The Chittagong Univ. J. Sci. 40 : 137-150, 2018

An Alternative Method of Construction and Analysis of Asymmetrical Factorial Experiment of the type 6×2^2 in Blocks of Size 12

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Abstract

This paper focuses on the construction and analysis of an extra ordinary type of asymmetrical factorial experiment which corresponds to fraction of a symmetrical factorial experiment as indicated by Das (1960). For constructing this design, we have used 3 choices and for each choice we have used 5 different cases. Finding the block contents for each case we have seen that there are mainly two different cases for each choice. In case of analysis of variance, we have seen that, for the case where the highest order interaction effect is confounded in 4 replications, the loss of information is same for all the choices. Again for the case where the highest order interaction effect is confounded in 3 replications, the loss of information is also same for all the choices and one effect which is confounded due to fractionation has the same loss of information for all the choices.

Keywords: *Asymmetrical Design; Confounding; Replications; Pseudo Factors; Fractionation; and Balanced Design.*

এ গবেষণায় একটি অপ্রতিসম বহু নিদানী পরীক্ষণের নির্মাণ এবং বিশ্লেষণ নিয়ে আলোচনা করা হয়েছে যেটি দাসের (১৯৬০) দ্বারা নির্দেশিত একটি প্রতিসম বহু নিদানী পরীক্ষণের ভগ্নাংশ। এ ডিজাইন নির্মাণের জন্য আমরা তিনটি চয়েস এবং প্রতিটি চয়েসের অধীনে পাঁচটি ভিন্ন কেস ব্যবহার করেছি। প্রতিটি কেসের জন্য ব্লক আধার বের করে আমরা দেখেছি যে, প্রতিটি চয়েসের জন্য প্রধানত দুইটি ভিন্ন কেস রয়েছে। ভেদাংক বিশ্লেষণের ক্ষেত্রে আমরা দেখেছি যে, কেস যেখানে উচ্চতম মিশ্র প্রভাব চারটি পুনরীকরণে বিজরিত হয় তার তথ্য লুপ্তির পরিমাণ সকল চয়েসের জন্য একই।আবার কেস যেখানে উচ্চতম মিশ্র প্রভাব তিনটি পুনরীকরণে বিজরিত হয় তার তথ্য লুপ্তির পরিমাণ ও সকল চয়েসের জন্য একই এবং একটি প্রভাব যেটি ভগ্নাংশকরণের জন্য বিজরিত হয় তার তথ্য লুপ্তির পরিমাণ সকল চয়েসের জন্য একই।

1. Introduction

Construction and analysis of confounded factorial designs are not as straight forward as those of symmetrical factorial design. M. N. Das (1960) [1] developed a method of constructing asymmetrical factorial design by considering it as a fraction of symmetrical factorial design. In this investigation we have constructed a balanced confounded design of the type 6×2^2 in blocks of size 12 by a simple modification of the method of M. N. Das (1960) [1], we have introduced 3 choices according to the pseudo factors combinations and in every choice we have considered 5 cases. It has been seen that for the case 1, case 2, case 3 and case 4 in every choice, a mixed effect alone is confounded and the design has been balanced in 4 replications and for case 5 in every choice, one mixed effect is confounded and one real factor interaction effect is confounded due to fractionation. Here, the design has been balanced in 3 replications. Reviewing the existing literature we have seen that a lot of works was done on construction and analysis of confounded asymmetrical factorial experiments. Several authors like R. H. E. Inskon (1963) [2], K. Kishen and B. N. Tyagi (1973) [3], C. P. Kartha (1967) [4], M. G. Sardana and M. N. Das (1965) [5], P. R. Sreenath and M. G. Sardana (1967) [6], M. N. Das and P. S. Rao (1967) [7], A. K. Banarjee and M. N. Das (1969) [8], B.V. Shah (1960b) [9], M. N. Das and N. C. Giri (1979) [10] gave the method of constructing and analyzing different types of confounded asymmetrical factorial experiments.

2. Materials and Methods

6×2^2 is an asymmetrical factorial design with 3 factors, one at 6 levels and other two factors each at 2 levels. We want to construct a confounded asymmetrical design in blocks of size 12.

For constructing the asymmetrical factorial experiment of the type 6×2^2 in blocks of size 12 we have, P=6×2^2=24=total number of treatment combinations and R=12=block size.

Now, according to M. N. Das (1960) [1] we know that, $\frac{P}{R}$ = N =s^k, where s is a prime number and k is any positive integer.

Hence we have, $\frac{P}{R}$= N=$\frac{24}{12}$=2=2^1=s^k . So, the factor at 6 levels is the factor of asymmetry and is denoted by X. The other 2 factors each at 2 levels are the 2 real factors and are denoted by A and B.

Here P_i>s that is 6>2, so we can write 2^{3-1}<6<2^3.So, we have to take 3 pseudo factorsX_1, X_2 and X_3 corresponding to X.

Therefore, M= t_0 + $\sum n_i$=2+3=5, [where M=number of factors, t_0 =number of real factors, $\sum n_i$=number of pseudo factors corresponding to all the factors of asymmetry] and the corresponding symmetrical factorial experiment is s^M=2^5 and the treatment combinations of this factorial experiment can be written as usual.

Now, since we have to consider 6 treatment combinations to define 6 levels of factor of asymmetry, we have to go for $\frac{1}{4}$ fraction of 2^3 factorial experiment which can be obtained by considering any one of the following 3 defining contrasts:

(1) I= X_1= X_2= X_1X_2

(2) I= X_1 =X_3 = X_1X_3

(3) I= X_2 = X_3 = X_2X_3

From the defining contrast, we can easily understand the omitted combinations of the pseudo factors. In designs where the number of omitted combinations of the pseudo factors corresponding to a factor of asymmetry has a common multiple with the total number of combinations of the pseudo factors, different choices of the initial replication may lead to different designs.

3. Results and Discussion

Case 1: Here we consider $X_1X_2X_3$AB for confounding to get the replication in every choice.

Case 2: Here we consider X_1X_3AB for confounding to get the replication in choice1 and X_1X_2AB for choice 2 and choice 3.

Case 3: Here we consider X_2X_3AB for confounding to get the replication in choice1 and choice 2 and X_1X_3AB for choice 3.

Case 4: Here we consider X_3AB, X_2AB and X_1AB for confounding to get the replication in choice1, choice 2 and choice 3 respectively.

Case 5: Here we consider X_1X_2AB, X_1X_3AB and X_2X_3AB for confounding to get the replication in choice1, choice 2 and choice 3 respectively.

3.1 Choice 1:

For this choice we have considered defining contrast I= X_1 = X_2= X_1X_2 and for case 1,case 2,case 3,case 4 and case 5 we have confounded the higher order interaction effects $X_1X_2X_3$AB, X_1X_3AB, X_2X_3AB, X_3AB and X_1X_2AB respectively to get the replications.

For case 1 where only one effect XAB corresponding to asymmetrical type is confounded in 4 replications, following the usual method of finding the block content and after recoding we get the block content for this case which is shown in Table 3.1.

Similarly, using the usual method of block content and after recoding we get same block content as like as case 1 for case 2, case 3 and case 4.But for case 5 where one effect XAB corresponding to asymmetrical type is confounded in 3 replications and one effect AB corresponding to asymmetrical type is confounded due to fractionation we get different block content which is shown in Table 3.2.

Table 3.1:

Replication-1		Replication-2		Replication-3		Replication-4	
Block-1	Block-2	Block-1	Block-2	Block-1	Block-2	Block-1	Block-2
000	001	000	001	000	001	000	001
011	010	011	010	011	010	011	010
101	100	100	101	101	100	100	101
110	111	111	110	110	111	111	110
201	200	201	200	200	201	200	201
210	211	210	211	211	210	211	210
301	300	301	300	301	300	301	300
310	311	310	311	310	311	310	311
400	401	401	400	400	401	401	400
411	410	410	411	411	410	410	411
500	501	500	501	501	500	501	500
511	510	511	510	510	511	510	511

Table 3.2:

Replication-1		Replication-2		Replication-3	
Block-1	Block-2	Block-1	Block-2	Block-1	Block-2
000	001	000	001	000	001
011	010	011	010	011	010
101	100	100	101	101	100
110	111	111	110	110	111
201	200	201	200	200	201
210	211	210	211	211	210
300	301	300	301	300	301
311	310	311	310	311	310
401	400	400	401	401	400
410	411	411	410	410	411
501	500	501	500	500	501
510	511	510	511	511	510

In choice 1, since the same block content is obtained for case 1, case 2, case 3 and case 4 where the higher order interactions corresponding to symmetrical factorial have no common letters as a whole corresponding to pseudo factors in the defining contrast and balanced design is obtained by confounding only one effect XAB in 4 replications. Therefore, we can proceed with analysis considering any one of these 4 cases.

On the other hand in case 5, the higher order interaction corresponding to symmetrical factorial has common letters corresponding to pseudo factors in the defining contrast and for that a balanced design is obtained in 3 replications confounding XAB and AB is confounded due to fractionation.

Following the usual method of ANOVA, for the case where balanced design is obtained in 4 replications we get the loss of information for XAB is $(1 - \frac{4}{5}) = \frac{1}{5}$ and for the case where balanced design is obtained in 3 replications we get the loss of information for XAB is $(1 - \frac{5}{7}) = \frac{2}{7}$ and for AB is $(1 - \frac{8}{9}) = \frac{1}{9}$.

3.2 Choice 2:

For this choice we have considered defining contrast I= X_1= X_3= X_1X_3 and for case 1, case 2, case 3,case 4 and case 5 we have confounded the higher order interaction effects $X_1X_2X_3$AB, X_1X_2AB, X_2X_3AB, X_2AB and X_1X_3AB respectively to get the replications.

For case 1 where only one effect XAB corresponding to asymmetrical type is confounded in 4 replications, following the usual method of finding the block content and after recoding we get the block content for this case which is shown in Table 3.3.

Table 3.3:

Replication-1		Replication-2		Replication-3		Replication-4	
Block-1	Block-2	Block-1	Block-2	Block-1	Block-2	Block-1	Block-2
000	001	000	001	000	001	000	001
011	010	011	010	011	010	011	010
101	100	100	101	101	100	100	101
110	111	111	110	110	111	111	110
201	200	201	200	201	200	201	200
210	211	210	211	210	211	210	211
300	301	301	300	300	301	301	300
311	310	310	311	311	310	310	311
401	400	401	400	400	401	400	401
410	411	410	411	411	410	411	410
500	501	500	501	501	500	501	500
511	510	511	510	510	511	510	511

Similarly, using the usual method of block content and after recoding we get same block content as like as case 1 for case 2, case 3 and case 4.But for case 5 where one effect XAB corresponding to asymmetrical type is confounded in 3 replications and one effect AB corresponding to asymmetrical type is confounded due to fractionation we get different block content which is shown in Table 3.4.

In choice 2, since the same block content is obtained for case 1, case 2, case 3 and case 4 where the higher order interactions corresponding to symmetrical factorial have no common letters as a whole corresponding to pseudo factors in the defining contrast and balanced design is obtained by confounding only one effect XAB in 4 replications. Therefore, we can proceed with analysis considering any one of these 4 cases.

Table 3.4:

Replication-1		Replication-2		Replication-3	
Block-1	Block-2	Block-1	Block-2	Block-1	Block-2
000	001	000	001	000	001
011	010	011	010	011	010
101	100	100	101	101	100
110	111	111	110	110	111
200	201	200	201	200	201
211	210	211	210	211	210
301	300	300	301	301	300
310	311	311	310	310	311
401	400	401	400	400	401
410	411	410	411	411	410
501	500	501	500	500	501
510	511	510	511	511	510

On the other hand in case 5, the higher order interaction corresponding to symmetrical factorial has common letters corresponding to pseudo factors in the defining contrast and for that a balanced design is obtained in 3 replications confounding XAB and AB is confounded due to fractionation.

Following the usual method of ANOVA, for the case where balanced design is obtained in 4 replications we get the loss of information for XAB is $(1 - \frac{4}{5})=\frac{1}{5}$ and for the case where balanced design is obtained in 3 replications we get the loss of information for XAB is (1 - $\frac{5}{7}$) =$\frac{2}{7}$ and for AB is (1 - $\frac{8}{9}$) = $\frac{1}{9}$.

3.3 Choice 3:

For this choice we have considered defining contrast I= X_2= X_3= X_2X_3 and for case 1,case 2,case 3,case 4 and case 5 we have confounded the higher order interaction effects $X_1X_2X_3$AB, X_1X_2AB, X_1X_3AB, X_1AB and X_2X_3AB respectively to get the replications.

For case 1 where only one effect XAB corresponding to asymmetrical type is confounded in 4 replications, following the usual method of finding the block content and after recoding we get the block content for this case which is shown in Table 3.5.

Table 3.5:

Replication-1		Replication-2		Replication-3		Replication-4	
Block-1	Block-2	Block-1	Block-2	Block-1	Block-2	Block-1	Block-2
000	001	000	001	000	001	000	001
011	010	011	010	011	010	011	010
101	100	101	100	101	100	101	100
110	111	110	111	110	111	110	111
201	200	200	201	201	200	200	201
210	211	211	210	210	211	211	210
300	301	301	300	300	301	301	300
311	310	310	311	311	310	310	311
401	400	401	400	400	401	400	401
410	411	410	411	411	410	411	410
500	501	500	501	501	500	501	500
511	510	511	510	510	511	510	511

Similarly, using the usual method of block content and after recoding we get same block content as like as case 1 for case 2, case 3 and case 4.But for case 5 where one effect XAB corresponding to asymmetrical type is confounded in 3 replications and one effect AB corresponding to asymmetrical type is confounded due to fractionation we get different block content which is shown in Table 3.6.

Table 3.6:

Replication-1		Replication-2		Replication-3	
Block-1	Block-2	Block-1	Block-2	Block-1	Block-2
000	001	000	001	000	001
011	010	011	010	011	010
100	101	100	101	100	101
111	110	111	110	111	110
201	200	200	201	201	200
210	211	211	210	210	211
301	300	300	301	301	300
310	311	311	310	310	311
401	400	401	400	400	401
410	411	410	411	411	410
501	500	501	500	500	501
510	511	510	511	511	510

In choice 3, since the same block content is obtained for case 1, case 2, case 3 and case 4 where the higher order interactions corresponding to symmetrical factorial have no common letters as a whole corresponding to pseudo factors in the defining contrast and balanced design is obtained by confounding only one effect XAB in 4 replications. Therefore, we can proceed with analysis considering any one of these 4 cases.

On the other hand in case 5, the higher order interaction corresponding to symmetrical factorial has common letters corresponding to pseudo factors in the defining contrast and for that a balanced design is obtained in 3 replications confounding XAB and AB is confounded due to fractionation.

Following the usual method of ANOVA, for the case where balanced design is obtained in 4 replications we get the loss of information for XAB is $(1 - \frac{4}{5}) = \frac{1}{5}$ and for the case where balanced design is obtained in 3 replications we get the loss of information for XAB is $(1 - \frac{5}{7}) = \frac{2}{7}$ and for AB is $(1 - \frac{8}{9}) = \frac{1}{9}$.

4. Conclusions

Considering all the choices, we have seen that the alias structures as well as block contents in different choices are different. Here, XAB is confounded in 4 replications for the case where highest order interaction has no common letters corresponding to pseudo factors in the defining contrast and through the usual method of analysis we have observed that, the loss of information for XAB is $\frac{1}{5}$ which is same for all the choices for this case. Again, XAB is confounded in 3 replications and AB is confounded due to fractionation for the case where highest order interaction has common letters corresponding to pseudo factors in the defining contrast and by the usual method of analysis we have seen that, the loss of information for XAB is $\frac{2}{7}$ which is also same for all the choices and one effect AB is confounded due to fractionation has loss of information $\frac{1}{9}$ for all the choices. As a result we can consider any one of the choices for constructing asymmetrical factorial experiment of the type 6×2^2 in 12 plots block. Since the loss of information in case of confounded design in 4 replications is less as well as no two factor interactions is confounded due to fractionation. Therefore, considering this case we can get most efficient design of the type 6×2^2 in blocks of size 12 in 4 replications.

Acknowledgments

This study can be successfully accomplish, the first person we are deeply to thank, Dr. Rabindra Nath Shil, Ex- Professor, Department of Statistics, University of Chittagong for his guidance, advises, critics and explanation in completing this article.

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Manuscript received on 16 October, 2017, Revised manuscript received on 10 July, 2018 and accepted on 12 September, 2018

The Chittagong Univ. J. Sc. Vol. 40, 2018