EFFECT OF GRAPHICAL METHOD FOR SOLVING MATHEMATICAL PROGRAMMING PROBLEM

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Abstract: *In this paper, a computer implementation on the effect of graphical method for solving mathematical programming problem using MATLAB programming has been developed. To take any decision, for programming problems we use most modern scientific method based on computer implementation. Here it has been shown that by graphical method using MATLAB programming from all kinds of programming problem, we can determine a particular plan of action from amongst several alternatives in very short time.*

Keywords: *Mathematical programming, objective function, feasible-region, constraints, optimal solution.*

1 Introduction

Mathematical programming problem deals with the optimization (maximization/ minimization) of a function of several variables subject to a set of constraints (inequalities or equations) imposed on the values of variables. For decision making optimization plays the central role. Optimization is the synonym of the word maximization/minimization. It means choosing the best. In our time to take any decision, we use most modern scientific and methods based on computer implementations. Modern optimization theory based on computing and we can select the best alternative value of the objective function. [1].But the modern game theory, dynamic programming problem, integer programming problem also part of the optimization theory having wide range of application in modern science, economics and management. In the present work I tried to compare the solution of Mathematical programming problem by Graphical solution method and others rather than its theoretic descriptions. As we know that not like linear programming problem where multidimensional problems have a great deal of applications, non-linear programming problem mostly considered only in two variables. Therefore for non-linear programming problems we have a opportunity to plot the graph in two dimension and get a concrete graph of the solution space which will be a step ahead in its solutions. We have arranged the materials of the paper in the following way: First I discuss about Mathematical Programming (MP) problem. In second step we discuss graphical method for solving mathematical programming problem and taking different kinds of numerical examples, we try to solve them by graphical method. Finally we compare the solutions by graphical method and others. For problem so consider we use MATLAB programming to graph the constraints for obtaining feasible region. Also we plot the objective functions for determining optimum points and compare the solution thus obtained with exact solutions.

2 Mathematical Programming Problems

The general Mathematical programming (MP) problems in n-dimensional Euclidean space R^n can be stated as follows:

Maximize *f(x)*

subject to

$$g_i(x) \leq 0 \;,\quad i = 1, 2, \;.....,m \qquad (1)$$

$$h_j\,(x) = 0 \;,\quad \mathrm{j} = 1, 2, \;....., \mathrm{p} \qquad (2)$$

$$x \in \mathrm{s} \qquad (3)$$

Where x = $(x_1, x_2, ..., x_n)^T$ is the vector of unknown decision variables and $f(x),\, g(x)$ $(i = 1,\, 2,\, 3, ...m)$ $h_j(x),\;\; (j = 1,\, 2,\, ...\, p)$ are the real valued functions. The function *f(x)* is known as objective function, and inequalities

(1) equation (2) and the restriction (3) are referred to as the constraints. We have started the MP as maximization one. This has been done without any loss of generality, since a minimization problem can always be converted in to a maximization problem using the identity min *f(x)* = *-max (-f(x))* (4)

i.e, the minimization of *f(x)* is equivalent to the maximization of *(-f(x))*. The set S is normally taken as a connected subset of R^n. Here the set S is taken as the entire space R^n. The set X={$x \in$ s, g_i *(x)=0, i*=1,2, …..,m, *j*=1,2, …..,p} is known to as the feasible reason, feasible set or constraint set of the program MP and any point $x \in$ x is a feasible solution or feasible point of the program MP which satisfies all the constrains of MP. If the constraint set x is empty (i.e. x=ϕ), then there is no feasible solution; in this case the program MP is inconsistent and it was developed by [2].

A feasible point $x^{\circ} \in$ x is known as a global optimal solution to the program MP if $f(x) \leq f(x^{\circ}), x \in$ x . By [3].

3 Graphical Solution Method

The graphical (or geometrical) method for solving Mathematical Programming problem is based on a well define set of logical steps. Following this systematic procedure, the given Programming problem can be easily solved with a minimum amount of computational effort and which has been introduced by [4]. We know that simplex method is the well-studied and widely useful method for solving linear programming problem. while for the class of non-linear programming such type of universal method does not exist. Programming problems involving only two variables can easily solved graphically. As we will observe that from the characteristics of the curve we can achieve more information. We shall now several such graphical examples to illustrate more vividly the differences between linear and non-linear programming problems.

Consider the following linear programming problems

Maximize $z = 0.5x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 1$

$2x_1 + x_2 \geq 6$

$0.5x_1 - x_2 \geq -4$

$x_1 \geq 1, x_2 \geq 0.$

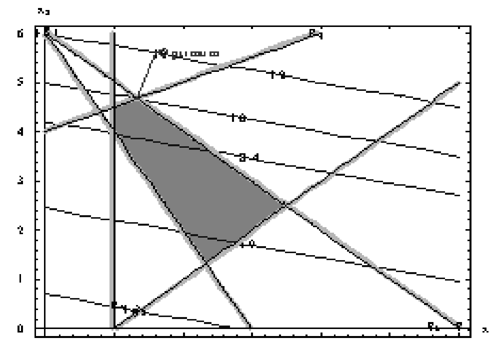

Fig.1 Optimal solution by graphical method

The graphical solution is show in Fig.1. The region of feasible solution is shaded. Note that the optimal does occur at an extreme point. In this case, the values of the variables that yield the maximum value of the objective function are unique, and are the point of intersection of the lines $x_1 + x_2 = 6,\ 0.5x_1 - x_2 = -4$ so that the optimal values of the variables $x_1{}^*$ and $x_2{}^*$ are $x_1{}^* = \frac{4}{3}$, $x_2{}^* = \frac{14}{3}$. The maximum value of the objective function is $z = 0.5 \times \frac{4}{3} + 2 \times \frac{14}{3} = 10$, which was by [5].

Now consider a non-linear programming problem, which differs from the linear programming problem only in that the objective function:

$z = 10\ (x_1 - 3.5)^2 + 20\ (x_2 - 4)^2$. (5)

Imagine that it is desired to minimize the objective function. Observe that here we have a separable objective function. The graphical solution of this problem is given in Fig.2

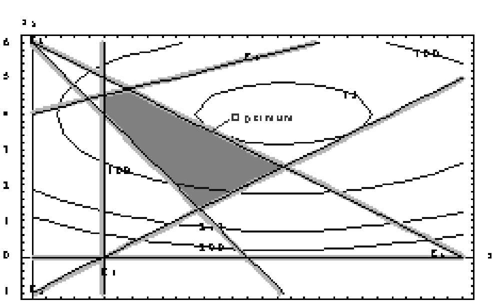

Fig. 2 Optimal solution by graphical method

The region representing the feasible solution is, of course, precisely the same as that for

the linear programming problem of Fig1. Here, however, the curves of constant z are ellipse with centers at the point (3.5, 4). The optimal solution is that point at which an ellipse is tangent to one side of the convex set. If the optimal values of the variables are x_1^* and x_2^*, and the minimum value of the objective function is z^*, then from Fig 1-2, $x_1^* + x_2^* = 6$, and $z^* = 10(x_1^* - 3.5)^2 + 20(x_2^* - 4)^2$.
Furthermore the slope of the curve $z^* = 10(x_1 - 3.5)^2 + 20(x_2 - 4)^2$ evaluated at $(x_1^*,\ x_2^*)$ must be –1 since this is the slope of $x_1 + x_2 = 6$. Thus we have the additional equation $x_2^* - 4 = 0.5(x_1^* - 3.5)$. We have obtained three equations involving x_1^*, x_2^* and z^*. The unique solution is $x_1^* = 2.50$, $x_2^* = 3.50$ and $z^*=15$. Now the point which yields the optimal value of the objective function lies on the boundary of the convex set of feasible solutions, but it is not an extreme point of this set. Consequently, any computational procedure for solving problems of this type cannot be one which examines only the extreme points of the convex set of feasible solutions. By a slight modification of the
objective function studied above the minimum value of the objective function can be made to occur at an interior point of the convex set of feasible solutions. Suppose, for example, that the objective function is -

$$z = 10(x_1 - 2)^2 + 20(x_2 - 3)^2$$

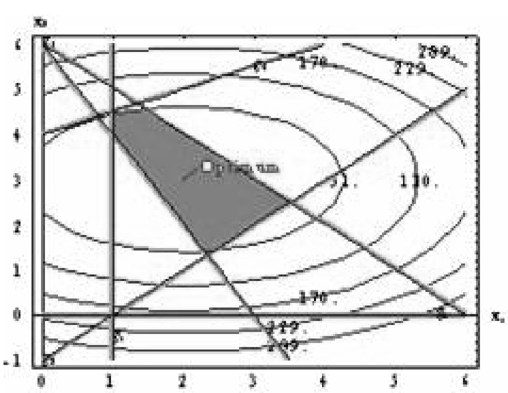

Fig. 3 Optimal solution by graphical method

and that the convex set of feasible solutions is the same as that considered above. This case is illustrated graphically in Fig.3. The optimal values of x_1, x_2, and z are $x_1^* = 2$, $x_2^* = 3$, and $z^* = 0$.Thus it is not even necessary that the optimizing point lie on the boundaries. Note that in this case, the minimum of the objective function in the presence of the constraints and non-negativity restrictions is the same as the minimum in the absence of any constraints or non-negativity restrictions. In such situations we say that the constraints and non-negativity restrictions are inactive, since the same optimum is obtained whether or not the constraints and non-negativity restrictions are included. Each of the examples presented thus far the property that a local optimum was a global optimum and was introduced by [5].
As a final example, I shall examine an integer linear programming problem.
Let us solve the problem

$$0.5x_1 + x_2 \le 1.75$$
$$x_1 + 0.30x_2 \le 1.50$$
$$x_1, x_2 \ge 0, \qquad x_1, x_2 \quad \text{intigers}$$
$$\text{Max } z = 0.25x_1 + x_2.$$

The situation is illustrated geometrically in Fig.3.4.The shaded region would be the convex set of feasible solutions in the absence of the integrality requirements. When the x_j are required to be integers, there only four feasible solutions which are represented by circles in Fig.4. If we solve the problem as a linear programming problem, ignoring the integrality

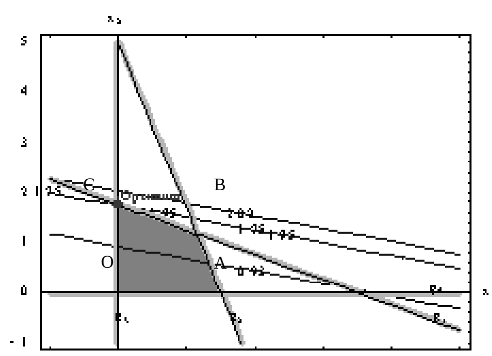

Fig. 4 Optimal solution by graphical method

requirements, the optimal solutions is $x^*_1 = 0$, $x^*_2 = 1.75$, and $z^* = 1.75$. However it is clear that when it is required that the x_j be integers, the optimal solution is $x_1^* = 1$, $x_2^* = 1$, and $z^* = 1.25$. Note that this is not the solution that would be obtained by the solving the linear programming problem and rounding the results to the nearest integers, which satisfy the constraints (this would give $x_1 = 0, x_2 = 1$). and $z = 0$.However, in the case of a NLP problem the optimal solution may or may not occur at one of the extreme points of the solution space, generated by the constraints and the objective function of the given problem.

Graphical solution algorithm: The solution NLP problem by graphical method, in general, involves the following steps:

Step 1: construct the graph of the given NLP problem.

Step 2: Identify the convex region (solution space) generated by the objective function and constraints of the given problem.

Step 3: Determine the point in the convex region at which the objective function is optimum maximum or minimum).

Step 4: Interpret the optimum solution so obtained. Which has been introduced by [2].

4 Solution of Various Kinds of Problems by Graphical Solution Method

4.1 Problem with objective function linear constraints non-Linear

$$\textit{Maximize} \quad Z = 2x_1 + 3x_2$$

Subject to the constraints

$$x_1^2 + x_2^2 \le 20$$
$$x_1 x_2 \le 8$$
$$x_1 \ge 0, \quad x_2 \ge 0.$$

Let us solve the problem by graphical method:

For this, first we are tracing the graph of the constraints of the problem considering inequalities as equations in the first quadrant (since $x_1 \ge 0, x_2 \ge 0$). We get the following shaded region as opportunity set OABCD.

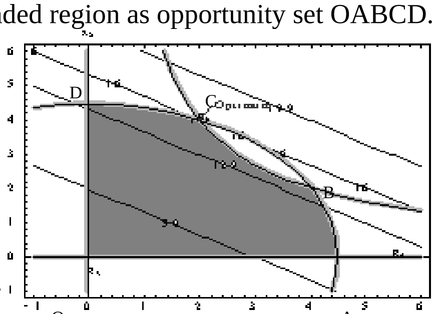

Fig. 5 Optimal solution by graphical method

The point which maximizes the value $z = 2x_1 + 3x_2$ and lies in the convex region OABCD have to find. The desired point is obtained by moving parallel to $2x_1 + 3x_2 = k$ for some k, so long as $2x_1 + 3x_2 = k$ touches the extreme boundary point of the convex region. According to this rule, we see that the point C (2, 4) gives the maximum value of Z. Hence we can find the optimal solution at this point by [6]

$$z_{Max} = 2.2 + 3.4$$
$$= 16 \quad at \quad x_1 = 2, \; x_2 = 4.$$

4.2 Problem with objective function linear constraints non-linear+linear

$$\textit{Maximize} \quad Z = x_1 + 2x_2$$
$$slt$$
$$x_1^2 + x_2^2 \le 1$$
$$2x_1 + x_2 \le 2$$
$$x_1, x_2 \ge 0.$$

Let us solve the above problem by graphical method:

For this we see that our objective function is linear and constraints are non-linear and linear. Constraints one is a circle of radius 1 with center (0, 0) and constraints two is a straight line. In this case tracing the graph of the constraints of the problem in the first quadrant, we get the following shaded region as opportunity set.

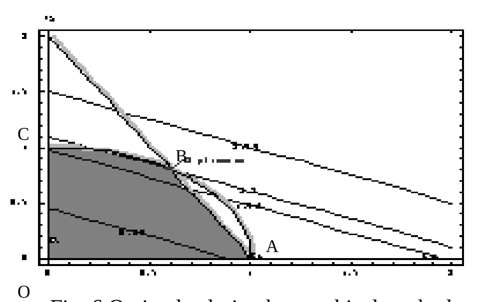

Fig. 6 Optimal solution by graphical method

Considering the inequalities to equalities

$$x_1^2 + x_2^2 = 1 \quad (6)$$
$$2x_1 + x_2 = 2 \quad (7)$$

Solving (6) *and* (7)

We get $(x_1, x_2) = (1, 0), \left(\frac{3}{5}, \frac{4}{5}\right)$

The extreme points of the convex region are O(0, 0), A (1,0) B$(3/5, 4/5)$ and C(0,1).

By moving according to the above rule we see that the line $x_1 + 2x_2 = k$ touches $(3/5,\ 4/5)$ the extreme point of the convex region. Hence the required solution of the given problem is

$$Z_{Max} = \frac{3}{5} + 2 \cdot \frac{4}{5} = \frac{3}{5} + \frac{8}{5} = \frac{11}{5} = 2.2$$

$$at \quad x_1 = \frac{3}{5},\ x_2 = \frac{4}{5}.$$

4.3 Problem with objective function non-linear constraints linear

$$Minimize\ Z = x_1^{\ 2} + x_2^{\ 2}$$

subject to the constraints:

$$x_1 + x_2 \geq 4$$
$$2x_1 + x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

Our objective function is non-linear which is a circle with origin as center and constraints are linear. The problem of minimizing $Z = x_1^{\ 2} + x_2^{\ 2}$ is equivalent to minimizing the radius of a circle with origin as centre such that it touches the convex region bounded by the given constraints. First we contracts the graph of the constraints by MATLAB programming [9] in the 1st quadrant since $x_1 \geq 0, x_2 \geq 0$.

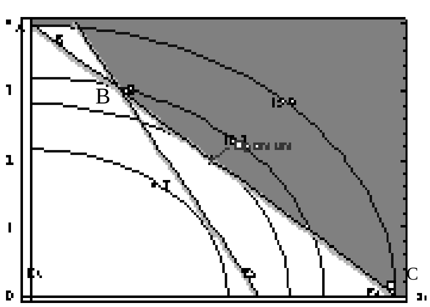

Fig. 7 Optimal solution by graphical method

Since $x_1 + x_2 \geq 4$ *and* $2x_1 + x_2 \geq 5$, the desire point must be some where in the unbounded convex region ABC. The desire point will be that point of the region at which a side of the convex region is tangent to the circle. Differentiating the equation of the circle

$$2x_1 dx_1 + 2x_2 dx_2 = 0$$

$$\Rightarrow \frac{dx_2}{dx_1} = -\frac{x_1}{x_2} \qquad (8)$$

Considering the inequalities to equalities

$$2x_1 + x_2 = 5 \ \ and \ \ x_1 + x_2 = 4$$

Differentiating, we get

$$2dx_1 + dx_2 = 0 \ \ and \ \ dx_1 + dx_2 = 0$$

$$\Rightarrow \frac{dx_2}{dx_1} = -2 \qquad \frac{dx_2}{dx_1} = -1 \qquad (9)$$

Now, from (8) and (9) we get

$$\frac{-x_1}{x_2} = -2 \Rightarrow x_1 = 2x_2$$

$$and \quad \frac{-x_1}{x_2} = -1 \Rightarrow x_1 = x_2 .$$

This shows that the circle has a tangent to it-

(i) the line $x_1 + x_2 = 4$ at the point (2,2)

(ii) the line $2x_1 + x_2 = 5$ at the point (2,1).

But from the graph we see that the point (2,1) does not lie in the convex region and hence is to be discarded. Thus our require point is (2,2).

$$\therefore Minimum\ \ Z = 2^2 + 2^2 = 8 \text{ at the point} (2,2).$$

5 Comparison of Solution by Graphical Method and Others

Let us consider the problem

Maximize $Z = 2x_1 + 3x_2 - x_1^{\ 2}$

Subject to the constraints:

$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

First I want to solve above problem by graphical solution method.

The given problem can be rewriting as:

Maximize $Z = -(x_1 - 1)^2 + 3\left(x_2 + \frac{1}{3}\right)$

Subject to the constraints

$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

We observe that our objective function is a parabola with vertex at (1, -1/3) and constraints are linear. To solve the problem graphically, first we construct the graph of the constraint in the first quadrant since $x_1 \geq 0$ and $x_2 \geq 0$ by considering the inequation to equation.

Here we contract the graph of our problem by MATLAB programming [9] According to our previous graphical method our desire point is at

(1/4, 15/8)

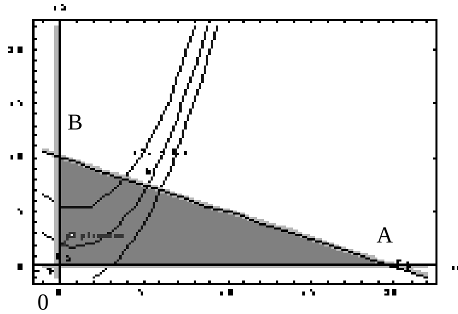

Fig. 8 Optimum solution by graphical method

Hence we get the maximum value of the objective function at this point. Therefore,

$$Z_{max} = 2x_1 + 3x_2 - x_1^2$$

$$= \frac{97}{16} \quad at \;\; x_1 = \frac{1}{4}\;,\; x_2 = \frac{15}{8}.$$

Let us solve the above problem by using [7] Kuhn-Tucker Conditions. The Lagrangian function of the given problem is

$$F(x_1, x_2, \lambda) \equiv 2x_1 + 3x_2 - x_1^2 + \lambda(4 - x_1 - 2x_2).$$

By Kuhn-Tucker conditions, we obtain

$(a)\;\; \frac{\partial F}{\partial x_1} \equiv 2 - 2x_1 - \lambda \le 0, \quad \frac{\partial F}{\partial x_2} \equiv 3 - 2\lambda \le 0$

$(b)\;\; \frac{\partial F}{\partial \lambda} \equiv 4 - x_1 - 2x_2 \ge 0$

$(c)\; x_1 \frac{\partial F}{\partial x_1} + x_2 \frac{\partial F}{\partial x_2} \equiv x_1(2 - 2x_1 - \lambda) + x_2(3 - 2\lambda) = 0$

$(d)\;\; \lambda \frac{\partial F}{\partial \lambda} \equiv \lambda(4 - x_1 - 2x_2) = 0 \;\;$ with $\lambda \ge 0$.

Now there arise the following cases:

Case (i) : Let $\lambda = 0$, in this case we get from

$\frac{\partial F}{\partial x_1} \equiv 2 - 2x_1 \le 0$ and $\frac{\partial F}{\partial x_2} \equiv 3 - 2 \cdot 0 \le 0$

$\Rightarrow 3 \le 0$ which is impossible and this solution is to be discarded and it has been introduced by [12].**Case (ii):** Let $\lambda \ne 0$. In this case we get

from $\lambda(4 - x_1 - 2x_2) = 0$

$$4 - x_1 - 2x_2 = 0 \Rightarrow x_1 + 2x_2 = 4 \qquad (10)$$

Also from $\frac{\partial F}{\partial x_1} \equiv 2 - 2x_1 - \lambda \le 0$

$\frac{\partial F}{\partial x_2} \equiv 3 - 2\lambda \le 0 \quad \therefore 2x_1 + \lambda - 2 \ge 0$

and $\;\; 2\lambda - 3 \ge 0 \Rightarrow \lambda \ge \frac{3}{2}$

If we take $\lambda = \frac{3}{2}$, then $\;2x_1 \ge \frac{1}{2}$

If we consider $\;2x_1 = \frac{1}{2}\;$ then $\;x_1 = \frac{1}{4}$.

Now putting the value of x_1 *in* (10), we get $\;x_2 = \frac{15}{8}$

$\therefore (x_1, x_2, \lambda) = \left(\frac{1}{4}, \frac{15}{8}, \frac{3}{2}\right)$. for this solution

$\frac{\partial F}{\partial x_1} \equiv 2 - 2 \cdot \frac{1}{4} - \frac{3}{2} = \frac{4 - 1 - 3}{2} = 0 \;\;$ satisfied

$\frac{\partial F}{\partial x_2} \equiv 3 - 2 \cdot \frac{3}{2} = 0 \;\;$ satisfied

$\frac{\partial F}{\partial \lambda} \equiv 4 - \frac{1}{4} - 2 \cdot \frac{15}{8} = \frac{16 - 1 - 15}{4} = 0 \;\;$ satisfied

$x_1 \frac{\partial F}{\partial x_1} + x_2 \frac{\partial F}{\partial x_2} \equiv \frac{1}{4} \times 0 + \frac{15}{8} \times 0 = 0 \;\;$ satisfied

$\lambda \frac{\partial F}{\partial \lambda} \equiv \frac{3}{2} \cdot 0 = 0 \;\;$ satisfied

Thus all the Kuhn-Tucker necessary conditions are satisfied at the point (1/4, 15/8)

Hence the optimum (maximum) solution to the given NLP problem is

$$Z_{max} = 2x_1 + 3x_2 - x_1^2$$

$$= \frac{97}{16} \quad \text{at } x_1 = \frac{1}{4},\; x_2 = \frac{15}{8}.$$

Let us solve the problem by Beale's method.

Maximize $\;f(x) = 2x_1 + 3x_2 - x_1^2$

Subject to the constraints:

$$x_1 + 2x_2 \le 4$$

$$x_1, x_2 \ge 0$$

Introducing a slack variables s, the constraint becomes

$$x_1 + 2x_2 + s = 4$$

$$x_1, x_2 \ge 0$$

since there is only one constraint, let s be a basic variable. Thus we have by [13]

$x_B = (s),\; x_{NB} = (x_1, x_2) \;\;$ with $s = 4$

Expressing the basic x_B and the objective function in terms of non-basic x_{NB}, we have $s=4-x_1-2x_2$ and $f = 2x_1+3x_2-x_1^2$. We evaluated the partial derivatives of f w.r.to non-basic variables at $x_{NB}=0$, we get

$$\left(\frac{\partial f}{\partial x_1}\right)_{x_{NB}=0} = (2 - 2x_1)_{x_{NB}=0} = 2 - 2 \cdot 0 = 2$$

$$\left(\frac{\partial f}{\partial x_2}\right)_{x_{NB}=0} = 3$$

since both the partial derivatives are positive, the current solution can be improved. As

$\frac{\partial f}{\partial x_2}$ gives the most positive value, x_2 will enter the basis. Now, to determine the leaving basic variable, we compute the ratios:

$$\min\left\{\frac{\alpha_{ho}}{|\alpha_{hk}|}, \frac{\gamma_{ko}}{\gamma_{kk}}\right\} = \min\left\{\frac{\alpha_{30}}{|\alpha_{32}|}, \frac{\gamma_{20}}{|\alpha_{22}|}\right\}$$

$$= \min\left\{\frac{4}{|-2|}, \frac{3}{0}\right\} = 2$$

since the minimum occurs for $\frac{\alpha_{30}}{|\alpha_{30}|}$, s will leave the basis and it was introduced by [8]. Thus expressing the new basic variable, x_2 as well as the objective function f in terms of the new non-basic variables (x_1 and s) we have:

$$x_2 = 2 - \frac{x_1}{2} - \frac{s}{2}$$

$$\text{and} \quad f = 2x_1 + 3\left(2 - \frac{x_1}{2} - \frac{s}{2}\right) - x_1^2$$

$$= 6 + \frac{x_1}{2} - \frac{3}{2}s - x_1^2$$

we, again, evaluate the partial derivates of f w. r. to the non-basic variables:

$$\left(\frac{\partial f}{\partial x_1}\right)_{x_{NB}=0} = \left(\frac{1}{2} - 2x_1\right)_{x_1=0} = \frac{1}{2}$$

$$\left(\frac{\partial f}{\partial s}\right)_{x_{NB}=0} = -\frac{3}{2}.$$

since the partial derivatives are not all negative, the current solution is not optimal, clearly, x_1 will enter the basis. For the next Criterion, we compute the ratios

$$\min\left\{\frac{\alpha_{20}}{|\alpha_{21}|}, \frac{\gamma_{10}}{|\gamma_{11}|}\right\} = \left\{\frac{2}{|-1/2|}, \frac{1/2}{|-2|}\right\} = \frac{3}{4}.$$

since the minimum of these ratios correspond to $\frac{|\gamma_{10}|}{|\gamma_{11}|}$, non-basic variables can be removed. Thus we introduce a free variable, u_1 as an additional non-basic variable, defined by

$$u_1 = \frac{1}{2}\frac{\partial f}{\partial x_1} = \frac{1}{2}\left(\frac{1}{2} - 2x_1\right) = \frac{1}{4} - x_1$$

Note that now the basis has two basic variables x_2 and x_1 (just entered). That is, we have

$$x_{NB} = (s, u_1) \text{ and } x_B = (x_1, x_2).$$

Expressing the basic x_B in terms of non-basic x_{NB}, we have, $x_1 = \frac{1}{4} - u_1$

$$\text{and} \quad x_2 = \frac{1}{2}(4 - x_1 - x_3) = \frac{15}{8} + \frac{1}{2}u_1 - \frac{1}{2}s.$$

The objective function, expressing in terms of x_{NB} is,

$$f = 2\left(\frac{1}{4} - u_1\right) + 3\left(\frac{15}{8} + \frac{1}{2}u_1 - \frac{1}{2}s\right) - \left(\frac{1}{4} - u_1\right)^2$$

$$= \frac{97}{16} - \frac{3}{2}s - u_1^2.$$

Now, $$\left(\frac{\partial F}{\partial s}\right)_{\substack{x_{NB}=0\\ u=0}} = -\frac{3}{2};$$

$$\left(\frac{\delta f}{\delta u_1}\right)_{\substack{x_{NB}=0\\ u=0}} = -2u_1 = 0$$

since $\frac{\partial f}{\partial u_1} \le 0$ for all x_j in x_{NB} and $\frac{\partial f}{\partial u} = 0$, the current solution is optimal. Hence the optimal basic feasible solution to the given problem is:

$$x_1 = \frac{1}{4}, \quad x_2 \frac{15}{8}, \quad Z^* = \frac{97}{16}$$

Similarly we can find that by Wolfe's algorithm the optimal point is at (1/4, 15/8). which was introduced by [14].

Thus for the optimal solution for the given QP problem is

$$\text{Max } Z = 2x_1 + 3x_2 - x_1^2$$

$$= 2\cdot\frac{1}{4} + 3\cdot\frac{15}{8} - \left(\frac{1}{4}\right)^2$$

$$= \frac{97}{16} \quad \text{at} \quad (x_1^*, x_2^*) = \left(\frac{1}{4}, \frac{15}{8}\right)$$

Therefore the solution obtained by graphical solution method, Kuhn-Tucker conditions, Beale's method and Wolf's algorithm are same. The computational cost is that by the graphical solution method using MATLAB Programming it will take very short time to determine the plan of action and the solution obtained by graphical method is more effective than any other methods we considered.

6 Conclusion

This paper has been presented a direct, fast and accurate way for determining an optimum schedule (such as maximizing profit or minimizing cost) The graphical method gives a physical picture of certain geometrical characteristics of programming problems. By using MATLAB programming graphical solution can help us to take any decision or determining a particular plan of action from amongst several alternatives in very short moment. All kinds of programming problem can be solved by graphical method. The limitation is that programming involving more than two variables i.e for 3-D problems can not be solved by this method. Non-linear programming problem mostly considered only in two variables. Therefore, from the above discussion, we can say that graphical method is the best to take any decision for modern game theory, dynamic programming problem science, economics, and management from amongst several alternatives.

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