Dhaka Univ. J. Sci. 61(2): 203-206, 2013 (July)

A General Method of Detecting Confounded Effect in p^n (p is prime) Factorial Experiments

Md. Kamrul Hasan Khan and M A Jalil*

Department of Statistics, Biostatistics and Informatics, Dhaka University, Dhaka-1000, Bangladesh

(Received : 12 January 2013; Accepted : 14 May 2013)

Abstract

A general detection method of confounded effect in a given plan of a p^n (p is prime) factorial experiment with a single effect confounded is proposed. The method is described by introducing some new equations called detection equations.

Keywords: Confounding plan, Detection of confounded effect, Detection equation.

I. Introduction

If we are given a plan of a p^n factorial experiment where a single factorial effect is confounded and then if we are to detect the confounded effect, we can do it by the manipulating method of symbolic equations considering its mod value at the level (p) of the experiment[1]. A quick and easier method of detecting the confounded effect was proposed in 1994[2]. The method works nicely in detection the confounded effect for a p^n factorial experiment when p equals 2 or 3. If $p > 3$, the method does not work in detecting the confounded effect. In this article, moderation is made in detection of the confounded effect by introducing some new equations called detection equations. The moderated method can be used in detecting the confounded effect for a p^n (p is prime) factorial experiment.

II. The Methods

Suppose we have a confounding plan with level combinations, confounded with a factorial effect in a p^n factorial experiment into p incomplete blocks and no information is given about which factorial effect is confounded. Now, to detect the confounded effect we have the manipulating method where the symbolic equations are solved equating the levels under mod p in a trial and error manner. If we have a plan of p^n factorial experiment where a single factorial effect is confounded will have p incomplete blocks. For detection which factorial effect is confounded, by manipulating method, we have to solve the following p equations in a trial and error fashion for each of the incomplete blocks:

$$\left.\begin{array}{l} a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0 \\ a_1x_1 + a_2x_2 + \cdots + a_nx_n = 1 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_1x_1 + a_2x_2 + \cdots + a_nx_n = (p-1) \end{array}\right\} \bmod p$$

where, x_i is the level of the i-th factor (F_i) and will take on values from 0 to $(p-1)$, a_i will also take on values from 0 to $(p-1)$ but not all equal to zero. There is a restriction that the coefficient of the first a_i that is not zero to be equal to unity[1].

For a quick and easier detection, an algebraic method was developed in 1994[2]. This method of detection of the confounded effect in a given plan of a p^n (p is prime) factorial experiment with a single factorial effect is described below.

Among the p incomplete blocks, the incomplete block with the level combinations of n factors each at their lowest level is to be ignored. Any of the rest $(p-1)$ incomplete blocks is to be considered for the detection purpose. From the considered block, select those treatment combinations where all but one at their lowest level. Let the n factors in a p^n f.e. be denoted by $F_1, F_2, \dots, F_n$ and in general, F_t be the t-th factor. The level combinations of a treatment combination with the i-th level for first factor, j-th level for 2nd factor, k-th level for 3rd factor and finally the r-th level for n-th factor can symbolically be written as[3]:

$$((F_1)_i(F_2)_j(F_3)_k \dots (F_n)_r) \qquad (1)$$

Now, to find the confounded effect the selected treatments are multiplied in such a way that the various level combinations of each factor are raised to its power. The arithmetic of this multiplication can be expressed as,

$$((F_1)_i(F_2)_j \dots (F_n)_r)\ ((F_1)_i(F_2)_j \dots (F_n)_r) \dots$$
$$((F_1)_i(F_2)_j \dots (F_n)_r) => F_1^{\Sigma i} \quad F_2^{\Sigma j} \quad \dots \ F_n^{\Sigma r} \qquad (2)$$

As stated the method works in detecting the confounded effect in a p^n (p is prime) factorial experiment. Practically, the method works when $p = 2\ or\ 3$ and it does not work in detecting the confounded effect when $p > 3$ as shown in the examples described below.

Let us consider a plan of 5^3 factorial experiment where the effect $F_1F_2F_3^2$ is confounded. Here $p = 5$ is a prime number; so the levels are $0, 1, 2, 3, 4$ and $n = 3$; the factors are $F_1, F_2\ and\ F_3$. The construction layout of the confounding plan becomes:

*Author for correspondence, *e-mail* : majalil@du.ac.bd

$$\begin{bmatrix}
0 & 0 & 0 & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} & \mathbf{4} & \mathbf{0} & \mathbf{0} \\
3 & 0 & 1 & 4 & 0 & 1 & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 1 & 2 & 0 & 1 \\
1 & 0 & 2 & 2 & 0 & 2 & 3 & 0 & 2 & 4 & 0 & 2 & \mathbf{0} & \mathbf{0} & \mathbf{2} \\
4 & 0 & 3 & \mathbf{0} & \mathbf{0} & \mathbf{3} & 1 & 0 & 3 & 2 & 0 & 3 & 3 & 0 & 3 \\
2 & 0 & 4 & 3 & 0 & 4 & 4 & 0 & 4 & \mathbf{0} & \mathbf{0} & \mathbf{4} & 1 & 0 & 4 \\
4 & 1 & 0 & \mathbf{0} & \mathbf{1} & \mathbf{0} & 1 & 1 & 0 & 2 & 1 & 0 & 3 & 1 & 0 \\
2 & 1 & 1 & 3 & 1 & 1 & 4 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 2 & 3 & 1 & 2 & 4 & 1 & 2 \\
3 & 1 & 3 & 4 & 1 & 3 & 0 & 1 & 3 & 1 & 1 & 3 & 2 & 1 & 3 \\
1 & 1 & 4 & 2 & 1 & 4 & 3 & 1 & 4 & 4 & 1 & 4 & 0 & 1 & 4 \\
3 & 2 & 0 & 4 & 2 & 0 & \mathbf{0} & \mathbf{2} & \mathbf{0} & 1 & 2 & 0 & 2 & 2 & 0 \\
1 & 2 & 1 & 2 & 2 & 1 & 3 & 2 & 1 & 4 & 2 & 1 & 0 & 2 & 1 \\
4 & 2 & 2 & 0 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 \\
2 & 2 & 3 & 3 & 2 & 3 & 4 & 2 & 3 & 0 & 2 & 3 & 1 & 2 & 3 \\
0 & 2 & 4 & 1 & 2 & 4 & 2 & 2 & 4 & 3 & 2 & 4 & 4 & 2 & 4 \\
2 & 3 & 0 & 3 & 3 & 0 & 4 & 3 & 0 & \mathbf{0} & \mathbf{3} & \mathbf{0} & 1 & 3 & 0 \\
0 & 3 & 1 & 1 & 3 & 1 & 2 & 3 & 1 & 3 & 3 & 1 & 4 & 3 & 1 \\
3 & 3 & 2 & 4 & 3 & 2 & 0 & 3 & 2 & 1 & 3 & 2 & 2 & 3 & 2 \\
1 & 3 & 3 & 2 & 3 & 3 & 3 & 3 & 3 & 4 & 3 & 3 & 0 & 3 & 3 \\
4 & 3 & 4 & 0 & 3 & 4 & 1 & 3 & 4 & 2 & 3 & 4 & 3 & 3 & 4 \\
1 & 4 & 0 & 2 & 4 & 0 & 3 & 4 & 0 & 4 & 4 & 0 & \mathbf{0} & \mathbf{4} & \mathbf{0} \\
4 & 4 & 1 & 0 & 4 & 1 & 1 & 4 & 1 & 2 & 4 & 1 & 3 & 4 & 1 \\
2 & 4 & 2 & 3 & 4 & 2 & 4 & 4 & 2 & 0 & 4 & 2 & 1 & 4 & 2 \\
0 & 4 & 3 & 1 & 4 & 3 & 2 & 4 & 3 & 3 & 4 & 3 & 4 & 4 & 3 \\
3 & 4 & 4 & 4 & 4 & 4 & 0 & 4 & 4 & 1 & 4 & 4 & 2 & 4 & 4
\end{bmatrix}$$

Since in block-1, the level combinations of all the factors are at their lowest level, block-1 is to be ignored. Any one of the rest four blocks can be considered. If we consider block-2, the required treatments as described in the method will be: (100), (003) and (010).

In the given plan the resulting confounded effect could be found with the equation (2) as,

$((F_1)_1(F_2)_0(F_3)_0)\ ((F_1)_0(F_2)_0(F_3)_3)\ ((F_1)_0(F_2)_1(F_3)_0)$

=> $F_1^{1+0+0}F_2^{0+0+1}F_3^{0+3+0} = F_1^1F_2^1F_3^3 = F_1F_2F_3^3$.

Here, in the given layout the factorial effect $F_1F_2F_3^2$ confounded. But when we are detecting the confounded effect using the quick method (Equation 2), we are getting $F_1F_2F_3^3$ as the confounded effect, which is not true. Using the other blocks we also get $F_1F_2F_3^3$ as the confounded effect. Clearly, the method fails to detect the confounded effect.

III. The Reason

In the construction of a confounded plan, symbolic equations are used to construct the level combinations of the incomplete blocks. But in the detection, we need the coefficients of symbolic equations, because the exponent of factors in the factorial effect to be confounded becomes the coefficient in the symbolic equations. For example, if in a 3^3 factorial experiment $F_1F_2F_3^2$ is the factorial effect to be confounded, the symbolic equation becomes as

$$x_1 + x_2 + 2x_3 = 0, = 1, =\ 2\ (mod\ 3)$$

Let from the considered block we select a treatment combination where all but x_i are at their lowest level. So using the detection method we will get the i-th factor in the confounded effect as $F_i^{x_i}$ but actually the i-th factor in the confounded effect becomes as $F_i^{a_i}$. For p^n factorial experiment we get the value of the coefficient, a_i, by solving any one of the following equations

$$\left.\begin{array}{l} a_i \times x_i = 1 \\ a_i \times x_i = 2 \\ \dots\dots\dots\dots\dots\dots \\ a_i \times x_i = (p-1) \end{array}\right\} \bmod p \qquad (3)$$

Here $a_i\ and\ x_i$ take values from $1, 2, \dots, (p-1)$.

So first we take any one equation from (3). Then we have to solve this equation for all values of $a_i\ and\ x_i$. Thus we will get $(p-1)$ equations. We name the equations in (3) as the detection equations.

IV. The Moderated Detecting Method

To detect confounded effect in a p^nfactorial experiment, where n denotes number of factors and p (prime) denotes the number of levels; first we take any one equation from (3). Then we shall solve this equation for all values of $a_i\ and\ x_i$. Thus we will get $(p-1)$ equations. Then using the method proposed by Jalil et. al.[2], we get a treatment effect. If x_i is the power of a factor in the treatment effect then we have to replace x_i by a_i to get the confounded effect. Similarly we have to replace the power of all factors obtained in the treatment effect. After interchanging the power of all factors we get the confounded effect.

V. Illustration

To illustrate the method, now we consider a plan of 5^3 factorial experiment where a single factorial effect is confounded. A plan of 5^3 factorial experiment where a single factorial effect is confounded is given below:

$$\begin{bmatrix}
0 & 0 & 0 & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} & \mathbf{4} & \mathbf{0} & \mathbf{0} \\
3 & 0 & 1 & 4 & 0 & 1 & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 1 & 2 & 0 & 1 \\
1 & 0 & 2 & 2 & 0 & 2 & 3 & 0 & 2 & 4 & 0 & 2 & \mathbf{0} & \mathbf{0} & \mathbf{2} \\
4 & 0 & 3 & \mathbf{0} & \mathbf{0} & \mathbf{3} & 1 & 0 & 3 & 2 & 0 & 3 & 3 & 0 & 3 \\
2 & 0 & 4 & 3 & 0 & 4 & 4 & 0 & 4 & \mathbf{0} & \mathbf{0} & \mathbf{4} & 1 & 0 & 4 \\
4 & 1 & 0 & \mathbf{0} & \mathbf{1} & \mathbf{0} & 1 & 1 & 0 & 2 & 1 & 0 & 3 & 1 & 0 \\
2 & 1 & 1 & 3 & 1 & 1 & 4 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 2 & 3 & 1 & 2 & 4 & 1 & 2 \\
3 & 1 & 3 & 4 & 1 & 3 & 0 & 1 & 3 & 1 & 1 & 3 & 2 & 1 & 3 \\
1 & 1 & 4 & 2 & 1 & 4 & 3 & 1 & 4 & 4 & 1 & 4 & 0 & 1 & 4 \\
3 & 2 & 0 & 4 & 2 & 0 & \mathbf{0} & \mathbf{2} & \mathbf{0} & 1 & 2 & 0 & 2 & 2 & 0 \\
1 & 2 & 1 & 2 & 2 & 1 & 3 & 2 & 1 & 4 & 2 & 1 & 0 & 2 & 1 \\
4 & 2 & 2 & 0 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 \\
2 & 2 & 3 & 3 & 2 & 3 & 4 & 2 & 3 & 0 & 2 & 3 & 1 & 2 & 3 \\
0 & 2 & 4 & 1 & 2 & 4 & 2 & 2 & 4 & 3 & 2 & 4 & 4 & 2 & 4 \\
2 & 3 & 0 & 3 & 3 & 0 & 4 & 3 & 0 & \mathbf{0} & \mathbf{3} & \mathbf{0} & 1 & 3 & 0 \\
0 & 3 & 1 & 1 & 3 & 1 & 2 & 3 & 1 & 3 & 3 & 1 & 4 & 3 & 1 \\
3 & 3 & 2 & 4 & 3 & 2 & 0 & 3 & 2 & 1 & 3 & 2 & 2 & 3 & 2 \\
1 & 3 & 3 & 2 & 3 & 3 & 3 & 3 & 3 & 4 & 3 & 3 & 0 & 3 & 3 \\
4 & 3 & 4 & 0 & 3 & 4 & 1 & 3 & 4 & 2 & 3 & 4 & 3 & 3 & 4 \\
1 & 4 & 0 & 2 & 4 & 0 & 3 & 4 & 0 & 4 & 4 & 0 & \mathbf{0} & \mathbf{4} & \mathbf{0} \\
4 & 4 & 1 & 0 & 4 & 1 & 1 & 4 & 1 & 2 & 4 & 1 & 3 & 4 & 1 \\
2 & 4 & 2 & 3 & 4 & 2 & 4 & 4 & 2 & 0 & 4 & 2 & 1 & 4 & 2 \\
0 & 4 & 3 & 1 & 4 & 3 & 2 & 4 & 3 & 3 & 4 & 3 & 4 & 4 & 3 \\
3 & 4 & 4 & 4 & 4 & 4 & 0 & 4 & 4 & 1 & 4 & 4 & 2 & 4 & 4
\end{bmatrix}$$

Here $p = 5$, which is a prime number; so the levels are $0, 1, 2, 3, 4$ and $n = 3$; the factors are F_1, F_2 *and* F_3. For $p = 5$ the values of a_i *and* x_i are $1, 2, 3, 4$. From equation (3) we can write

$$a_i \times x_i = 1$$

$$a_i \times x_i = 2$$

$$a_i \times x_i = 3$$

$$a_i \times x_i = 4.$$

Now we have to take any one equation from these four equations. Here we take

$$a_i \times x_i = 1. \tag{4}$$

Solving this equation for all values of a_i *and* x_i, we get

$$1 \times 1 = 1$$

$$2 \times 3 = 6 = 1\ (mod\ 5)$$

$$3 \times 2 = 6 = 1\ (mod\ 5)$$

$$4 \times 4 = 16 = 1\ (mod\ 5).$$

Since block-1 contains the level combination of all factors each at their lowest level, so block-1 is ignored. Any one of the remaining four blocks can be considered.

For block-2

If we consider block-2 the required treatments as described in the method will be: (100), (003) and (010).

In the given plan the resulting confounded effect could be found with the equation (2) as,

$((F_1)_1(F_2)_0(F_3)_0)\ ((F_1)_0(F_2)_0(F_3)_3)\ ((F_1)_0(F_2)_1(F_3)_0)$

=> $F_1^{1+0+0} F_2^{0+0+1} F_3^{0+3+0} = F_1^1 F_2^1 F_3^3 = F_1 F_2 F_3^3$.

Since the power of F_1 in the treatment effect is 1, we shall have to replace 1 by 1 using the solutions of the equation (4). In F_2 the power is 1, so we shall have to replace 1 by 1 and in F_3 the power is 3, so we shall have to replace 3 by 2.

So the confounded effect is $F_1^1 F_2^1 F_3^2 = F_1 F_2 F_3^2$.

Similarly from block-3, block-4 and block-5 we get $F_1 F_2 F_3^2$ as the confounded effect.

Now if we take

$$a_i \times x_i = 2. \tag{5}$$

Solving this equation for all values of a_i *and* x_i, we get

$$1 \times 2 = 2$$

$$2 \times 1 = 2\ (mod\ 5)$$

$$3 \times 4 = 12 = 2\ (mod\ 5)$$

$$4 \times 3 = 12 = 2\ (mod\ 5).$$

Since block-1 contains the level combination of all factors each at their lowest level, so block-1 is ignored. Any one of the remaining four blocks can be considered.

For block-3

If we consider block-3 the required treatments as described in the method will be: (200), (001) and (020).

In the given plan the resulting confounded effect could be found with the equation (2) as,

$((F_1)_2(F_2)_0(F_3)_0)\ ((F_1)_0(F_2)_0(F_3)_1)\ ((F_1)_0(F_2)_2(F_3)_0)$

=> $F_1^{2+0+0}F_2^{0+0+2}F_3^{0+1+0} = F_1^2F_2^2F_3^1 = F_1F_2F_3^3\ (mod\ 5)$.

Since the power of F_1 in the treatment effect is 1, we shall have to replace 1 by 2 using the solutions of the equation (5). In F_2 the power is 1, so we shall have to replace 1 by 2 and in F_3 the power is 3, so we shall have to replace 3 by 4.

So the confounded effect is $F_1^2F_2^2F_3^4 = F_1F_2F_3^2\ (mod\ 5)$.

Similarly from block-2, block-4 and block-5 we get $F_1F_2F_3^2$ as the confounded effect.

Now if we take

$$a_i \times x_i = 3. \tag{6}$$

Solving this equation for all values of $a_i\ and\ x_i$, we get

$$1 \times 3 = 3$$
$$2 \times 4 = 8 = 3\ (mod\ 5)$$
$$3 \times 1 = 3\ (mod\ 5)$$
$$4 \times 2 = 8 = 3\ (mod\ 5).$$

Since block-1 contains the level combination of all factors each at their lowest level, so block-1 is ignored. Any one of the remaining four blocks can be considered.

For block-2

If we consider block-2 the required treatments as described in the method will be: (100), (003) and (010).

In the given plan the resulting confounded effect could be found with the equation (2) as,

$((F_1)_1(F_2)_0(F_3)_0)\ ((F_1)_0(F_2)_0(F_3)_3)\ ((F_1)_0(F_2)_1(F_3)_0)$

=> $F_1^{1+0+0}F_2^{0+0+1}F_3^{0+3+0} = F_1^1F_2^1F_3^3 = F_1F_2F_3^3$.

Since the power of F_1 in the treatment effect is 1, we shall have to replace 1 by 3 using the solutions of the equation (6). In F_2 the power is 1, so we shall have to replace 1 by 3 and in F_3 the power is 3, so we shall have to replace 3 by 1.

So the confounded effect is $F_1^3F_2^3F_3^1 = F_1F_2F_3^2\ (mod\ 5)$.

Similarly from block-3, block-4 and block-5 we get $F_1F_2F_3^2$ as the confounded effect.

Now if we take

$$a_i \times x_i = 4. \tag{7}$$

Solving this equation for all values of $a_i\ and\ x_i$, we get

$$1 \times 4 = 4$$
$$2 \times 2 = 4$$
$$3 \times 3 = 9 = 4\ (mod\ 5)$$
$$4 \times 1 = 4 = 4\ (mod\ 5).$$

Since block-1 contains the level combination of all factors each at their lowest level, so block-1 is ignored. Any one of the remaining four blocks can be considered.

For block-3

If we consider block-3 the required treatments as described in the method will be: (200), (001) and (020).

In the given plan the resulting confounded effect could be found with the equation (2) as,

$((F_1)_2(F_2)_0(F_3)_0)\ ((F_1)_0(F_2)_0(F_3)_1)\ ((F_1)_0(F_2)_2(F_3)_0)$

=> $F_1^{2+0+0}F_2^{0+0+2}F_3^{0+1+0} = F_1^2F_2^2F_3^1 = F_1F_2F_3^3\ (mod\ 5)$.

Since the power of F_1 in the treatment effect is 1, we shall have to replace 1 by 4 using the solutions of the equation (7). In F_2 the power is 1, so we shall have to replace 1 by 4 and in F_3 the power is 3, so we shall have to replace 3 by 3.

So the confounded effect is $F_1^4F_2^4F_3^3 = F_1F_2F_3^2\ (mod\ 5)$.

Similarly from block-2, block-4 and block-5 we get $F_1F_2F_3^2$ as the confounded effect.

VI. Conclusion

In this article, a general detection method has been developed for p^n (p is prime) factorial experiment where a single factorial effect is confounded. The method is restricted to p^n factorial experiment when p is prime.

References

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