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Blur Face Recognition using Blur Metric and Some Variants of Supervised Distance Preserving Projection

Mohiuddin Muhi, Sohana Jahan*, and Md. Anwarul Islam Bhuiyan

Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh

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Abstract

This paper is focused on face recognition techniques in uncontrolled scenarios, specifically on the recognition of face images with blur effects. At first, the blur level of the testing image is determined using recently proposed blur metric. This blur metric value is used to blur the training set of gallery images using Gaussian filter. The blur level of training images is the same as that of the testing image. Two variants of Supervised Distance Preserving Projection (SDPP), SDPP as Semidefinite Least Square (SLS-SDPP) and Regularized Supervised Distance Preserving Projection (RSDPP), are used for extracting effective features of training and testing images. K-Nearest Neighbor classifier is used for matching. Numerical experiments were carried out on two benchmarking face data ORL and Yale. The performances of SLS-SDPP and RSDPP are compared with one of the leading methods Eigenface method. Experimental results show that the combination of blur metric and the feature extraction methods achieved outstanding performance in recognizing blur images of different levels and also outperforms the base methods and Eigenface method.

Keywords: Blur Metric, K-NN, SLS-SDPP, RSDPP, Eigenface.

I. Introduction

Face recognition is one of the most challenging research areas in pattern recognition and computer vision, which has multiple applications in biometrics, information security, access control, and surveillance systems. Its application can also be found in areas such as video coding, crowd surveillance, video conferencing, and intelligent human–computer interfaces[1]. In recent years, almost every infrastructure, industry, and institution have invested heavily to maintain public safety[2].

A general face recognition problem is as follows:

Given two sets of face images. One set known as training set in which each image is labelled with the respective person's identity. Another set is testing set which consists of face images that are unlabelled and are from the same group of people. The target is to determine the identity of each person from the test.

Face recognition techniques can identify facial features that can give high compatibility in machine-readable travel documentation[3]. In last 30 years, face recognition techniques have gone through a lot of theoretical advancement[4]. Despite the advancements, many internal and external factors still strongly influence the effectiveness of face recognition. For instance, occlusion, poor illumination, blur image, and low-quality image have an enormous effect on the accuracy of face recognition[5]. That is, in uncontrolled acquisition conditions (blocking effects, blur effects, lighting conditions, facial pose change in large scale) the performances of face recognition system drop drastically. The blur of the image expresses a loss of detail and decline of the edge sharpness in the content space.

Blur is one of the important factors of image quality degradation that has a significant impact on face recognition performance. An image may have blur effect for several reason such as movement of the camera or the subjects during the exposure[6], if camera is not in focus[7], or if the device used to take image is of poor quality. For example, analogue web camera or the auto-focus of the digital pocket camera does not focus into the face area properly[8]. An accurate estimation of blur is the first stage in reconstructing a sharp image from its blurry form. Therefore, it is necessary to establish a metric for determining an image's blurriness value[9].

The majority of research has concentrated on different methods for different uncontrolled scenario such as lighting circumstances, or for overcoming the problem caused by huge facial position changes[10]. Despite the fact that blur is a major contributing factor to image quality degradation, there has been very little research on the subject. Most face recognition algorithms are not sufficiently robust in such conditions, resulting in a relatively low recognition rate[2].

In this paper, we have worked on face recognition techniques with blur images. We have used a no-reference blur metric (BluM) estimation approach discussed in[2,11] for identifying blur level of the test images. In a face recognition system, an image can be thought as a high dimensional vector. Each of the coordinates of the vector corresponds to a pixel value of the image. Working directly on these high dimensional vectors may have huge storage demands as well as would lead to high computational costs. Also using all the features of an image for recognition may be misleading. So, pre-processing of the data is necessary for effective feature selection and hence for better classification. In this paper, for feature extraction, we have implemented two variants of dimension reduction method SDPP[12], RSDPP[13] and SLS-SDPP[14].

*Author for correspondence. e-mail: jahansohana@gmail.com

For projecting n high dimensional data points $\{x_1, x_2, \ldots, x_n\}$ to a lower-dimensional space, SDPP minimizes the stress $F(W) = \frac{1}{n}\sum ijG_{ij}\left(d_{ij}^2(W) - \delta_{ij}^2\right)^2$ to determine the transformation matrix W. RSDPP incorporates a regularization term, $\lambda vec(\mathrm{W})^T\ vec(\mathrm{W})$, to the formulation of SDPP. The modified cost function is given by: $J(W) = \frac{1}{n}\sum_{ij} G_{ij}\left(d_{ij}^2(W) - \delta_{ij}^2\right)^2 + \lambda vec(\mathrm{W})^T\ vec(\mathrm{W})$ In SLS-SDPP the author modified the objective function of (SDPP) by incorporating the total variance $\sum_{j=1}^{n} \| z_i \|^2$ of transformed co-variates and maximizes the function $P(W) = \sum_{i=1}^{n} \| z_i \|^2 - \frac{v}{n}\sum_{ij} G_{ij}\left(d_{ij}^2(W) - \delta_{ij}^2\right)^2$
The performances of these algorithms are compared with one of the leading methods Eigenface.

We have implemented these algorithms to extract effective features of testing images with blur effects. We have applied these feature extraction algorithms to two cases.

i. Without any previous information of blur level (RSDPP, SLS-SDPP, Eigenface).

ii. Using the blur metric value of the test images before extracting the features (BluM+RSDPP, BluM-SLS-SDPP, BluM+Eigenface).

Numerical experiments carried out on two well established face data set Yale and ORL. Using the blur metric value in pre-processing step showed remarkable performance of both SLS-SDPP and RSDPP over the base methods and Eigenface method.

The following sections of the article are organised as follows:

At first, we have discussed the blur metric estimation procedure in the next section. In the following section, we have briefly discussed the feature extraction algorithms SDPP, SLS-SDPP, RSDPP, and Eigenface method. In section III, Numerical experiment is documented. Here we have included a short description of Yale and ORL datasets followed by the pre-processing step where we have explained the generation of the training set and testing set of different blur levels. A comparison of the performance of BluM+RSDPP, BluM+SLS-SDPP, BluM+Eigenface is done with the base methods. Our findings are summarized in section IV.

II. Methodologies

Blur level estimation using Blur Metric

Blurring has always been a big concern in image processing applications. There are several types of blurring. For instance, the relative motion between the camera and the scene causes the motion blur, and lens imperfections or defocused camera causes out-of-focus blur[15]. A clear image has a high-frequency information which can be lost for the blur effect. So, a low-pass filter may be used to reproduce it. An increase in blur level of an image results the neighboring pixels to be in the same gray level. Thus, blurring a sharp image have a huge impact on the pixels gray levels. In this case the neighboring pixels shift significantly[11]. Blur is more noticeable at edges and in textured regions[16]. In this paper, for identifying blurred faces, we have implemented an approach which is based on a no-reference blur metric (BluM)[2,11]. The methodologies of determining blur metrics can be described as follows.

Let T be an image of size of $m \times n$ pixels. First step to generate a blurred image B by blurring the clear image T. To do this, two strong low-pass filter is chosen, a horizontal (h_h) and a vertical (h_h) to generate B_{Hor} and B_{Hor}, where, $h_v = \frac{1}{8}\times[11111111]$; $h_h = transpose(h_v) = h_v{}'$; $B_{Ver} = h_v * T$; $B_{Hor} = h_h * T$

Then, differences between neighboring pixels are evaluated, by computing the absolute difference images D_T_{Ver} and D_T_{Hor} D_B_{Ver} and D_B_{Hor} as follows:

$$D\_T_{Ver}(i,j) = Abs\left(T(i,j) - T(i-1,j)\right); 1 \le i \le m-1, 0 \le j \le n-1$$

$$D\_T_{Hor}(i,j) = Abs\left(T(i,j) - T(i,j-1)\right); 1 \le j \le n-1, 0 \le i \le m-1$$

$$D\_B_{Ver}(i,j) = Abs\left(B_{Ver}(i,j) - B_{Ver}(i-1,j)\right); 1 \le i \le m-1, 0 \le j \le n-1$$

$$D\_B_{Hor}(i,j) = Abs\left(B_{Hor}(i,j) - B_{Hor}(i,j-1)\right); 1 \le j \le n-1, 0 \le i \le m-1$$

Following the blurring stage, the neighboring pixels variations are evaluated. Large variance indicates a sharp initial picture or frame, whereas little variation indicates a blurry or less clear initial image or frame. This variation is measured by:

$$V_{Ver} = Max\left(0, D\_T_{Ver}(i,j) - D\_B_{Ver}(i,j)\right); 1 \le i \le m-1, 0 \le j \le n-1$$

$$V_{Hor} = Max\left(0, D\_T_{Hor}(i,j) - D\_B_{Hor}(i,j)\right); 1 \le j \le n-1, 0 \le i \le m-1$$

Then, the sum of the coefficients of D_T_{Ver}, D_T_{Hor}, D_V_{Ver} and D_V_{Hor} are computed to evaluate the differences from the original image and then normalized within a given range of 0 to 1:

$$s\_T_{Ver} = \sum_{i,j=1}^{m-1,n-1} D\_T_{Ver}(i,j),\quad s\_T_{Hor} = \sum_{i,j=1}^{m-1,n-1} D\_T_{Hor}(i,j)$$

$$s\_V_{Ver} = \sum_{i,j=1}^{m-1,n-1} D\_V_{Ver}(i,j),\quad s\_V_{Hor} = \sum_{i,j=1}^{m-1,n-1} D\_V_{Hor}(i,j)$$

$$b\_T_{Ver} = \frac{s\_T_{Ver} - s\_V_{Ver}}{s\_T_{Ver}},\quad b\_T_{Hor} = \frac{s\_T_{Hor} - s\_V_{Hor}}{s\_T_{Hor}}$$

Then, the blur level is chosen as the maximum of the vertical one and the horizontal one.

$$blur\_T = Max\left(b\quad T_{Ver}, b\_T_{Hor}\right)$$

This produced ranging from 0 to 1. With 0 and 1 being the greatest and worst blur perception quality, respectively. Fig.1 represents samples of blur images of different blur level of a gallery image T with their respective blur metric values $blur\_T$

1 2 3 4 5 6

Fig.1. A Sample of blur images with their respective blur metric values

SDPP and Its Variants for Feature Extraction

SDPP

SDPP[12] is a dimensionality reduction technique proposed recently by Zhu *et al*. (2013). The method is used to extract important features by reducing the dimension of higher dimensional data. This is done by projecting data to a lower-dimensional space.

Given n data points $\{x_1, x_2, \ldots, x_n\}$, where $x_i \in \mathbb{R}m$ and their responses/class level $\{y_1, y_2, \ldots, y_n\}$. SDPP projects the data through the linear function $f: \mathbb{R}m \rightarrow \mathbb{R}r$ defined by $f(x) = W^T x$,where $W \in \mathbb{R}mr$ is known as transformation matrix. The method minimizes the difference between the distances $d_{ij}^2(W)$ of data in the projected space and the distances δ_{ij}^2 of the responses. Thus, the objective function of SDPP can be written as:

$$F(W) = \frac{1}{n}\sum_{ij} G_{ij}\left(d_{ij}^2(W) - \delta_{ij}^2\right)^2 \tag{2}$$

where, G_{ij} is the neighborhood graph defined by

$$G_{ij} = \begin{cases} 1 & if\ i \sim j\ (k\text{-}\ N\ Nneighbor) \\ 0 & otherwise. \end{cases}$$

$d_{ij}^{\ 2}(W)$ represents Euclidean metric $d_{ij}^{\ 2}(W) = \|z_i - z_j\|^2$. that is used to characterize the pairwise distances in Z space. The response function δ_{ij} has the following form:

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \sim j(x_i\ and\ x_j\ belongs\ to\ same\ class) \\ 1 & \text{otherwise,} \end{cases}$$

RSDPP

RSDPP proposed by Alencar *et al*.[13] is a modified version of the SDPP. RSDPP incorporates a regularization term, $\lambda vec(\mathrm{W})^T vec(\mathrm{W})$, to the formulation of SDPP. Thus, the target is to minimize modified cost function given by:

$$J(W) = \frac{1}{n}\sum_{ij} G_{ij}\left(d_{ij}^2(W) - \delta_{ij}^2\right)^2 + \lambda vec(\mathrm{W})^T vec(\mathrm{W}) \tag{3}$$

where, the regularization parameter is λ and $vec(.)$ is an operator which is used to represent the matrix into a vector. Substituting $D_{ij} = d_{ij}^2(W)$ and $\Delta_{ij} = \delta_{ij}^2$, the objective function can be written as:

$$J(W) = \frac{1}{n}\sum_{ij} G_{ij}\left(D_{ij} - \Delta_{ij}\right)^2 + \lambda vec(\mathrm{W})^T vec(\mathrm{W}) \tag{4}$$

The gradient of J with respect to W is equal to ∇ given by:

$$\nabla_W J = \frac{4}{n} P^T (S - R) PW + 2\lambda W .$$

Here, the ith row of P represents the data point x_i, R is a symmetric matrix with $R = Q + Q^T$ where, $Q = G \odot (D - \Delta)$ and S is a diagonal matrix whose diagonal entries are $S_{ii} = \sum_j R_{ij}$, Conjugate-Gradient (CG) optimization[3] technique is used to minimize Eq. (4) and Eq. 2 of RSDPP and SDPP respectively.

SLS-SDPP

SLS-SDPP[14] proposed by Jahan[14] modified the objective function of (SDPP) by incorporating the total variance $\sum_{i=1}^{n} \| z_i \|^2$ of transformed co-variates and maximizes the function

$$\max P(W) = \sum_{i=1}^{n} \| z_i \|^2 - \frac{v}{n}\sum_{ij} G_{ij}\left(d_{ij}^2(W) - \delta_{ij}^2\right)^2 \tag{5}$$

After some algebraic manipulations[14,17] the objective function takes the Matrix optimization form

$$\max \psi, X - \frac{1}{n}\|U\|^2$$

$$s.t.\ \mathcal{A}X - U = b$$

$$X \in \mathcal{S}_+^m$$

SLS_SDPP uses a two block Alternating Direction Method of Multiplier (ADMM) to get the required result. SLS-SDPP[17] is used to face recognition problem along with blur effect.

Eigenface

Eigenface method[18,19] is one of the oldest methods which very effective for image recognition. For reduction the dimension of the image space, it uses Principal Component Analysis (PCA)[20,21]. The method uses the total variance matrix $S \in S^n$ defined by: $S = \sum_{i=1}^{n}(x_i - \mu)(x_i - \mu)^t$ where $\mu \in \mathbb{R}$ is the mean image of all samples, to determine the matrix W that maximizes $|W^T SW|$ which is known as the total scatter matrix of the projected sample. Here W, the transformation matrix is an $n \times m$ matrix. Each of the m columns of W represent m eigenvectors of S. These eigenvectors correspond to m largest eigenvalues of S and is n dimensional which is same as the original images. The eigenvectors are known as Eigenfaces.

Numerical Experiment

This section illustrates experimental results obtained by RSDPP, SLS-SDPP and Eigenface method.

At first, we have determined the blur level of the test image using BluM and got the blur metric value. We used the information regarding this blur metric value to blur the training set of images up to that level of the test image. After that, feature extraction techniques RSDPP, SLS-SDPP, and Eigenface discussed in the previous section are individually implemented to extract effective features of testing and training images. Finally, the K-Nearest Neighbor[20,21] algorithm is used to determine the class information of the testing image. We have used the 1-Nearest Neighbordor our experiment. We have calculated the identification/recognition rate by taking the ratio of the number of correct recognitions to the number of test images. In the next section, we will observed that the each feature extraction method shows and improvement in their performance if the blur metric is implemented in pre-processing step. The results are well documented in tables. Improvement of SLS-SDPP and RSDPP are shown in graphs as well.

Data set description

We have implemented our approach on two well recognized face data Yale and ORL databases. Yale face database is constructed in Computer Science and Engineering Department of University of generated at AT&T laboratories Cambridge. The processed data sets are collected from Cai et al[22]. All the face images are manually cropped to make of size of 64×64 pixels, each pixel with 256 gray levels. Each pixel information corresponds to a dimension. So, an image is represented by 4096-dimensional vector.

Olivetti Research Laboratory (ORL) dataset

The ORL data set contains frontal images of 40 subjects. Each subject has 10 different images of size 92×112. The images have different variations such as, they were taken at different times having different lighting condition, some has distinct facial expressions or facial details (open eyes/closed eyes, smiling face/ not smiling face, with glasses / without glasses). For each of these images, background was chosen dark homogeneous. A sample of ORL faces are shown in Fig. 2(a)

Yale dataset

The Yale face database contains images of 15 individuals with 11 images per person. Thus, the dataset has in total 165 grayscale frontal images of size 243×320. The images have varying facial expressions: sad, happy, surprised, sleepy and wink. Images are taken placing the light in different point (left, centre, right), one normal image under ambient lighting, each person has one image with glasses Fig. 2(b) depicts image samples of an individual of Yale face data.

Fig. 2. (a) Sample images of ORL dataset

Fig. 2. (b) Sample images of Yale dataset

Pre-processing step

At first a set of blurred images are generated artificially from Yale and ORL datasets. Gaussian filter is used for the degradation step. This is done by convolving the images using a rotationally symmetric Gaussian filter $F_{g(s,h)}$, where the standard deviation s takes one of the values $s = 0;1;2;\ldots;5$ and $h = 3;5;7;9;11$. Here h represents the size of a square matrix. For different values of the parameters s and h, original gallery $T_{(0;0)}$ were convolved with the given filter $F_{g(s,h)}$ and thus, was split into 25 others galleries $T_{i(s;h)}, i = 1,2,\ldots,25$. The Blur Metric BluM discussed in section 2.2 is used to get a blur level $b_{i(s;h)}$ of each blurry image $T_{i(s;h)}$.

Fig. 3. Blur image $T_{(s;h)}$ of different level

$$B_m = [b_{(s,h)}] = \begin{bmatrix} 0.6293 & 0.6847 & 0.6899 & 0.6900 & 0.6910 \\ 0.6400 & 0.7586 & 0.8346 & 0.8620 & 0.8690 \\ 0.6413 & 0.7674 & 0.8686 & 0.9261 & 0.9463 \\ 0.6417 & 0.7699 & 0.8748 & 0.9404 & 0.9579 \\ 0.6419 & 0.7710 & 0.8757 & 0.9410 & 0.9502 \end{bmatrix}$$

$s = 1,2,\ldots,5$

$h = 3,5,\ldots,11$

Blur Metric value $b_{(s,h)}$ for different blur level of the images in Fig.3

Figure. 3. represents blur faces of different level of an image (Gallery 1) of ORL dataset. The 5×5 matrix B contains the blur metric values of the corresponding blur faces shown in Fig 3.

Selection of Blurred Gallery Set for Matching

At first, the level blur b_t ranging from 0 to 1 is estimated for each test image is estimated. To select the blurred gallery set, this estimated value b_t is compared to all the $b_{(s,h)}$ which are computed during the pre-processing step. Thus, the gallery set is selected by minimizing the function

$$(s,h) = \underset{s,h}{\boldsymbol{arg\ min}} \left\| b_{(s,h)} - b_t \right\| \qquad (6)$$

These optimum values of s and h . are used to determine the blurred gallery set which will be used for feature extraction using dimension reduction techniques and then for recognition. Finally, feature extraction algorithms RSDPP, SLS-SDPP and Eigenface method are used to extract the effective features. For matching purposes, we have used the K-NN[20,21] algorithm. These steps are shown in the flowchart of Fig. 4

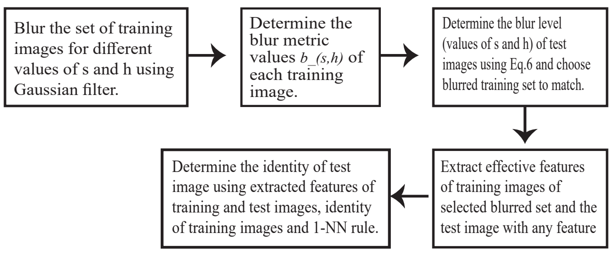
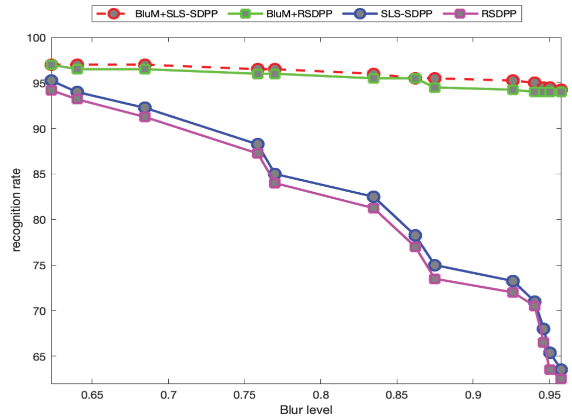


Fig. 4. Basic steps of blur face recognition procedure using blur metric

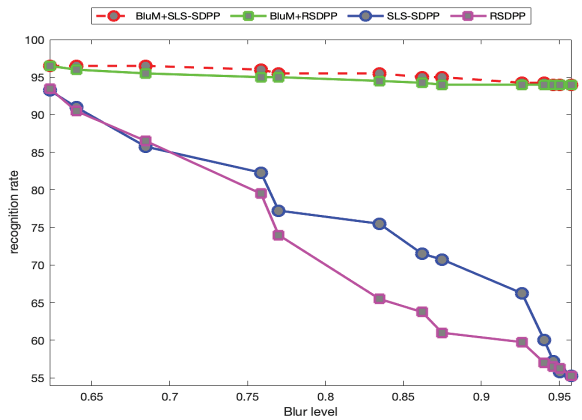
Experimental Result

We have divided the data set into training set and testing set. This split is done by in different ways such as taking 5 images per person for training and rest for testing. Similarly, 6, 7 or 8 images can be chosen randomly with class level for training. The split is done is such a way that there is no overlap of images across training and testing samples.

In Fig. 5, the performance of SLS-SDPP and RSDPP are shown. Here we have used 8 training images per person for both the data set. BluM+SLS-SDPP indicates blur metric is used for selecting blurred gallery for matching and then SLS-SDPP used for feature extraction. A similar approach was applied for BluM+RSDPP. It can be clearly observed that using blur metric to select gallery images for recognition, significantly improves the efficiency of both the algorithms in terms of identification. For the ORL data set, with the increase of blur level, the recognition rate obtained by SLS-SDPP and RSDPP dropped drastically. It is observed from Table 1 that the recognition rate dropped from $\sim 95\%$ to $\sim 63\%$ for SLS-SDPP with the increase of blur level and for RSDPP this drop rate is from $\sim 94\%$ to $\sim 62\%$. A similar drop in recognition rate is seen for the Yale data set. Using BluM, both the methods achieved a very good recognition rate, almost constant for any blur level, varying between $\sim 97\%$ and $\sim 94\%$ for ORL data and $\sim 96\%$ to $\sim 94\%$ for Yale dataset. Thus, the combination of BluM with these feature extraction methods makes the algorithm more tolerant to blur effect. The results are well documented in Table 1 and Table 2.



(a) ORL



(b) Yale

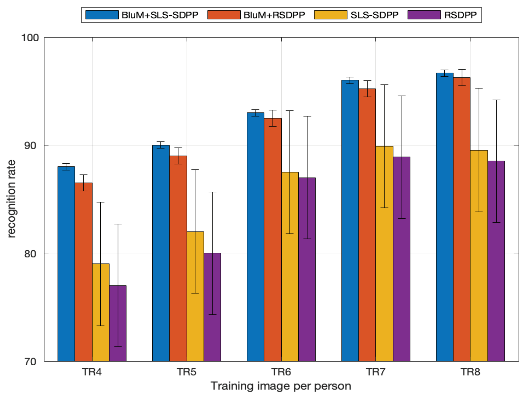
Fig. 5. (a) and (b) represent performance of the methods SLS-SDPP and RSDPP with and without using blur metric for different level of blur images. For each of the data sets, efficiency of SLS-SDPP and RSDPP improved significantly if blur metric used to select the blur level of the test image.

Table 1. Recognition rate along different level of blur of ORL data

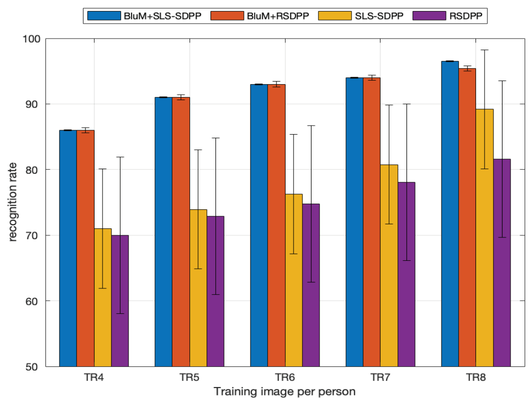
Blur metric Value	BLuM+ SLS-SDPP	SLS-SDPP	BLuM+ RSDPP	RSDPP	BLuM+ Eigenface	Eigenface
0.62	97%	95.25%	97%	94.2%	90%	84.2%
0.75	96.5%	88.25%	96%	87.25%	89.25%	83%
0.83	96%	82.5%	95.5%	81.25 %	88%	82.5 %
0.92	95.25%	73.25%	94.25%	72%	87.5%	78.5%
0.95	94.2%	63.50%	94%	62.5%	87%	74%

Table 2. Recognition rate along different level of blur of Yale data

Blur metric Value	BLuM+ SLS-SDPP	SLS-SDPP	BLuM+ RSDPP	RSDPP	BLuM+ Eigenface	Eigenface
0.62	96.5%	93.25%	96.5%	93%	82%	64%
0.75	96%	82.3%	95%	79.5%	81.5%	63.5%
0.83	95%	75.5%	94.5%	65.5 %	81%	63.5 %
0.92	94.25%	66.25%	94%	59.75%	80.75%	63%
0.95	94%	55.25%	94%	55%	80%	63%



(a) ORL



(b) Yale

Fig. 6. Bar diagrams (a) and (b) represent average recognition rate along different numbers of training images of ORL data and Yale data. Here TRp stands for p images per person taken for training purposes. Error bar represents the consistency of performance of the algorithms for different sets of training images.

In Fig. 6 we can see the performance of the methods for different numbers of training images. It is clear that all the four approaches improve with the increase in training data size. For the Yale dataset the recognition rate obtained by BluM+SLS-SDPP varies from 86% to 97% and for ORL this variation is from 88% to 97.5% for different numbers of training samples, whereas these percentage values are much lower for SLS-SDPP and RSDPP for a smaller training sample. The error bar BluM+SLS-SDPP and BluM+RSDPP imply that the recognition rate does not deviate much from the mean. Therefore, for any number of training images, BluM+SLS-SDPP and BluM+RSDPP show consistent performance. Fig. 6 clearly shows that, BluM+SLS-SDPP gives the best performance than any other approaches.

III. Conclusion

Face recognition problem is one of the most important and challenging areas of research. Though many existing methods solved this problem satisfactorily in constrained scenarios, their performance drop drastically in uncontrolled situations such as for blur images. An image may have blur for various reasons. For example, for movement of camera or subject during capture, for camera without focus, or if image is obtained by cropping from a large image, etc. Researchers are working on different techniques to get better identification of blurry images.

In this paper,

- we have worked on face recognition problem with images having blur effect.
- the blur level of a testing image is determined by a blur metric.
- for feature extraction, two variants RSDPP and SLS_SDPP of recently proposed dimension reduction method SDPP are used.

Findings

- The performances of the variants of SDPP are compared with that of the Eigenface method.
- Numerical experiments show that the combination of blur metric and the feature extraction technique (BluM+RSDPP, BluM+SLS-SDPP, BluM+Eigenface) increase the recognition rate significantly and also end up intolerant to any blur level. This means that the blur effect does not degrade the recognition rate that much.
- Thus, the implementation of blur metric shows a consistent performance of each of these methods.
- BluM+RSDPP, BluM+SLS-SDPP outperforms BluM+Eigenface and also improves the base methods RSDPP, SLS-SDPP, Eigenface. BluM +SLS-SDPP showed the best performance.

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