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Effects of Mixed Convection and Radiation Parameter on MHD Heat Transfer Flow over a Curved Stretching Sheet

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Abstract

Objective of this study is to analysis the MHD flow over a curved stretching surface. In this model we incorporated the radiation, mixed convection and partial slip parameter. Using a curvilinear coordinate system developed a mathematical model and the system of basic governing equations are converted into ordinary differential equations with appropriate transformations. The obtained results are solved by using bvp4c solver. The numerical; results for the velocity and temperature as well as the physical quantities such as skin friction and rate of heat transfer are determined and discussed through graph. Results shown that the velocity and temperature decreases with increasing values of curvature parameter and but for magnetic parameter velocity decreases whereas temperature profile increase. Comparison with previous literature are also shown in tabular form and found excellent agreement.

Keywords: MHD, curved stretching sheet, heat transfer, radiation parameter, variable thermal conductivity.

I. Introduction

Fluid flow and heat interchange with mixed convection and radiation over a curved stretching sheet is aremarkable theme, due to its applications to several over an unsteady stretching sheet make a significantobservation to the researchers. The analysisof the physical quantities change has received for vital important for modern fluid dynamics as well as bioengineering and medical equipment.

The concept of boundary layer over a stretching sheet first introduced by Tsou et al.[1]. Thereafter, Using different conditionsthe work of Tsou[1] has been extended by various authors such as (using suction and blowing by Gupta and Gupta[2]and variable thermal conductivity by Soundalgekar and Murty[3]. Abbas et al.[4]reported the analysis the MHD flow over a curved stretching sheet and they concluded that with increasing curvature parameter the exchange of heat from sheet to the fluid is slower. After that Abbas et al.[5] also analyzed the nanofluid with radiation and heat generation over a curved stretching sheet and they concluded that with increasing radiation parameter temperature distribution increased. Vajravelu and Hadjinicolaou[6] discussed the electrical conducting fluid with free stram velocity and internal heat generation. Hayat et al.[7] reported the viscous fluid with magnetic field and chemical reaction over a curved sheet.

Murtaza et al.[8]examined the MHD flow with variable thermal conductivity over a curved sheet.A laminar liquid flim with variable fluid properties are analyzed by Khan et al.[9]Hayat et al.[10] analyzed the Jeffery fluid with temperature dependent thermal conductivity. Rahman et al.[11] the micropolar fluid with variable fluid properties.

Noghrehabadi et al.[12] examined the nanofluid with thermophoresis and partial slip effect. Wange[13] investigated the exact solution of Navier Stokes equation with partial slip condition. The fluid flow and heat transfer for different condition with partial slip over a stretching sheet have been examined by many authors (such as Fang et al. [14], Sajid et al.[15], Sharma et al.[16], Noghrehabadi et al.[17], Das[18]).

Hayet et al.[19] explored the influence of radiation and mixed convection on MHD fluid with stagnation point.Ibharim and Anbessa[20] analyzed the nanofluid with variable thermal conductivity and mixed convection. Bayones et al.[21] presented the MHD flow with mixed convection wityh stretching surface.Daniel et al.[22] studied the mixed convection nanofluid with electrical conduction and thermal radiation. Fenuga et al.[23]investigated the MHD flow with mixed convection over a nonlinear sheet. Jamaludin[24] analyzed the cross fluid with mixed convection and thermal radiation with shrinking sheet.Rehman et al.[25] studied the combined effect of physical properties with mixed convection on MHD fluid over a unsteady stretching sheet.Latif et al.[26] analyzed the effect of thermal radiation and mixed convection with chemical reaction over a curved stretching surface and concluded that the temperature enriches with growing thermal radiation parameter.

The goal of current study is to discuss the effects of MHD flow over a curved stretching sheet with the presence of pressure, buoyancy force and variable thermal conductivity. The variation in this paper is the presence of radiation.

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II. Mathematical Formulation of the Problem

Consider MHD flow of an incompressible viscous fluid with mixed convection and thermal radiation passing over a curved stretching sheet.Here we used the curvilinear Coordinate's . where fluid flow is the s direction and r direction is perpendicular to it. R denoted radius of the curvature. The stretching velocity of the sheet is where (a>0) is a constant. Here we consider a uniform magnetic field whose strength B_0 is situated in the r-direction.

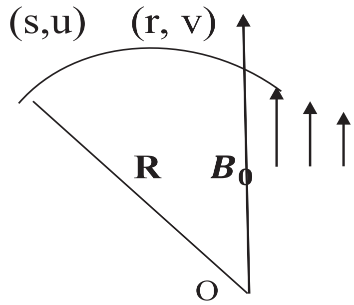


Fig.1. Model geometry

Under the above assumption, governing equations and the boundary conditions of the model

$$\frac{\partial}{\partial r}\{(r+R)v\}+R\frac{\partial u}{\partial s}=0 \quad (1)$$

$$\frac{u^2}{r+R}=\frac{1}{\rho}\frac{\partial p}{\partial r} \quad (2)$$

$$v\frac{\partial u}{\partial r}+\frac{R}{r+R}u\frac{\partial u}{\partial s}+\frac{uv}{r+R}=\frac{1}{\rho}\frac{R}{r+R}\frac{\partial p}{\partial s}+\vartheta\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r+R}\frac{\partial u}{\partial r}-\frac{u}{(r+R)^2}\right)-\frac{\sigma B_\circ^2 u}{\rho}+g\beta(T-T_\infty)(3)$$

$$\rho c_p(v\frac{\partial T}{\partial r}+\frac{R}{r+R}u\frac{\partial T}{\partial s}+\sigma B_\circ^2 u^2=\frac{1}{r+R}\frac{\partial}{\partial r}\left\{(r+R)k\frac{\partial T}{\partial r}\right\}-\frac{1}{r+R}\frac{\partial}{\partial r}(r+R)q_r+Q_\circ(T-T_\infty)(4)$$

where the velocity components and v along the x - axis and y axis respectively. c_p is specific heat at constant pressure, ρ is the fluid density, σ is electrical conductivity.

Boundary conditions

$$\left.\begin{array}{c} u=as+L\left[\frac{\partial u}{\partial r}-\frac{u}{r+R}\right], v=0, T=T_\infty at r=0 \\ u\to 0, \frac{\partial u}{\partial r}\to 0, T\to T_\infty as r\to\infty \end{array}\right\} (5)$$

III. Mathematical Analysis

Introducing the following transformations:

$$u=asf', v=-\frac{R}{R+r}\sqrt{av}\,f, p=\rho a^2 s^2 p(\eta);\ \eta=\sqrt{\frac{a}{v}}\,r, \theta=\frac{T-T_\infty}{T_w-T_\infty};$$

$$k=k_\infty\big(1+b(T-T_\infty)\big)=k_\infty(1+m\theta)(6)$$

Here k_∞ is fluidthermal conductivity; m is the thermal conductivity parameter. is fluid temperature. is fluid velocity. is curvature parameter and is magnetic number.

Transformation of Equation

Substituting the equation (6) into (2) to (5), the following dimensionless equations obtained

$$p'=\frac{f'^2}{\eta+k}(7)$$

$$f'''+\frac{k}{\eta+k}ff''+\frac{k}{(\eta+k)^2}ff'+\frac{1}{\eta+k}f''-\frac{k}{\eta+k}f'^2-\frac{1}{(\eta+k)^2}f'-\frac{2k}{\eta+k}p-M_1 f'+\lambda_1\theta=0\ (8)$$

$$(1+m\theta)\theta''+(1+m\theta)\frac{\theta'}{\eta+k}+m\theta'^2+p_r\frac{k}{\eta+k}f\theta'-M_1\lambda f'^2+\frac{R_d}{\eta+k}\theta'+R_d\theta''-p_r\lambda_1=0(9)$$

Transformation of Boundary condition:

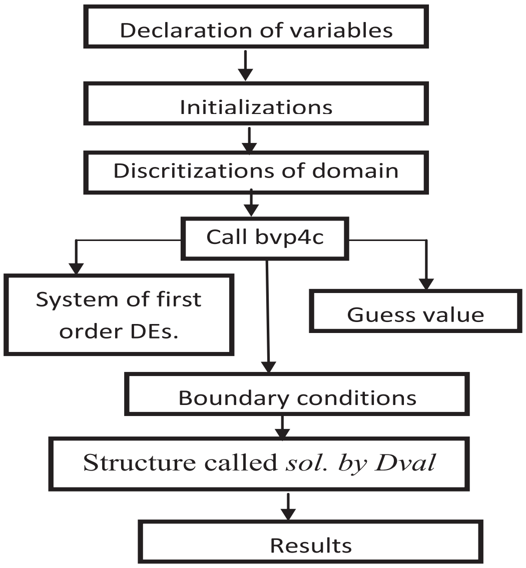
$$f(0)=1, f'(0)=1+\kappa\left[f''(0)-\frac{f'(0)}{k}\right], \theta(0)=1, f'(\infty)=0, f''(\infty)=0, \theta(\infty)=0 \quad (10)$$

Where $k=\sqrt{\frac{a}{v}}\,R, M_1=\frac{\sigma B_0^2}{\rho a}, p_r=\frac{\mu c_p}{k}, \lambda_1=\frac{g\beta(T_w-T_\infty)}{a^2 s}, q_r=\frac{-4\sigma^*\partial T^4}{3k^*\partial r}, T^4=4T_\infty^3 T-3T_\infty^4, R_d=\frac{16\sigma^* T_\infty^3}{3k^* k_\infty}, \lambda=\frac{a^2 s^2 \rho c_{p\mu}}{k_\infty(T_w-T_\infty)}$

which are defined as a curvature parameter, magnetic number,prandtl number and mixed convection parameter, radiative heat flux, dimensionless temperature, radiation parameter and viscous dissipation parameterrespectively.

IV. Numerical Method for Solution

In order to solve equations (7), (8), (9) subject to the boundary condition (5), we reformed the boundary value problem to be in a compatible form for the solver(bvp4c) function in MATLAB software.The following algorithm of bvp4c are used in this problem.



Algorithm of bvp4c

For this we needed to convert equation (7),(8) and (9) along with equations (5) into first order differential equations by assuming new variables. We estimated the initial variable in the following way

$$f = y_1, f' = y_2, f'' = y_3\,, p = y_4\,, \theta = y_5\,, \theta' = y_6\,.$$

Then the equations and boundary conditions were converted into a scheme of first order ordinary differential equations as given below

$$f' = y_2,$$

$$f'' = y_3 = y_2'$$

$$f''' = y_3' = -\frac{k}{\eta+k} y_1 y_3 - \frac{k}{(\eta+k)^2} y_1 y_2 - \frac{1}{\eta+k} y_3 + \frac{k}{\eta+k} y_2^2 + \frac{1}{(\eta+k)^2} y_2 + \frac{2k}{\eta+k} y_4 + M_1 y_2 - \lambda_1 y_5 \left(\frac{y_2^2}{\eta+k}\right) \quad (11)$$

$$\theta'' = \frac{-1}{1+my_5+r_d}\left(\frac{(1+my_5)y_6}{\eta+k} + my_6^2 + \frac{p_r k}{\eta+k} y_1 y_6 - M_1 \lambda y_2^2 + r_d \frac{y_6}{\eta+k} - p_r \lambda_1\right.$$

(12) Subject to the boundary conditions:

$$y_1(0) = 1, y_2(0) = 1 + K\left[y_3(0) - \frac{y_2(0)}{k}\right]$$

$$y_5 = 1, y_2(\infty) = 0, y_3(\infty) = y(\infty) = 0 (13)$$

Equations (11) and (12) are solved numerically with boundary value problem. All the process were completedwith the help of bvp4c solver available in MATLAB software.

V. Results and Discussion

To ensure the accuracy and validity of our study, we compared the present numerical Skin Friction values with those of Murtaza etal.[8]with various values of Curvature parameter k. The comparison shows an excellent agreement as presented in Table 1.

Table 1. Comparison with for changed values of curvature parameter k.

Curvature parameter k	Murtaza et al.[8]	Present
5	1.1591	1.158626
10	1.0745	1.079578
20	1.0363	1.031457
30	1.0241	1.023579
40	1.0181	1.017797
50	1.0145	1.019574
100	1.0070	1.004381
200	1.0030	1.003293

The results of the numerical calculations for the solution of the system (8) and (9) with boundary conditions (10) are placed in this section.

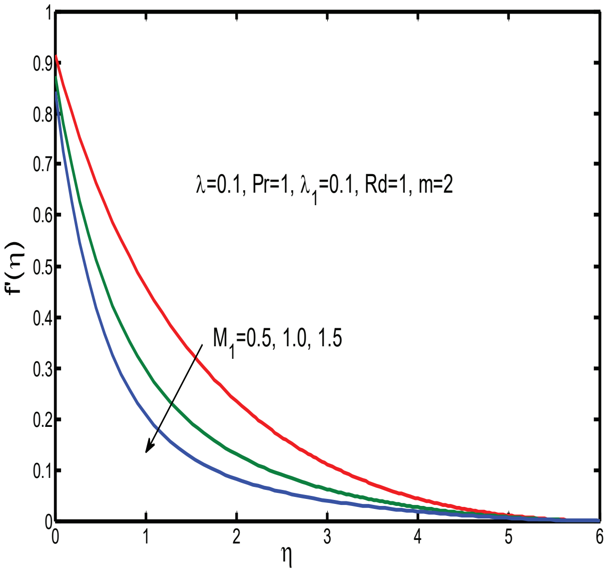


Fig. 2. Influence M_1 of on f'

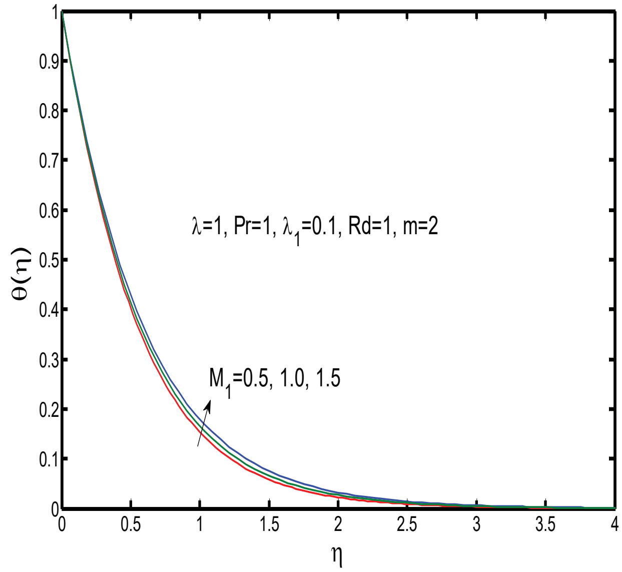


Fig. 4. Influence of M_1 on θ

When assigning values to the numerous parameters for the problem under consideration.We adopted values published in previous studies as follows:

Prandtl number p_r =1,2,3; Thermal conductivity m = 0.1,1,2 ; Magnetic number M_1= 0.5,1,1.5 Radiation parameter R_d= 0.1, 1, 2 ; Mixed convection parameter λ_1= 0.5,1, 2; Curvature parameter K = 1,3,5,10,20 ; the viscous dissipation number λ=0.1.

Figures 2-4 depict the velocity, pressure and temperature distribution for several values of magnetic number From the figures it is observed that the velocity decreases with increasing values of magnetic number whereas the pressure distribution and temperature distribution increase. This happen due to the Lorentz force which is also called the resistance force which performs counter to the flow. That's why slow down the flow with increasing the magnetic force. The resistance force increases mean the contract of fluid particles increases that's why temperature increases.

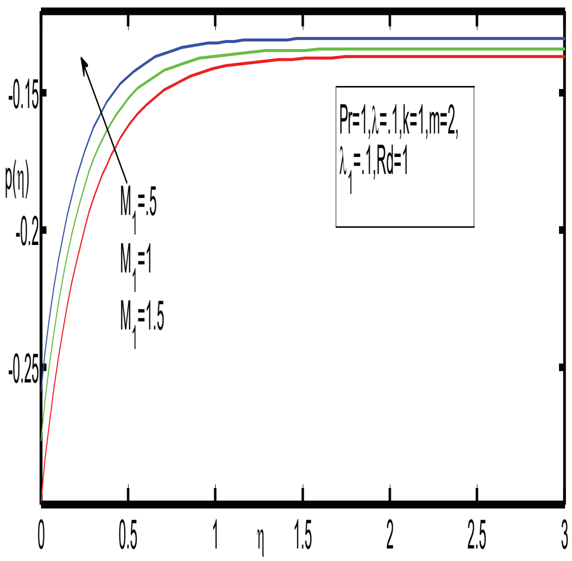


Fig. 3. Influence of M_1 on p

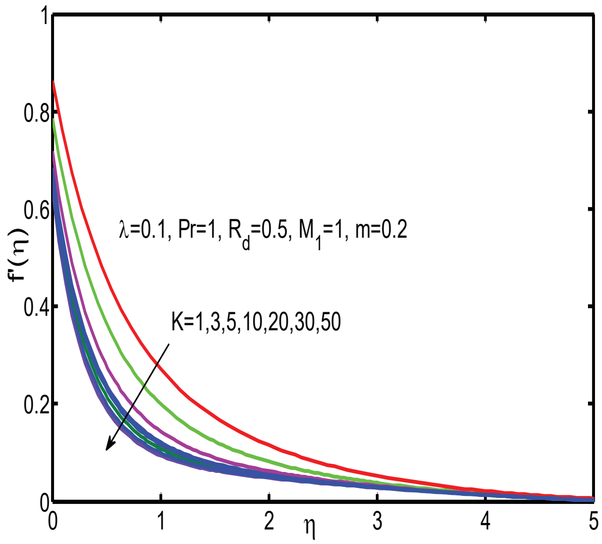


Fig. 5. Influence of k on f'

Figures 5-7 depict the velocity, pressure and temperature profile for various values of curvature parameter. From the figure it is examined that the velocity and temperature reduces with increasing values of curvature parameter and whereas pressure profile increase. It is noticed that, for bigger value of curvature parameter, reduce the velocity distribution as well as the momentum boundary layer. From fig 7 it can be seen that the temperature declines with rising of radius of curvature parameter. Note that for increasing value of radius of curvature parameter, the curve sheet converted to the flat sheet and rate of heat transfer from the sheet to fluid is slower than curved sheet.

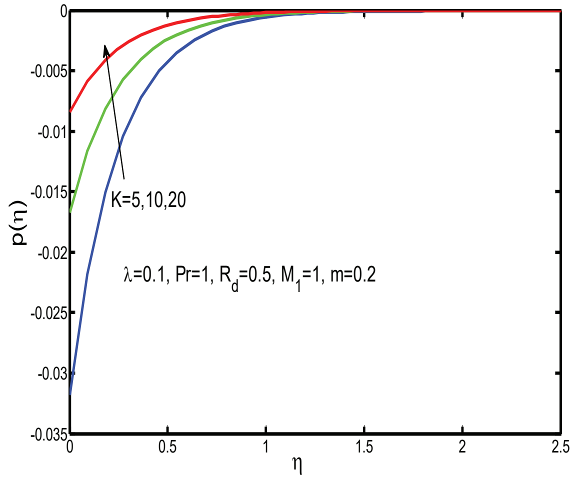


Fig. 6. Influence of k on p

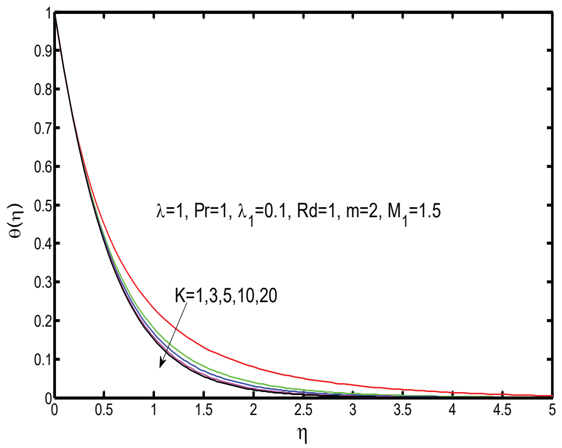


Fig. 7. Influence of k on θ

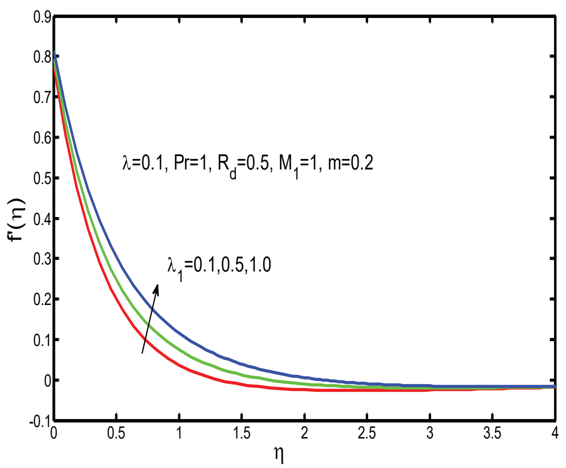


Fig. 8. Influence of λ_1 on f'

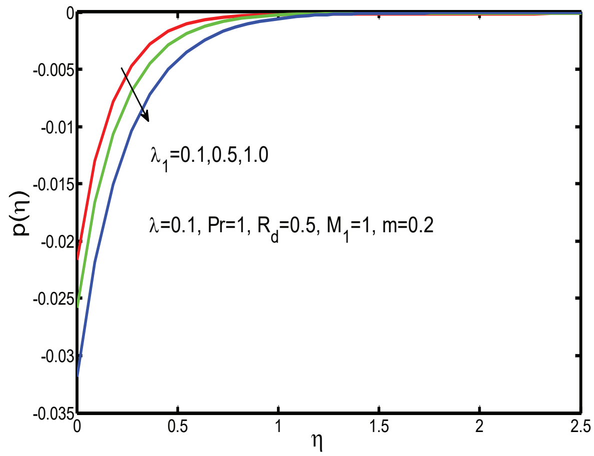


Fig. 9. Influence of λ_1 on p

Figures 8-10 depict that the velocity, temperature and pressure profile for various values of mixed convection parameter.From the figure we observed that the velocity increased with increasing values of mixed convection parameter. Whereas temperatureand pressuredecrease with mixed convection parameter.

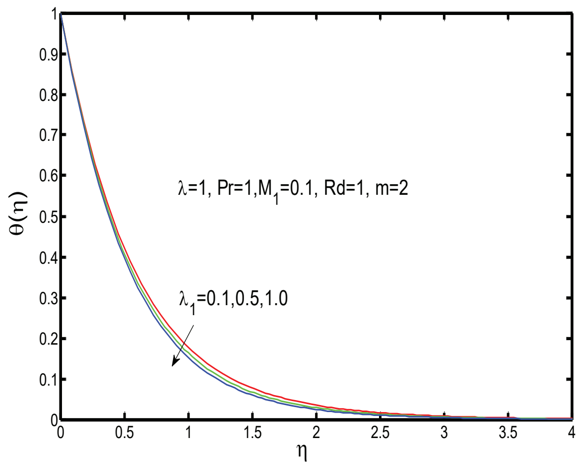


Fig. 10. Influence of λ_1 on θ

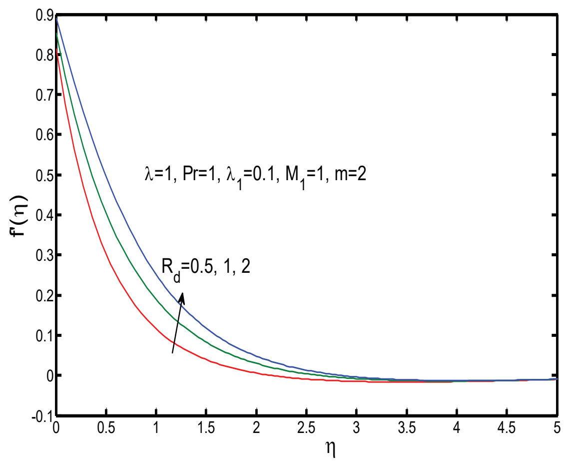


Fig. 11. Influence of R_d on f'

Figures 11-13 depict that the velocity, temperature and pressure profile for various values of radiation parameter. From these figure we detected that the velocity and temperature profile rises with increasing values of radiation parameter.

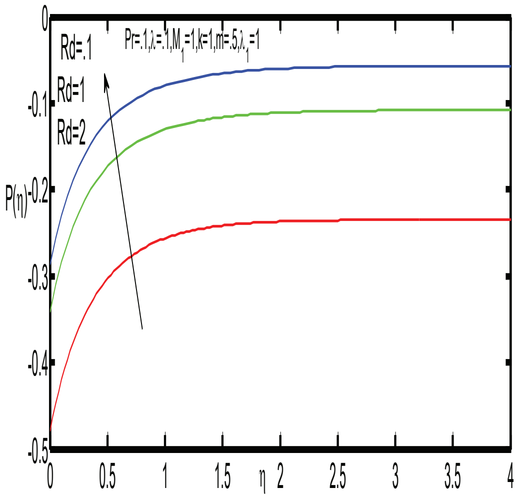


Fig. 12. Influence R_d of on p

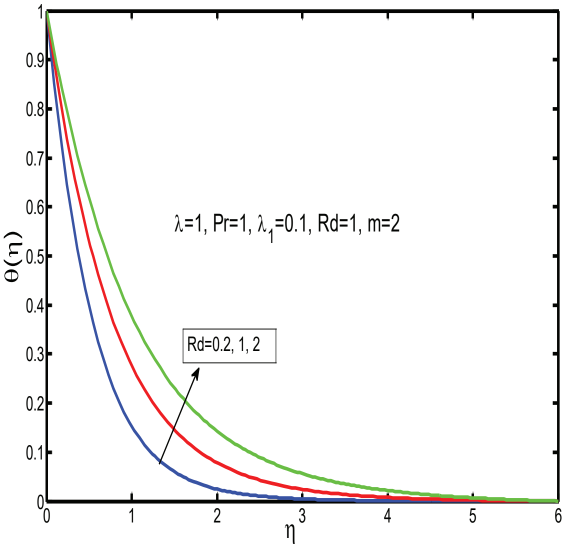


Fig. 13. Influence of R_d on θ

Figures 14-20 represent the skin friction and rate of heat transfer with magnetic parameter and curvature parameter for several values of curvature parameter, mixed convection parameter. From these figures we observed that skin friction coefficient is a decreasing function with magnetic parameter and its behavior of decreasing is nonlinear. But the rate of heat transfer is increase with curvature parameter and its increase is linearly. Whereas with curvature parameter skin friction and rate of heat transfer is increase nonlinearly as well as mixed convection parameter see figs. 17-20.

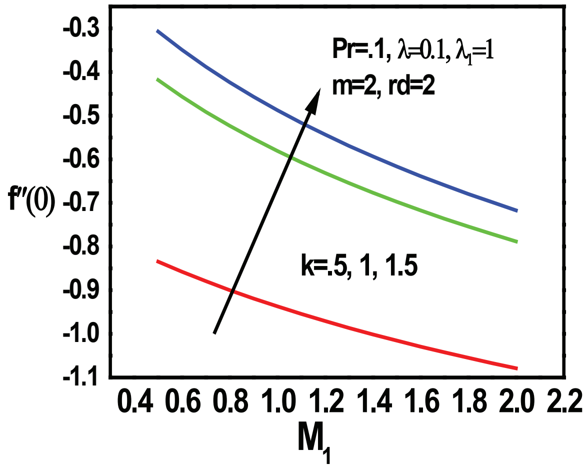


Fig. 14. $f''(0)$ with M_1 for Different values of k

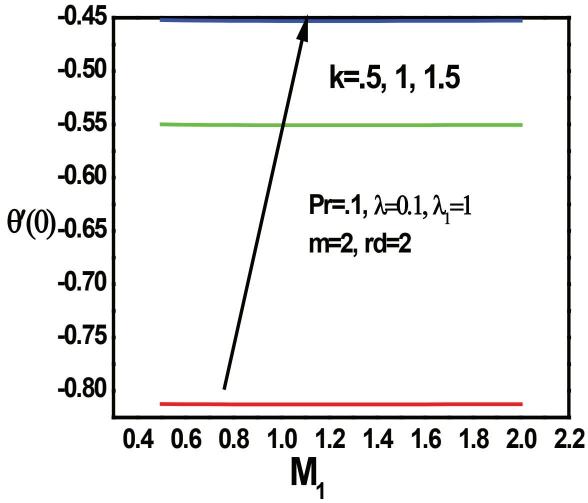


Fig. 15. θ' (0) with M_1 For different values of k

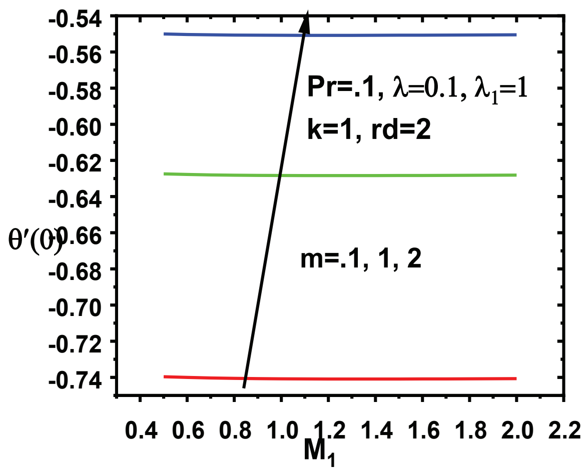


Fig. 16. θ' (0) with M_1 for different values of m

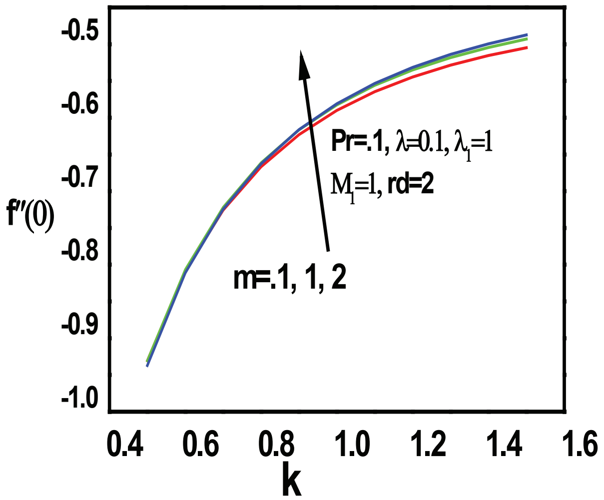


Fig. 17. *f''* (0) with k for different values of m.

VI. Conclusions

In the present analyzed, we examined the MHD flow and heat transfer over a curved stretching sheet with the presence of pressure, buoyancy force, variable thermal conductivity and radiation parameter.

The following observations can be made from the present findings:

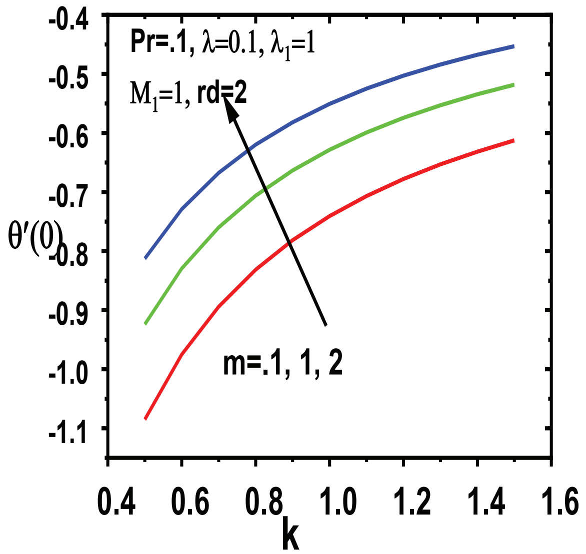


Fig.18. θ' (0) with k for different values of m

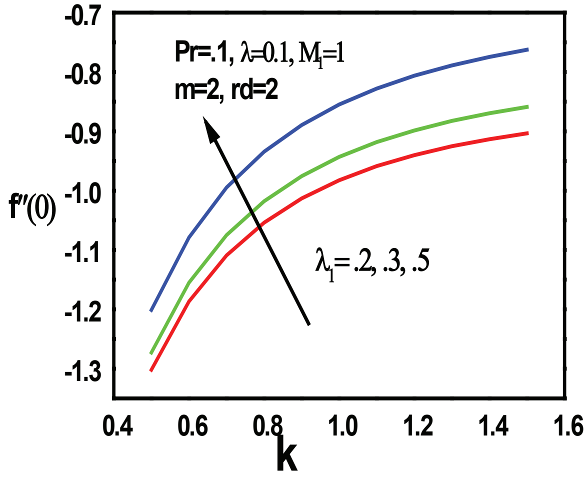


Fig. 19. *f''* (0) with k for different values of λ_1

1. Velocity profilereduces with magnetic number whereas pressure distribution and temperature profile increase with increasing values of magnetic number.
2. Velocity and temperature profile decrease with growing values of curvature parameter whereaspressure profile increases with increasing values of curvature parameter.
3. Velocity profile rises with rising values of mixed convection parameter whereas pressure profilesand temperature distributionsdecline with rising values of mixed convection parameter.
4. The effect of radiation parameter, velocity, pressure and temperature profilesare induces.

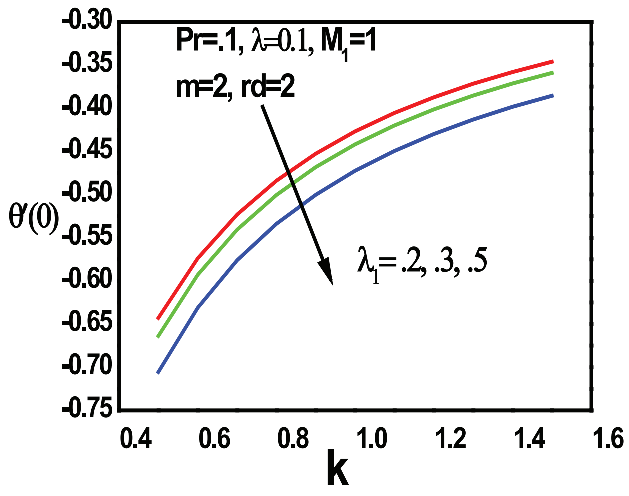


Fig. 20. θ'(0) with k for different values of λ_1

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