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A Proposed Technique for Solving Linear Fractional Bounded Variable Problems

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Abstract

In this paper, a new method is proposed for solving the problem in which the objective function is a linear fractional Bounded Variable (LFBV) function, where the constraints functions are in the form of linear inequalities and the variables are bounded. The proposed method mainly based upon the primal dual simplex algorithm. The Linear Programming Bounded Variables (LPBV) algorithm is extended to solve Linear Fractional Bounded Variables (LFBV).The advantages of LFBV algorithm are simplicity of implementation and less computational effort. We also compare our result with programming language MATHEMATICA.

Key words: Bounded Variable, Lower & Upper Bound, Linear Fractional, Pseudo Code, Computer Algebra.

I. Introduction

Linear fractional programming (LFP) problems (i.e. ratio objective that have numerator and denominator) have attracted considerable research and interests, since they are useful in production management, financial and corporate planning, health care and hospital planning.

The field of LFP, largely developed by Hungarian mathematician B. Martors and his associates in the 1960's, is concerned with problem of optimization. Several methods to solve this problem are proposed. In (1962), Charnes and Cooper[12] have proposed their method that depends on transforming the LFP to equivalent linear programs (LP). Another method called up dated objective function method derived from Bitran and Novaes (1973) is used to solve the LFP by solving a sequence of linear programs only re-computing the local gradient of the objective function. Also some aspects concerning duality and sensitivity analysis in LFP was discussed by Bitran and Magnant I (1976). Singh. C. (1981) in his paper made a useful study about the optimality condition in LFP. Swarup[13] extended the usual simplex method of Dantzig[10,11] for solving LFP problems. The above mentioned articles deal with variables of the type ≥0. But when considering real-world applications of LFP, it may occur that one or more unknown variables x_j not only have a non- negativity constraints but are constrained by upper-bound constraints. Also they may have lower-bound constraints. In this case all of the methods described above may fail. Beside this we face a large number of LFBV in our real life. For solving such problems a method is discussed in ERIK B. BAJALINOV [1]. But this method is laborious. This reference also says, 'Obviously, because of the increased size of the problem obtained, the approach is undesirable computationally '. For this reason we try to find another procedure which takes less computational effort. So, in this paper, we propose a new method for solving LFBV problems.

The proposed method depends mainly on solving LFBV problems in which the one or more of the variables are bounded. We use the concept of LP bounded variable (LPBV) method to solve this problem.

The rest of the paper is organized as follows. In Section II, we discuss on Glossary background of LPBV &LFBV. In Section III and IV, we briefly discuss on the existing LPBV and LFBV methods respectively. In Section V, we propose our alg1orithm and in Section VI another algorithm and its computer code is given to calculate the result within a short time. Also, in Section VII, we compare the methods considered in this research. In Section VIII, we give a conclusion remarks about the proposed method and the implementation code.

II. Preliminaries

In this section, we briefly discuss some definitions of LP, LPBV and LFBV. Problems of LFP arise when there appears a necessity to optimize the efficiency of some activity. An application of LFP to solving real-world problems connected with optimizing efficiency could be as useful as in the case of LP.

Standard Form of LP

An LP problem may be defined as the problem of maximizing or minimizing.

The standard LP problem can be expressed in a compact form as:

$$\text{Maximize (Minimize):} \quad Z = cx \qquad (1.1)$$
$$\text{subject to} \quad A\,x = b \qquad (1.2)$$
$$x \geq 0 \qquad (1.3)$$
$$b \geq 0 \qquad (1.4)$$

Where A = $(a_1, a_2, \dots \dots, a_m, a_{m+1}, \dots a_n)$ is a $m \times n$ matrix, $b \in \Re^m$, x, $c \in \Re^n$, x is a $(n \times 1)$ column vector, and c is a $(1 \times n)$ row vector.

Bounded variable LP

In LP models, variables may have explicit positive upper and lower bounds. For example, in production facilities lower and upper bounds can represent the minimum and maximum demands for certain products.

Define the upper bounded LP models as

Maximize z ={CX | (A,I)X=b,0≤ X ≤ U}

The bounded algorithm uses only the constraints (A,I)X=b, X ≥ 0 explicitly, while accounting for X ≤ U implicitly through modification of the simplex feasibility condition.

Standard Form of LFP

An LFP is said to be standard form if all constraints (2.2) are equations and all x ≥ 0 i.e

$$\text{Maximize} \quad Z = \frac{ax+\alpha}{dx+\beta}$$

$$\text{subject to} \quad Ax = b, \quad x \geq 0$$

Where A = $(a_1, a_2, \dots \dots, a_m, a_{m+1}, \dots a_n)$ is a $m \times n$ matrix, $b \in \Re^m$, x, c, $d \in \Re^n, \alpha, \beta \in \Re$.

It is assumed that the feasible region

$S = \{x \in \Re^n : Ax \leq b, x \geq 0\}$

is nonempty and bounded and the denominator $dx + \beta \neq 0$.

Let us consider a standard LFP defined as,

$$\text{(LFP)} \quad \text{Maximize } F(x) = \frac{c^T x + \alpha}{d^T + \beta} \quad (2.1)$$

$$\text{subject to} \quad Ax=b \quad (2.2)$$

$$x \geq 0 \quad (2.3)$$

Where x, c, d $\in \mathbb{R}^n$; b$\in \mathbb{R}^m$; α & $\beta \in R$; A is an m×n matrix and the superscript T denotes transpose.

Bounded variable LFBV

Consider the following LFBV problems

$$\max : Q(x) = \frac{p(x)}{D(x)} = \frac{\sum_{j=1}^{n} p_j x_j + p_0}{\sum_{j=1}^{n} d_j x_j + d_0} \quad (2.4)$$

subject to

$$\sum_{j=1}^{m} a_{ij} x_j = b_i, i = 1,2,3,\dots\dots\dots\dots, m \quad (2.5)$$

$$l_j \leq x_j \leq u_j, \ j = 1,2, \dots\dots\dots\dots\dots, n \quad (2.6)$$

Where $l_j \leq u_j, \ j = 1,2, \dots\dots\dots\dots\dots, n$ (2.7)

Let us assume that D(x)>0 for all $x = (x_1, x_2, \dots \dots, x_m, x_{m+1}, \dots x_n)T \in$ S, S denotes a feasible set defined by the constraints (2.5) and (2.6). We assume also that the feasible set S is non-empty and bounded.

Lower and Upper –bound

It may occur that one or more unknown variables x_j not only have a non- negativity constraints but are constrained by upper-bound constraints i.e.

$l_j \leq x_j \leq u_j$, for some j$\in$ J= {1,2,................,n}

Since constraints of this form provide lower and upper bounds on variables, u_j are usually called upper-bound and l_j are usually called lower-bound of the constraints.

III. Existing Method on LPBV Problems

In this section, we briefly discuss LPBV. For this, we first discuss the general a LP problems.

Consider the following LP problem,

Maximize Z=CX

subject to (A,I)X=b

L≤ $X \leq U$

Where U=$\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n+m} \end{pmatrix}$ and L=$\begin{pmatrix} l_1 \\ l_2 \\ \vdots \\ l_{n+m} \end{pmatrix}$, U$\geq L \geq 0$

The elements of L and U for an unbounded variable are 0 and ∞.The problem can be solved by the regular simplex method, in which case the constraints are put in the form

$$(A,I)X=b$$

$$X+X'=U,$$

$$X-X''=L, X, X', X'' \geq 0$$

Where X' and X'' are slack and surplus variables. This problem includes 3(m+n) variables and 3m+2n constraints equations. However, the size can be reduced considerably through the of special techniques that ultimately reduce the constraints to the set (A,I)X=b.

Consider first lower bound constraints and we get $C = L + X''$.

which, eliminate X from all remaining constraints. The new variables of the problem thus become X' and X''. There is no fear in this case that X may violate the non-negativity constraints, since both L and X'' are non negative. The real difficulty occurs with the upper bounded variables.

Rather than include the constraints X+X''=U in the simplex tableau, one can account for their effect by modifying the feasibility condition for their effect by modifying the feasibility condition of their simplex method.

In developing the new feasibility condition, two main points must be considered:

1. The non negativity and upper-bound constraints for the entering variable.
2. The non negativity and upper bound constraints for those basic variables that may be affected by introducing the entering variables.

The method is discussed in detail in reference 4 & 6.

IV. Existing Method on LFBV Problems

In this section, we briefly discuss the existing method of solving LFBV and also include a numerical example.

The given vector $X = \{x_{1,} x_2, \ldots\ldots\ldots, x_n\}$ is a basic feasible solution (BFS) of LFP problem (2.4)-(2.6) if vector x satisfies system satisfies the system

$$\sum_{j \in j_B} A_j X_j = b$$

$$l_j < x_j < U_j, \forall j \ni J_B$$

$$l_j = x_j or$$

$$U_j = x_j \forall j \ni J_B$$

Similarly, for the ordinary simplex method we have to

introduce $\Delta'_j = \sum_{i=1}^{m} P_{si} X_{ij} - P_j$

$$\Delta''_j = \sum_{i=1}^{m} d_{si} X_{ij} - d_j$$

$\Delta_j(x) = \Delta'_j - Q(x)\Delta''_j$ for j=1,2,……………..,n

Where the coefficients x_{ij} are given from the following systems of linear equations

$$A_j = \sum_{i=1}^{m} A_{si} X_{ij}, j = 1,2,3, \ldots\ldots\ldots\ldots n$$

Suppose that we have some basis B and corresponding to it BFS vector x with the following index partitioning

$$l_j < x_j < U_j, \forall j \ni J_B = \{s_1, s_2, \ldots\ldots\ldots s_m\},$$

$$l_j = x_j \forall j \in j_N^l j_n,$$

$$U_j = x_j \forall j \ni J_N^u j_n,$$

Where $J_N^l \cup J_N^u = j_n$

and $j_n \cup j_n = j = \{1,2,3,4, \ldots\ldots\ldots n\}$

Assume that the vector x is not optimal and $\Delta_k(x) < 0, k \in j_n$. In accordance with the general scheme of the ordinary simplex method, it means we have to enter vector A_k into the basis and perform the simplex iteration.

The main difference between the ordinary simplex method with bounded variables is that when updating BFS we have to use the following rule.

$$X_\gamma(\theta) = \begin{cases} X_{si} - \theta . x_{ij}, \gamma = s_i, i = 1,2, \ldots\ldots m \\ X_\gamma + \theta, \gamma \in J_N, \gamma = K; \\ X_\gamma if, \gamma \in J_N, \gamma \neq K \end{cases} \quad \ldots\ldots\ldots(*)$$

$$l_{si} \le x_{si} < U_{si}, i = 1,2, \ldots\ldots\ldots\ldots\ldots, m.$$

$$l_k \le x_k(\theta) < U_k, k = 1,2, \ldots\ldots\ldots\ldots, m.$$

It is obvious that the latest may be rewritten as follows:

$\theta \le \frac{x_{si} - l_{si}}{x_{ij}}$, for those index i that $x_{ij} > 0$.

$\theta \ge \frac{x_{si} - l_{si}}{x_{ij}}$, for those index i that $x_{ij} < 0$.

$\theta \le \frac{x_{si} - u_{si}}{x_{ij}}$, for those index i that $x_{ij} < 0$.

$\theta \ge \frac{x_{si} - u_{si}}{x_{ij}}$, for those index i that $x_{ij} > 0$.

$$\theta \le u_k - x_k$$

$$\theta \ge l_k - x_k$$

From the latter we obtain $\theta_{\min} \le \theta_{\max}$.

$\theta_{\min} = \max\{l_k - x_{k,} \max_{x_{ij}<0} \frac{x_{si} - l_{si}}{x_{ij}}, \max_{x_{ij}>0} \frac{x_{si} - u_{si}}{x_{ij}}\}$ and

$\theta_{\max} = \min\{u_k - x_{k,} \min_{x_{ij}>0} \frac{x_{si} - l_{si}}{x_{ij}}, \min_{x_{ij}<0} \frac{x_{si} - u_{si}}{x_{ij}}\}$.

Note that in the non-degenerate case $\theta_{\max} > 0$ and $\theta_{\min} < 0$ (Bajalinov[1]).

Numerical Example 1

Consider the following linear fractional bounded program (BAJALINOV[1])

$$Maximize = \frac{5x_1 + x_2 + 10}{4x_1 + 2x_2 + 11}$$

subject to:

$$5x_1 + x_2 + x_3 = 20$$

$$4x_1 - x_2 + x_4 = 14$$

$$2 \le x_1 \le 5,\ 4 \le x_2 \le 12,\ 0 \le x_3 \le 25,\ 0 \le x_4 \le 18$$

Solution

$$x_1 = 16/5, x_2 = 4, x_3 = 0, x_4 = 6/5.$$

V. Proposed Algorithm to Solve LFBV Problems

In this Section, we propose an algorithm on LFBV and also include a number of numerical examples to clarification of the procedure.

Step 1: If the R.H.S. of any constraints is negative make it positive by multiplying the constraint by -1.

Step 2: Then convert the (LFBV), (Maximize) to its standard form by inserting slack and surplus variables to the constraints. If the constraint set is in a canonical form, go to step 3. If the constraint set is not in canonical form go to Step 9.

Step 3: If any variable is at positive lower bound, it should be substituted at its lower bound.

Step 4: Now, one has to compute z1, z2, relative profit factor (cj-zj1), relative cost factor (dj-zj2) and the ratio Δj,

Where, z1= cB xB + α

z2=dB xB + β

zj1=cB aj

zj2 = dB aj

and Δj = z2(cj-zj1) - z1(dj-zj2)

Step 5: For maximization problem if Δj ≤o for all non-basic variables at their upper bound optimum basic feasible solution is attained. If, not go to step-6.

*Step 6***:** Select the most positive Δj.

Step 7: Let x_j be the non basic variable at zero level which is selected to enter the solutions. Compute the quantities

$$\theta_1 = \min_i \left\{ \frac{(x_B)_i^*}{\alpha_i^j}, \alpha_i^j > 0 \right\} or$$

$$\theta_1 = \min_i \left\{ \frac{(x_B)_i^*}{\alpha_i^j}, (x_B^*)_i < 0 \,\&\, \alpha_i^j < 0 \right\}$$

$$\theta_2 = \min_i \left\{ \frac{(U_B)_i^* - (x_B^*)_i}{-\alpha_i^j}, \alpha_i^j < 0 \right\}$$

$$\theta = \min\{\theta_1, \theta_2, U_j\}$$

Here U_j is the upper bound for the variable x_j.

Step 8: Set $(x_B)_r$ be the variable corresponding to $\theta = \min\{\theta_1, \theta_2, U_j\}$ and follows the following Sub Step.

Sub Step (a): If $\theta = \theta_1$, $(x_B)_r$ leaves the solution and x_j enters by the using the regular row operations of the simplex method.

Sub Step (b): If $\theta = \theta_2$, $(x_B)_r$ leaves the solution and x_j enters by the using the regular row operations of the simplex method.

Sub Step (c): If $\theta = x_j$, x_j is substituted at its upper bound by $U_j - x'_j$ but remains non-basic.

Step 9: If the constraint set is not set is not in a canonical form. Then follow the following sub step.

Sub Step1: Introduce artificial variables wherever it required.

Sub Step 2: If any variable is at positive lower bound, it should be substituted at its lower bound.

Sub Step 3: Then write it an artificial linear objective function as in Minimization type (Minimize: w1+w2+.....). In Phase I solve the problem as a linear program.

Sub Step 4: Compute relative profit factor (cj-zj1).

Sub Step 5: For minimization problem if cj-zj1≥ 0 for all non-basic variables.

Sub Step 6: When it is feasible and then final basic variables of (LP) will be used to solve LFBV using the original problem using Step 4 to Step 8.

Solution Using Proposed Algorithm

Since x_1 and x_2 has positive lower bound so we substituted at its lower bound.

Let, $x_1 = 2 + y_1$, *then* $0 \le y_1 \le 3$

and $x_2 = 4 + y_2, \text{ then } 0 \le y_2 \le 8$

Now substituted these value the given problem becomes

$$Maximize = \frac{5y_1 + y_2 + 24}{4y_1 + 2y_2 + 24}$$

subject to

$$5y_1 + y_2 + x_3 = 6 \quad 4y_1 - y_2 + x_4 = 6$$

$$0 \le y_1 \le 3,\ 0 \le y_2 \le 8,\ 0 \le x_3 \le 25,\ 0 \le x_4 \le 18$$

Table. 1. Initial Table for LFBV

cB ↓	dB ↓	Cj →	⑤	1	0	0
		Dj →	4	2	0	0
		xBi →	y1	y2	x3	x4
1	2	y2=6	5	1	1	0
0	0	x4=6	4	0	-1	1
z1=30	z2=30	Z=1				
		cj-zj1	0	0	-1	0
		dj-zj2	-6	0	-2	0
		Δj →	180↑	0	30	0

$$\theta_1 = \min\left\{\frac{6}{5}, \frac{6}{4}\right\} = 6/5_1$$

Since, $(\alpha^j{}_i > 0, \theta_2 = \infty,$

U_1 =upper bound of y_1=3

$$\theta = \min\{\theta_1, \theta_2, U_j\} = 6/5 = \theta_1 .$$

So the entering variable is y_1 in replace of y_2.

Table. 2. Optimal table

cB ↓	dB ↓	Cj →	5	1	0	0
		Dj	4	2	0	0
		xBi →	y1	y2	x3	x4
5	4	y1=6/5	1	1/5	1/5	0
0	0	x4=6/5	0	-4/5	-9/5	1
z1= 30	z2=1 64/5	Z= 150/164				
		cj-zj1	0	0	-1	0
		dj-zj2	0	6/5	-4/5	0
		Δj →	0	-36	-44/5	0

This is now optimal and feasible solution. By using this, we have reached a solution y1=6/50, x4 = 6/5 with Z max = 150/164 which is correct optimal value .Now the optimal solution in terms of the original variables x1, x2, x3 & x4 is found as follows:

x1=2+6/5=16/5, (4)(16/5)-x3 + 6/5 =14, x3=0 , x2=4 and x4=6/5. The obtained result is identical with the Section 4 method.

In the above section, we discussed of our propose LFBV algorithm with an example. In the following Section, we develop an algorithm for solving LFBV and then use a computer program to solve LFBV problems.

VI. **Solving LFBV Using Computer Algebra**

In this section, we present our computational procedure in terms of some steps for solving LFBV for the programming language MATHEMATICA[8,9].

Algorithm for Solving LFBV

In this section, we give an Algorithm corresponding to the *MATHEMATICA* code.

Step 1: Express the LFBV to its standard form.

Step 2: Find all $m \times m$ sub-matrices of the coefficient matrix A by setting $n - m$ variables equal zero.

Step 3: Test whether the linear system of equations has unique solution or not.

Step 4: If the linear system of equations has got any unique solution, find it.

Step 5: Dropping the solutions with negative elements. Determine all basic feasible solutions.

Step 6: Calculate the values of the objective function for the basic feasible solutions found in step 5.

Step 7: For the maximization of LFBV the maximum value of Z is the optimal value of the objective function and the basic feasible solution which yields the optimal value is the optimal solution.

Pseudo Code

In this section, we give a short presentation corresponding to the above algorithm.

We demonstrated our computer algebra using pseudo code to find the LFBV.

Begin of the pseudo code:

► Finding All basic Variables

begin the function basic[AA_,bb]

Initialize ← Matrix AA, set bb

For k ← 1 to Length[ss] // “ss is the list of columns positionfor construct m×m matrix i.e. for the basis matrix”

  find **{** all the basic solutions

define the function bs [k_]

  find **{** all the basic feasible solutions

Initialize ← MatrixAA, cost coefficients set cc

define the function optimal[AA_,bb_,cc]

find **{** all the Optimal solution and the Optimal value of LFBV.

End of the pseudo code.

MATHEMATICA Code on LFBV

We develop a code for solving LFBV using the programming language MATHEMATICA. It is available in our hand. If anyone interest to see this code please contract with authors.

Input for Numerical Example 1

A={{5,1,1,0,0,0,0,0,0,0},
{4,0,-,1,0,0,0,0,0,0},
{1,0,0,0,-1,0,0,0,0,0},
{1,0,0,0,0,1,0,0,0,0},
{0,1,0,0,0,0,-1,0,0,0},
{0,1,0,0,0,0,0,1,0,0},
{0,0,1,0,0,0,0,0,1,0},
{0,0,0,1,0,0,0,0,0,1}};

b={20,14,2,5,4,12,25,18};

c= {5,1,0,0,0,0,0,0,0,0};

d= {4,2,0,0,0,0,0,0,0,0};

α =10;

β = 12;

basic[A,b]

optimal[A, b, c]

Output for Numerical Example 1

The possible all basic solution is:

{{2,4,6,12,0,3,0,8,19,6},
{16/5,4,0,6/5,6/5,9/5,0,8,
25,84/5},
{2,10,0,6,0,3,6,2,25,12}}

The optimal value of the objective function is 75/82.

The optimal solution is {16/5,4,0,6/5,6/5,9/5,0,8,25,84/5}.

Numerical Example 2

Consider the following linear fractional bounded program (Bajalinov [1]).

$$Maximize = \frac{x_1 + 3x_2 + 6}{2x_1 + 3x_2 + 12}$$

subject to

$$x_1 + 2x_2 \geq 10$$

$$2x_1 + 3x_2 \leq 60$$

$$5 \leq x_1 \leq 15,\ 4 \leq x_2 \leq 30.$$

Solution Using Our Propose Method

First, we will find basic variable for the LFBV minimize subject to the same set of constraints. Then we have the following simplex table.

Min: w

Subject to, $x_1 + 2x_2 - s_1 + w = 10$

$$2x_1 + 3x_2 + s_3 = 60$$

$$x_1, x_2, s_1, s_3, w \geq 0$$

Table. 3. Finding basic variable final table

C_B	c_j	0	0	0	0	1	B
	Basis	x_1	x_2	s_1	s_3	W	
0	x_2	.5	1	-.5	0	.5	5
.5	s_3	.5	0	3/2	1	-3/2	45
$c_j^* = c_j - z_j$		0	0	0	0	1	Z= 0

Since all $c_j^* = c_j - z_j \geq 0$ so we get the basic variable. We use this basic variable in the original LFBV problem to solve the original problem.

After the above calculations we get the following result.

x1=5+y1

x2=4+y2

0≤y1≤10

and 0≤y2≤ 26.

Converting the problem in standard form, we have

Maximize $Z= \frac{y_1 + 3y_2 + 23}{2y_1 + 3y_2 + 34}$

subject to

$$y_1 + 2y_2 - s_1 + w = -3$$

$$2y_1 + 3y_2 + s_3 = 38$$

$$y_1 \geq 0,\ y_2 \geq 0,\ s_1 \geq 0,$$

$$s_3 \geq 0, w \geq 0$$

Table. 4. Initial Table for LFBV

cB	dB		y1	y2	s_1	s3	w
		Cj	1	3	0	0	0
		Dj	2	3	0	0	0
		xBi	y1	y2	s_1	s3	w
3	3	Y2= -3	-1/3	0	(1)	-1	-2/3
0	0	S3=38	2/3	1	0	0	1/3
z1=14	z2=25	Z=14/25					
		cj-zj1	0	1	1	-1	0
		dj-zj2	0	-1	2	-2	0
		Δj →	0	0	9↑	-9	0

Table. 5. Optimal table

cB	dB		y1	y2	s_1	s3	w
		Cj	1	3	0	0	0
		Dj	2	3	0	0	0
		xBi	y1	y2	s_1	s3	w
0	0	S1= 85/3	-1/3	0	1	-1	-2/3
3	3	Y2=38/3	2/3	1	0	0	1/3
z1=61	z2=72	Z=61/72					
		cj-zj1	-1	0	0	0	-1
		dj-zj2	0	0	0	0	-1
		Δj →	-133	0	0	0	-11

We have y2=38/3, s1= 85/3 with Z max = 61/72 which is correct optimal value.

Finally, the optimal solution in terms of the original variables x1& x2 is found as follows: x2=4+y2=4+(38/3)=50/3;y1=0; x1=5+y1=5.

This shows that our solution is identical with the exact solution.

Input for Numerical Example 2

A={{1,2,-1,0,0,0,0,0},{2,3,0,1,0,0,0,0},

{1,0,0,0,-1,0,0,0},{1,0,0,0,0,1,0,0},

{0,1,0,0,0,0,-1,0},{0,1,0,0,0,0,0,1}}

;b={10,60,5,15,4,30};

c= {1,3,0,0,0,0,0,0};

d= {2,3,0,0,0,0,0,0};

α =6; β = 12;

basic[A,b];

optimal[A, b, c]

Output for Numerical Example 2

The possible all basic solution is:

{{5,4,13,18,10,0,0,26},

{5,4,3,38,0,10,0,26},

{15,10,25,0,10,0,6,20},

{5,50/3,85/3,0,0,10,38/3,40/3}}

The optimal value of the objective function is 61/72.

The optimal solution is {5,50/3,85/3,0,0,10,38/3,40/3}.

In the above section, we showed input and output for solving LFBV using the programming language MATHEMATICA. In the following Section, we show a comparison chart using iteration and CPU time.

VII. Comparison for Solving LFBV Problems

In this section, we give a comparison chart to show the efficiency of our algorithm and computer technique with the existing method. To find the run time of our implementation code we use “TimeUsed[ ] ” command. We use the following computer configuration. Processor: Intel(R) Pentium(R) Dual CPU E2180@2.00GHZ 2.00GHZ, Memory(RAM):1.00 GB, System type: 32-bit operating system.

Table. 6. Comparison table

		Iteration use	Computer Time taken
Numerical Example 1	Bajalinov LFBV	Three	1.106 Sec
	Propose method of LFBV	One	
Numerical Example 2	Bajalinov LFBV	Three	0.329 Sec
	Propose method of LFBV	Two	

VIII. Conclusions

In this paper, we developed a new method for solving the problem in which the objective function is a linear fractional Bounded Variable (LFBV) function, where the constraints functions are in the form of linear inequalities and the variables are bounded. The advantages of LFBV algorithm are simplicity of implementation and less computational effort. We developed an all basic feasible based computer technique for solving such problems by using the programming language MATHEMATICA[8,9]. We also compared the results obtained by our methods with that of the other existing method.

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