Dhaka Univ. J. Sci. 65(1): 15-20, 2017 (January)

A Markov Renewal Chain Model for Forecasting Earthquakes in Bangladesh Region

Md. Mohsan Khudri[1] and Md. Asaduzzaman*[2]

[1] *School of Business,Uttara University, Dhaka-1230, Bangladesh*
[2] *Institute of Statistical Research and Training, Dhaka University, Dhaka- 1000, Bangladesh*

(Received: 2 November 2015; Accepted: 16 May 2016)

Abstract

In this paper we have proposed a Weibull Markov renewal process to model earthquakes occurred in and around Bangladesh from 1961 to 2013. The process assumes that the sequence of earthquakes is a Markov chain and the sojourn time distribution is a Weibull random variable that depends only on two successive earthquakes. We estimated the parameters of the models along with transition probabilities using maximum likelihood method. The transient behavior of earthquake occurrences was investigated in details and probability forecasts were calculated for different lengths of time interval using the fitted model. We also investigated the stationary behavior of earthquake occurrences in Bangladesh region.

Keywords: earthquake forecasting; recurrence time; Markov renewal process; sojourn time distribution; Bangladesh

I. Introduction

Bangladesh has become tectonically active due to its position adjacent to the very active Himalayan front and continuing deformation in nearby parts of South-east Asia. The country is exposed to strong shaking from a variety of earthquake sources that may produce tremors of magnitude 8 or greater on Richter scale. Therefore, many cities of Bangladesh are vulnerable to major earthquake disaster (Cummins[1]; Sarker et al. [2]).

In Bangladesh, devastating large earthquakes occur less often. Nevertheless, if it occurs, it may affect larger areas and have substantial long-term economic effects. On the contrary, moderate sized earthquakes occur in every few years. Thereupon, the government has given special emphasis on growing awareness about earthquake among mass people. The government has also improved the national earthquake monitoring system. However, practically the country is still far behind from the least preparedness level to face such a hazard. This study has developed a forecasting model based on a stochastic counting process approach deeming the earthquakes occurred in and around Bangladesh during the period of 1961 to 2013.

Earthquake modeling and forecasting through stochastic processes have received immense attention in the past couple of years (e.g., Vere-Jones[3]; Ogata[4],Ogata[5], Ogata[6]; Zhuang et al. [7]; Ogata et al. [8]; Schoenberg[9]; Alvarez[10]; Garavaglia and Pavani[11]). Ogata[5] investigated trigger and epidemic type models for evaluating aftershock sequences emerging from earthquakes using a Japanese data set. Subsequently, adding the spatial components, i.e. latitude and longitude, Ogata[6] proposed a point-process model for earthquake occurrences. Zhuang et al.[7] proposed a space-time branching process model to decluster earthquake occurrences. Besides these intensity based models, Markov renewal models have also become popular for modeling earthquake occurrences (e.g., Alvarez[10]; Garavaglia and Pavani[11]). A sequence of earthquakes for a given seismic region can be modeled by either Poisson or renewal model. Since the Poisson process is inherently memory-less, it may not be appropriate to capture the characteristics of earthquake occurrences. A Markov renewal process is preferable as it assumes that the sequence of earthquakes is pertained to the phases of accumulation and release of energy characterizing a given seismogenetic source. Therefore, a Markov renewal model fits this conjecture better in comparison with other approaches available in the literature. The physics of earthquake generation states that the risk of an immediate strong earthquake increases after a certain elapsed time. Under these assumptions, Alvarez[10] developed a Markov Renewal model to forecast earthquakes in Turkey assuming stationarity of the process. While emphasizing on transient forecasting of earthquakes in Turkey, Garavaglia and Pavani[11] proposed a modified version of the Markov renewal model developed by Alvarez[11]. Markov renewal models have also been employed in many areas, for instance, in natural hazards analysis (Foufoula-Georgiou and Lettenmaier[12]; Asaduzzaman and Latif[13]), survival analysis (Dabrowska et al. [14]), transportation (Gilbert et al. [15]), engineering (Ghosn and Moses[16]), etc.

In this paper, we have developed a stationary Weibull Markov renewal model to forecast earthquake occurrences in Bangladesh. The following section narrates the Markov renewal chain (MRC), the Weibull Markov renewal model and its likelihood construction and parameter estimation. A brief description about source of data, variables and exploratory analyses of the variables is given in section 3. A Weibull MRP model is fitted using the data and results are discussed in section 4. The detailed probability forecasts of earthquake occurrences are specified and asymptotic properties are also demonstrated in this section. Finally, some concluding remarks are mentioned in section 5.

II. Markov Renewal Chain

Consider a random system that evolves in time and visits some states from a finite state space $E = \{1, \dots, S\}$. Let $\mathcal{J} = \{\mathcal{J}_n : n \geq 0\}$ be a chain with state space E which represents the system state at the *n*th jump, and $\mathcal{S} = \{\mathcal{S}_n : n \geq 0\}$ to be the time until the *n*th jump has occurred with $n \in \mathcal{N}$. We suppose that $\mathcal{S}_0 = 0$ and $0 < \mathcal{S}_1 < \mathcal{S}_2 < \dots < \mathcal{S}_n < \mathcal{S}_{n+1} \dots$ We define $\mathcal{X}_n = \mathcal{S}_n - \mathcal{S}_{n-1}$ for all $n \geq 0$, and $\mathcal{X}_0 = 0$ as such the random variable $\mathcal{X} = \{\mathcal{X}_n : \geq 0\}$ is the sojourn time in state $\mathcal{J}_{n-1}$ before the *n*th jump. The process $(\mathcal{J}, \mathcal{X}) = \{(\mathcal{J}_n, \mathcal{X}_n) : n \geq 0\}$ is said to be a Markov renewal chain (MRC) for all $n \geq 0, i, j \in$ E, and $\mathcal{X}_n \in \mathcal{R}^+$ if it satisfies the following condition

* Author for correspondence. e-mail: asad@isrt.ac.bd

$$\mathbb{P}(\mathcal{J}_{n+1} = j, \mathcal{S}_{n+1} - \mathcal{S}_n = x_{n+1} | \mathcal{J}_{0,\dots,}\mathcal{J}_n; \mathcal{S}_{0,\dots,}\mathcal{S}_n) = \mathbb{P}(\mathcal{J}_{n+1} = j, \mathcal{S}_{n+1} - \mathcal{S}_n = x_{n+1} | \mathcal{J}_n = i)$$

Assuming $(\mathcal{J}, \mathcal{S})$ as a homogeneous Markov renewal chain, we see that $\{\mathcal{J}_n : n \geq 0\}$ is also a homogeneous Markov chain known as the embedded Markov chain (EMC) associated with the MRC $(\mathcal{J}, \mathcal{S})$. The transition probability matrix of $\mathcal{J}_n$, denoted by $p = \{(p_{ij}), i, j \in \text{E}\}$, is defined as follows:

$$p_{ij} := \mathbb{P}(\mathcal{J}_{n+1} = j | \mathcal{J}_n = i), i, j \in E, n \geq 0,$$

Assuming that the process is homogeneous by time and aperiodic (i.e. ergodic), there exists one and only one stationary distribution $\pi = (\pi_{1,\dots,}\pi_{\mathcal{M}})$ where

$$\pi_j = \lim_{n\to\infty} \mathbb{P}(\mathcal{J}_n = j | \mathcal{J}_0 = i) = \lim_{n\to\infty} p_{ij}{}^{(n)}$$

is independent of *i*. Then π_j is the unique non-negative solution of

$$\pi_j = \sum_{j=1}^{\mathcal{M}} \pi_j p_{ij}, \qquad \sum_{j=1}^{M} \pi_j = 1, \;\; i, j \in E. \tag{1}$$

In MRC, we are interested in two types of sojourn time distributions: the conditional distributions depend on the next state to be visited and the sojourn time distributions in a given state. The conditional distributions of sojourn times can be given by

1. $f_{ij}(\cdot)$, the conditional distribution of X_{n+1},
   $f_{ij}(x_{n+1}) = \mathbb{P}(X_{n+1} = x_{n+1} | \mathcal{J}_n = i, \mathcal{J}_{n+1} = j)$,
   and
2. $F_{ij}(\cdot)$, the conditional cumulative distribution of X_{n+1},

$$F_{ij}(x_{n+1}) = \mathbb{P}(X_{n+1} \leq x_{n+1} | \mathcal{J}_n = i, \mathcal{J}_{n+1} = j) = \sum_{l=0}^{k} f_{ij}(l).$$

Let $\mathbf{Q} = (\mathbf{Q}(\boldsymbol{k}); \mathbf{k} \in \boldsymbol{\mathcal{N}})$ be the cumulative semi-Markov kernel defined, for all $i, j \in E$ and $k \in \mathcal{N}$, by

$$\mathcal{Q}_{ij}(k) = \mathbb{P}(J_{\mathcal{N}+1} = j, X_{n+1} \leq k | J_n = i) = \sum_{l=0}^{k} q_{ij}(l).$$

Then we obtain, $f_{ij}(k) = \frac{q_{ij}(k)}{p_{ij}}, \; p_{ij} \neq 0$ (2)

Using equation (2) the semi-Markov kernel verifies the following relation

$$q_{ij}(k) = p_{ij} f_{ij}(k), \textit{ For all } i, j \in E \textit{ and } k \in \mathbb{N} \textit{ such that } p_{ij} \neq 0. \tag{3}$$

The sojourn time distributions in a given state is given by

1. $h_i(\cdot)$, the sojourn time distribution in state i,
   $h_i(x_{n+1}) := \mathbb{P}(X_{n+1} = x_{n+1} | J_n = i)$
   and
2. $H_i(\cdot)$, the sojourn time cumulative distribution in state i,

$$H_i(x_{n+1}) := \mathbb{P}(X_{n+1} \leq x_{n+1} | \mathcal{J}_n = i) = \sum_{l=1}^{k} h_i(l).$$

Now using the conditional expectation, we can express the unconditional distribution functions in terms of the conditional distribution functions (see Pyke[17]; Limnios and Oprian[18] for details) in simplified notation as follows:

$$H_i(x) = \sum_{j=1}^{M} p_{ij} F_{ij}(x). \tag{4}$$

The means of the conditional (v_{ij}) and unconditional (η_i) distributions of sojourn times can be expressed respectively as follows

$$v_{ij} = \int x dF_{ij}(x) \text{ and } \eta_i = \int x dH_i(x); \quad i, j = 1, \dots, S$$

The equation (4) leads to the following relation:

$$\eta_i = \sum_{j=1}^{M} p_{ij} v_{ij}. \tag{5}$$

Likelihood Estimation of Weibull Markov Renewal Model

Let $(j_{0,} j_1, x_1, \dots, x_{\tau-1,} j_\tau, x_\tau)$ be a realization of the Markov renewal process over the time window $[0, T]$, where τ represents the number of states visited during $[0, T]$ and $\mathcal{J}_\tau$ indicates the last event. The sojourn time between the last event and T is x_τ that can be treated as censored, i.e. $x_\tau > [T - (x_1 + \dots + x_{\tau-1})]$. Then the conditional likelihood, given $\mathcal{J}_0 = j_o$, can be expressed using equation (3) as follows

$$L(j_0) = \left[\prod_{i=0}^{\tau-1} p_{j_i, j_{i+1}} f_{j_i, j_{i+1}}(x_{i+1})\right] \cdot \sum_{k=1}^{S} p_{j_\tau k} \left[1 - F_{j_\tau k}(x_\tau)\right].$$

The corresponding log-likelihood function can be expressed by the following equation

$$l(j_0) = \sum_{i=0}^{\tau-1} \ln\left(p_{j_i j_{i+1}}\right) + \sum_{i=0}^{\tau-1} \ln\left[f_{j_i j_{i+1}}(x_{i+1})\right] + \ln\left[\sum_{k=1}^{S} p_{j_\tau, k}\left(1 - F_{j_\tau, k}(x_\tau)\right)\right]. \tag{6}$$

We have chosen the Weibull distribution to model inter-occurrence times of earthquakes due to having the capability of generalization that makes it able to examine the fit of the nested sub- models. For a Weibull MRP, the probability density function of inter-occurrence times for transition from i to j are given by

$$f_{ij}(x) = \frac{\alpha_{ij}}{\mu_{ij}} \left(\frac{x_{ij}}{\mu_{ij}}\right)^{\alpha_{ij}-1} \exp \left[\left(-\frac{x_{ij}}{\mu_{ij}}\right)^{\alpha_{ij}}\right], \;\; \alpha_{ij}, \mu_{ij} > 0, i, j \in \{1,2, \dots, S\}. \tag{7}$$

where α_{ij} and μ_{ij} are shape and scale parameters, respectively. Using equation (7) in equation (6), we obtain the conditional log-likelihood function of a Weibull Markov renewal model that takes the following form

$$l(j_0) = \sum_{j=1}^{\tau-1} \ln p_{ij} + \sum_{i=1}^{\tau-1} \ln \left[\frac{\alpha_{ij}}{\mu_{ij}} \left(\frac{x_{ij}}{\mu_{ij}} \right)^{\alpha_{ij}-1} \right] - \sum_{i=1}^{\tau-1} \left(\frac{x_{ij}}{\mu_{ij}} \right)^{\alpha_{ij}} +$$
$$\ln k{=}1spj\tau,\ k\exp -x\tau\mu j\tau,k\alpha j\tau,k$$
(8)

The maximum likelihood estimates of the parameters p_{ij}, α_{ij} and μ_{ij} are obtained by maximizing the conditional log-likelihood function given in equation (8).

Probability of Occurring an Event during a Time Interval

Once a Markov renewal model is fitted, it is possible to forecast the probability that the next state of the process is j after time t^* being known that the last state was i and the time t_0 passed by the last occurred event. Under these postulates, the probability of the next event can be given by

$$\mathbb{P}(t^*, j|t_0, i) = \mathbb{P}(\mathcal{J}_{n+1} = j,\ X_{n+1} < t_0 + t^* | J_n = i, X_{n+1} \geq t_0)$$
$$i, j \in \{1,2, \dots, S\} \quad (9)$$

where J_n is the state of the last event, J_{n+1} is the state of the next event, X_{n+1} is the time already passed by the last occurrence to the moment at which the forecast is made, $t^*$is the time period for which the forecast would be obtained. Therefore, equation (9) becomes

$$\mathbb{P}_{ij}^{(t_0, t_0+t^*)} = \frac{[F_{ij}(t_0+t^*) - F_{ij}(t_0)]p_{ij}}{\sum_{k=1}^{S}[1-F_{ik}(t_0)]p_{ik}}. \quad (10)$$

Average Recurrence Time

The expected number of steps to return to the state j for the first time, starting from the same state j is defined as the mean recurrence time for the state j. The mean return time of a Markov renewal process for state i can be obtained by

$$p_i = \frac{1}{\pi_i} \sum_{k=1}^{s} \pi_k \eta_k. \quad (11)$$

Where $\eta_k, k = 1,2, \dots s$ is defined in equation (5), and $\pi = (\pi_1, \dots, \pi_M)$ is the unique stationary distribution of the embedded Markov chain $\{J_n, n \geq 0\}$.

III. Data Source, Variables and Exploratory Statistics

The data used in this study were collected from Advanced National Seismic System (ANSS) catalog which is hosted by the Northern California Earthquake Data Center (NCEDC – http://www.ncedc.org/anss/cata log-search.html). The earth-quakes occurred in Bangladesh and its surrounding area (18°N – 29°N Latitude and 86° E – 95° E Longitude) during the period of 1961 to 2013 were taken into account to model earthquake occurrences.

Table 1. Frequency distribution of earthquakes

Type	Symbol	Magnitude(M_b)	Number of earthquakes
Small	S	< 4.5	875
Medium	M	4.5 - 5.75	693
Large	L	> 5.75	28
Total			1596

The number of earthquakes in the aforementioned period was 1596. We have mainly focused on two variables, namely, the inter-event time (the time interval between two successive events) and magnitude measured in terms of body wave magnitude scale. The body wave magnitude (M_b) refers to the way of determining the size of an earthquake using the amplitude of the initial P-wave to calculate the magnitude. In general, this measurement scale of magnitude is used to calculate the severity of those earthquakes which are measured at distances greater than 600 km.

In accordance with the severity measured on M_b, we define earthqaukes of our dataset into three categories: small earthquakes($M_b \leq 4.5$), medium earthquakes($4.5 < M_b <$ *5.75* and large earthquakes *Mb>5.75*. According to this classification, the data contain 875 small earthquakes, 693 medium earthquakes and 28 large earthquakes (Table 1). The magnitudes of earthquakes are plotted against time in Fig 1. The histogram of magnitude of earthquakes (Fig 2) shows that moderate size earthquakes occurred frequently in Bangladesh.

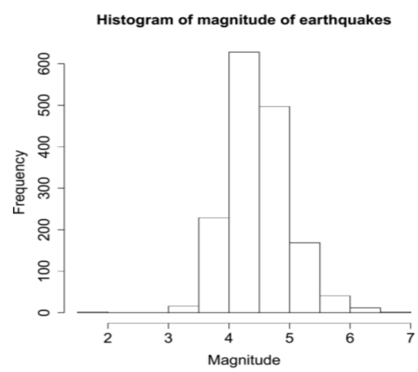

Fig. 1. Histogram of magnitude of earthquakes

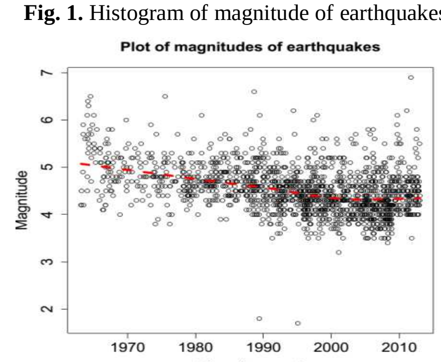

Fig. 2. Plot of magnitudes of earthquakes against time

The following matrices present the number of observed transitions from one state to another state and mean inter-occurrence times (rounded in days):

$$\text{Number of transitions} = \begin{matrix} & S & M & L \\ S & 559 & 307 & 8 \\ M & 307 & 369 & 17 \\ L & 8 & 17 & 3 \end{matrix},$$

$$\text{Mean inter-occurence times} = \begin{matrix} & S & M & L \\ S & 10 & 12 & 11 \\ M & 15 & 17 & 26 \\ L & 11 & 18 & 43 \end{matrix}$$

It is observed from the transition matrix that maximum transitions occurred in state S to S and minimum number of transitions occurred in state L to L. Consequently, the mean inter-occurrence time became shorter between two smaller earthquakes (S→S) whereas it became longer between two large earthquakes (L→L).

IV. Fitting an MRC and Forecasting Earthquakes

We have provided some descriptive measures in the previous section to understand the nature of earthquake occurrences in Bangladesh. The following assumptions are made while fitting an MRC to our dataset.

1. The sequence of earthquakes is a Markov chain and the inter-occurrence time depends only on the types of the last and the next event;
2. sojourn or inter-occurrence time is a random variable which follows Weibull distribution; and
3. The longer the inter-occurrence time for transition from the state *i* to the state j is, the higher the probability that the transition happens.

The definition of the states visited by the process during its evolution is required to know to apply a Markov renewal model. In this study, three different states have been defined based on the severity of earthquakes: small (S) earthquakes with magnitude less equal to 4.5, medium (M) earthquakes with magnitude ranging from 4.5 to 5.75 and large (L) earthquakes with magnitude greater equal to 5.75. These three states S, M and L constitute the Markov chain of the Markov renewal process. With a view to satisfying the above assumptions made, we have opted to treat the process from a parametric perspective by proposing that inter-occurrence times follow Weibull distribution with specific scale and shape parameters depending on the type of transitions. Owing to three types of earthquake category (S, M, L) = {1,2,3}, the number of possible transitions is nine.

Table 2. Tests for a sequence of nested models

Model	Description	Number of parameters	log *L*	*P* value
Weibull MRC (Model 1)	Full Weibull	24	-1406	-
Reduced Weibull MRC (Model 2)	$\alpha_{11}=\alpha_{12}=\alpha_{21}=\alpha_{22}=\alpha_{23}$ and $\alpha_{31}=\alpha_{32}=\alpha_{33}$	18	-1410	0.173
Reduced Weibull MRC (Model 3)	$\alpha_{11}=\alpha_{12}=\alpha_{21}=\alpha_{22}=\alpha_{23}$; $\alpha_{31}=\alpha_{32}=\alpha_{33}$ and $\mu_{12}=\mu_{13}=\mu_{31}$	16	-1410	0.830
Reduced Weibull MRC (Model 4)	$\alpha_{11}=\alpha_{12}=\alpha_{21}=\alpha_{22}=\alpha_{23}$; $\alpha_{31}=\alpha_{32}=\alpha_{33}$ and $\mu_{12}=\mu_{13}=\mu_{31}$ $\mu_{21}=\mu_{22}$	15	-1412	0.051
Reduced Weibull MRC (Model 5)	$\alpha_{11}=\alpha_{12}=\alpha_{21}=\alpha_{22}=\alpha_{23}$; $\alpha_{31}=\alpha_{32}=\alpha_{33}$; $\mu_{12}=\mu_{13}=\mu_{31}$ $\mu_{21}=\mu_{22}$	14	-1418	<0.001

Therefore, a full Weibull MRC model for the data used in this study requires six parameters for transition probability matrix corresponding to nine transitions (the remaining three parameters can be obtained under the restriction that the row sum equals to one), nine scale and nine shape parameters corresponding to nine different transitions for the inter-occurrence times. We have also fitted different models by reducing the number of parameters of the Weibull distribution. Five such important models are given in Table 2.

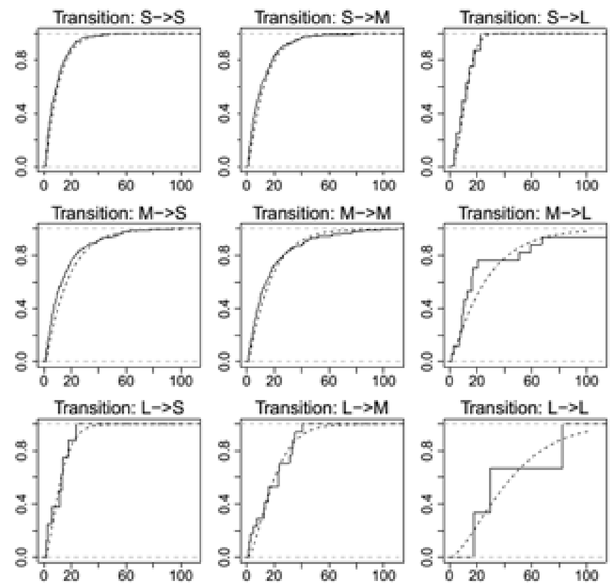

Fig. 3. Comparison between empirical (step-function) and estimated (dashed line) distributions of the inter-event times of earthquakes

We have performed the likelihood ratio test and p-values show that Model 1 (full Weibull) can be transformed to Model 4 by appropriately choosing the scale and shape parameters. However, 6 further reduction of parameters (Model 5) is not feasible as it decreases the likelihood significantly (*p*-value < 0.001). Therefore, we deem Model 4 the best model for further analysis. A graphical comparison is also made to check the fit of the Weibull distributions through plotting empirical distributions along with the estimated distributions of the inter-event times of earthquakes for each transition type (Fig 3). As the fitted line is closer to the empirical line, it may be inferred that the data do not contradict the choice of Model 4.Using the fitted model (Model 4), we obtain the following estimates of parameters of the transition probability matrix (TPM) P, shape parameters α and scale parameters μ.

$$\hat{P} = \begin{array}{c} \\ S \\ M \\ L \end{array} \begin{array}{c} \begin{array}{ccc} S & M & L \end{array} \\ \begin{pmatrix} 0.640 & 0.351 & 0.009 \\ 0.443 & 0.532 & 0.025 \\ 0.286 & 0.607 & 0.107 \end{pmatrix} \end{array}$$

$$\hat{\alpha} = \begin{array}{c} \\ S \\ M \\ L \end{array} \begin{array}{c} \begin{array}{ccc} S & M & L \end{array} \\ \begin{pmatrix} 1.07 & 1.07 & 1.94 \\ 1.07 & 1.07 & 1.07 \\ 1.43 & 1.43 & 1.43 \end{pmatrix} \end{array},$$

$$and\ \hat{\mu} = \begin{matrix} \\ S \\ M \\ L \end{matrix} \begin{matrix} \begin{matrix} S & M & L \end{matrix} \\ \begin{pmatrix} 9.88 & 12.37 & 12.37 \\ 16.88 & 16.88 & 26.43 \\ 12.37 & 20.56 & 46.97 \end{pmatrix} \end{matrix}.$$

Fitted mean inter-occurrence times (in days) for each transition type are given in the following matrix

$$\text{Mean inter–occurence times(in days)} = \begin{matrix} \\ S \\ M \\ L \end{matrix} \begin{matrix} \begin{matrix} S & M & L \end{matrix} \\ \begin{pmatrix} 10 & 12 & 11 \\ 16 & 16 & 26 \\ 11 & 19 & 43 \end{pmatrix} \end{matrix}$$

The estimates of transition probabilities ($\widehat{\text{P}}$) show that the likelihood of occurring a small earthquake (S) is higher than that of a medium earthquake (M) or a large earthquake (L) given that the last event is S. We see that the next earthquake will be of type S with 64% chances or of type M with 35% chance or of type L with 0.9% chance if the last earthquake is a small one according to the TPM. Medium earthquakes (M) are more likely to occur being known that the last event is M or L. The stationary probabilities (equation 1) of a small, a medium and a large earthquake are 55%, 43% and 2%, respectively. It may be noted that the mean observed inter-occurrence times nearly coincides to the fitted mean inter-occurrence times for each category of earthquakes.One of the major objectives of this study is to obtain the probability forecast (equation 10) of next event using the fitted model. The probability of occurring a specific type of earthquake (S/M/L), being known that the last earthquake was S or M or L, has been evaluated for different values of elapsed time (t_0) and time ahead (t^*)(Table 3). The probabilities of occurring an S immediately (t0 = 0) after the last earthquake that was an S are 0.369; 0.450; 0.464 and 0.467 for t^*= 15; 30; 45 and 60 days, respectively. We see that the probability of occurring an S is higher than the probability of occurring an M or an L knowing that last event was an S for different values of t_0 and t^*. On the other hand, the probability of occurring an L is higher than that of an S or an M given that the last earthquake was an L. The situation remains the same for all the values of t_0 and t^* considered in this study.

Table 3. Probability of occurrence of next event for different *t and different *t₀* given the last state**

		Forecast probability								
		Last event in S			Last event in M			Last event in L		
t_0	t* (days)	S-S	S-M	S-L	M-S	M-M	M-L	L-S	L-M	L-L
0 days	15	0.34	0.23	0.16	0.14	0.21	0.17	0.05	0.08	0.14
	30	0.45	0.30	0.21	0.20	0.30	0.28	0.06	0.14	0.31
	45	0.46	0.32	0.21	0.22	0.34	0.34	0.07	0.17	0.46
	60	0.47	0.32	0.21	0.23	0.35	0.37	0.07	0.17	0.58
30 days	15	0.39	0.24	0.21	0.15	0.23	0.19	0.06	0.13	0.26
	30	0.46	0.31	0.21	0.21	0.31	0.29	0.07	0.16	0.45
	45	0.47	0.32	0.21	0.23	0.34	0.35	0.07	0.17	0.58
	60	0.47	0.32	0.21	0.23	0.35	0.38	0.07	0.17	0.66
60 days	15	0.39	0.23	0.21	0.15	0.23	0.19	0.06	0.14	0.31
	30	0.46	0.31	0.21	0.21	0.31	0.30	0.07	0.17	0.51
	45	0.47	0.32	0.21	0.23	0.34	0.36	0.07	0.17	0.63
	60	0.47	0.32	0.21	0.23	0.35	0.40	0.07	0.17	0.69

The forecast probabilities that the transition of the type M to M will happen within 15 and 30 days immediately after the last event that was an M are 0.209 and 0.301, respectively. The probability of occurring an M is higher than that of an S or an L when the last earthquake was an M for $t_0 = 0$ and $t^* = 15$ and 30. For $t_0 = 0$ and $t^* = 15$ and 60, the probability of occurring an L is higher than that of an S or an M when the last event was an M. Both situations remain the same for $t_0 = 30$and 60 days.

Table 4. Probability of occurrence of next event evaluated for different *t with *t₀=45* days given last event was an *S***

	Forecast probability		
t*	S-S	S-M	S-L
15 days	0.391	0.246	0.209
30 days	0.455	0.305	0.209
45 days	0.466	0.320	0.209
60 days	0.467	0.323	0.209

It is observed that the likelihood of occurring any type of earthquake goes high with the increase of t^*. In accordance with our data set, the last earthquake occurred was an S. Therefore, it would be interesting to forecast the next event given that the last event was an S for different values of $t^*$and $t_0 = 45$ since 45 days have been elapsed since the last earthquake. Results are presented in Table 4. The probability of occurring an S within the next 15 days is the highest which is 0.391 given that the last event was a small earthquake (S).The corresponding probabilities of happening an M and an L are 0.246 and 0.209, respectively and the situation remains the same for $t^* = 30, 45\ and$ 60 days.

The average recurrence times (equation 11) for each type of earthquake (S/M/L) using Markov renewal model have been computed (Table 5). The results indicates that the mean recurrence period for a large earthquake is the highest which is about 2 years and it becomes the lowest for a small earthquake(24 days) .

Table 5. Recurrence periods for each type of earthquake

Types of earthquake	Average recurrence period (in days)
Small (S)	24
Medium (M)	31
Large (L)	758

V Conclusions

Earthquake occurrence has become a great threat for Bangladesh as several major cities of the country are exposed to high risk of large earthquakes. A Weibull Markov renewal model has been proposed to capture the earthquake occurrences in Bangladesh. The model is capable of testing several nested hypotheses, for instance, whether the process can further be reduced to a sub-model. An optimal model has been chosen to obtain probability forecasts of different types (small, moderate or large) of earthquakes for various lengths of time interval using the dataset of earthquakes occurred from 1961 to 2013. The results indicate that the probability of occurring moderate earthquakes is considerably high in the Bangladesh area. Furthermore, the country has also a high risk of occurring large earthquakes in every two years. We believe that the results emerged from this paper would be helpful to the planners for drawing inference and taking necessary measures to face earthquake hazards.

References

1. Cummins, P. R., 2007. The potential for giant tsunamigenic earthquakes in the northern Bay of Bengal. *Nature* **449,** 75–8.
2. Sarker, J. K., M. A. Ansary, M. S. Rahman and A. M. M. Safiullah, 2009. Seismic hazard assessment for Mymensingh, Bangladesh. *Environmental Earth Sciences.* **60,** 643–653.
3. Vere-Jones, D., 1995. Forecasting earthquakes and earthquake risk. *International Journal of Forecasting* **11,** 503–538.
4. Ogata, Y., 1995. Evaluation of probability forecasts of events. *International Journal of Forecasting* **11,** 539–541
5. Ogata, Y., 1988. Statistical Models for Earthquake Occurrences and Residual Analysis for Point Processes. *Journal of the American Statistical Association.* **83,** 9–27.
6. Ogata, Y., 1998. Space-time point-process models for earthquake occurrences. *Annals of the Institute of Statistical Mathematics.* **50,** 379–402.
7. Zhuang, J., Y., Ogata and D. Vere-Jones, 2002. Stochastic Declustering of Space-Time Earthquake Occurrences. *Journal of the American Statistical Association.* **97,** 369–380.
8. Ogata, Y., K. Katsura and M. Tanemura, 2003. Modelling heterogeneous space-time occurrences of earthquakes and its residual analysis. *Journal of the Royal Statistical Society. Series C: Applied Statistics.* **52,** 499–509.
9. Schoenberg, F. P., 2003. Multidimensional Residual Analysis of Point Process Models for Earthquake Occurrences. *Journal of the American Statistical Association.* **98,** 789–795.
10. Alvarez, E. E., 2005. Estimation in Stationary Markov Renewal Processes, with Application to Earthquake Forecasting in Turkey. *Methodology and Computing in Applied Probability. Probab.***7,** 119–130.
11. Garavaglia, E. and R. Pavani, 2009. About Earthquake Forecasting by Markov Renewal Processes. *Methodology and Computing in Applied Probability.* **13,** 155–169.
12. Foufoula-Georgiou, E. and D. P. Lettenmaier, 1987. A Markov Renewal Model for rainfall occurrences. *Water Resources Research.* **23,** 875–884.
13. Asaduzzaman, M. and A. H. M. M. Latif, 2014. A parametric Markov renewal model for predicting tropical cyclones in Bangladesh. *Natural Hazards.* **73,** 597–612.
14. Dabrowska, D.M., G.W. Sun and M. M. Horowitz, 1994. Cox Regression in a Markov Renewal Model: An Application to the Analysis of Bone Marrow Transplant Data. *Journal of the American Statistical Association.* **89,** 867–877.
15. Gilbert, G., G. L. Peterson and J. L. Schofer, 1972. Markov Renewal Model of Linked Trip Travel Behavior. *Transportation Engineering Journal of ASCE.* **98,** 691–704.
16. Ghosn, M. and F. Moses, 1985. Markov Renewal Model for Maximum Bridge Loading *Journal of Engineering Mechanics.* **111,** 1093–1104.
17. Pyke, R., 1961. Markov Renewal Processes: Definitions and Preliminary Properties. *The Annals of Mathematical Statistics.* **32,** 1231–1242.
18. Limnios, N. and G. Oprişan, 2001. *Semi-Markov Processes and Reliability*. Birkhäuser Boston.