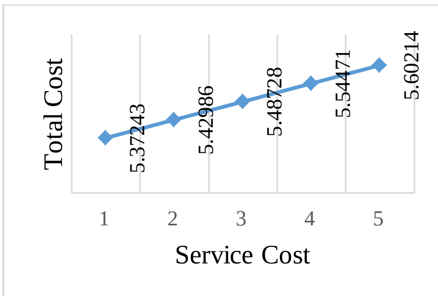
Graph-2: Total cost vs Holding cost per unit for reworks items



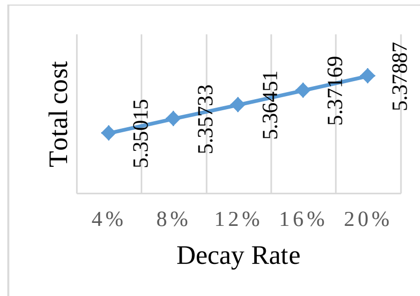
Graph-3: Total Cost vs Cost Per order.



Graph-4: Total Cost vs Lost Sale Per unit Fresh Items.



Graph-5: Total Cost vs Service Cost Per Unit For Return Items.



Graph-6: Total Cost vs Recay Rate For Fresh Items.

4. Discussion

From graphs, it is observed that all costs related to inventory system raise total cost. One unit increment of holding cost for fresh items increase about 3 units of total where the same cost increases about 1.15 units for return items. Ordering cost per order increases total cost 0.054 units. Per unit lost sale increase total cost 0.023 units, servicing cost per unit raises total cost about 0.057 unit and 1% of decay rate of fresh items increase total cost about 0.00117 unit. Though per unit service cost increase total cost more rapidly than that of lost sale, reworks facilities helps to keep inventory at lower level. It also upholds the goodwill of the organization.

5. Conclusions

There are some costs releted to inventory management system. All costs increase total cost more or less but when one unit of holding cost for fresh item increase total cost is increased rapidly than that of return items. So we should take care of holding cost for both items to minimize expected total cost of the system.

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Appendix

$\pi^{(0,0)}$ →0.0281319	$\pi^{(3,0)}$ →0.1125270
$\pi^{(0,1)}$ →0.0210989	$\pi^{(3,1)}$ →0.0843956
$\pi^{(0,2)}$ →0.0158242	$\pi^{(3,2)}$ →0.0632967
$\pi^{(0,3)}$ →0.0118681	$\pi^{(3,3)}$ →0.0474725
$\pi^{(1,0)}$ →0.0281319	$\pi^{(4,0)}$ →0.0843956
$\pi^{(1,1)}$ →0.0210989	$\pi^{(4,1)}$ →0.0632967
$\pi^{(1,2)}$ →0.0158242	$\pi^{(4,2)}$ →0.0474725
$\pi^{(1,3)}$ →0.0118681	$\pi^{(4,3)}$ →0.0356044
$\pi^{(2,0)}$ →0.0562637	$\pi^{(5,0)}$ →0.0562637
$\pi^{(2,1)}$ →0.0421978	$\pi^{(5,1)}$ →0.0421978
$\pi^{(2,2)}$ →0.0316484	$\pi^{(5,2)}$ →0.0316484
$\pi^{(2,3)}$ →0.0237363	$\pi^{(5,3)}$ →0.0237363

Manuscript received on 23 March, 2017, Revised manuscript received on 26 February, 2018 and accepted on 27 February, 2018