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An Integrated Forecasting Technique with Modified Weight Measurement

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Abstract

Forecasting has long been part of our life since early of the history of human being. In the middle of 20th century forecasting becomes a part of every business and financial sectors. Nowadays every successful firm has to make its own forecasts with an acceptable error as there is no chance of zero error. The situation becomes more complicated if the observed data is more diverted from the existing pattern. In such situation it becomes more difficult to fit it into a suitable forecasting model.Then it requires to combine several forecasts to reach a better forecast.In this paper, we willdevelop a sophisticated forecasting technique bycombining the weighted average method with Linear Programming (LP) model by developing an alternative technique to calculate the weights. We will carry out our analysis by using Microsoft Excel, statistical data analysis tool R and MATHEMATICA. We will demonstrate our model by numerical examples.

I. Introduction

In this section, we give an anatomy of our research in a whole. This section consists of several relevant topics such as a brief history of forecasting, importance of forecast in everyday life. We also discuss about the relevant research and methodology that will be used to carry out this research work. Finally, the paper outline is presented.

From ancient time to modern era people make forecasts[1]. Ancient people were not aware of that they made forecast because of lacking the proper knowledge but they did it for their sake. They had to predict where they should go to collect their food. They had to predict the weather etc.

In ancient time people used to predict weather from the cloud patterns[1]. Atmospheric changes, signs of rain and the movement of wind were also indicator of forecasts. The modern era of weather forecasting gets the first scientific involvement when the electric telegraph was invented in 1835. In 1922, the first numerical method for forecasting was developed by L. F. Richardson[2].

In the twentieth century, local or international business expands all over the world especially after World War II. Business organizations are always in search of new markets and a good position in that markets. For this, they need to estimate future demands as accurate as possible. Appropriate forecasting techniques can play a major role to estimate the future demands, costs and supply.The operations researchersare continuing to work to develop better forecasting models.Reeves andLawrence (1991)developed a multi-objective IPand LP modelto minimize the sum of the absolute errors[3]. Zhou et al. (1999) presented a multi-criteria multi-constraint (MC^2) LPmodel for combining forecasts[4] for telecommunications industry in south China. Chen (2017) combined three individual forecasts to justify the performance of returns on investment[5]. Their model results into a better model than the individual forecasting models.Zhou and Hao (1999), Shish and Tsokos (2008) and Granger and Ramanathan (1984) analyzed Exponential smoothing and weighted average methods[6, 7]. Zhou and Yang (1995), Karmakar (1984) and Clemen (1986) studied Linear programming for forecasting[8, 9]. Maurice et al. (2013), Abdullah (2012) and Chatfield and Yar (1988) analyzed the statistical tools and ARRIMA[10-12].We have learnt about the works of various researchers in this section. Their research encourages us to do some works in this field.

Importance of Forecasting

This section describes the importance of forecasting in the field of business managements. Operations of modern business management depends of reliable forecasting. It helps the manager of a business organization plan its resources to minimize cost and maximize profit. To do so organizations try to get rid of lack of forecasting or faulty forecasting.

Forecasts help a business organizations so that they can develop a unified overall plans to mesh divisional and departmental plans. By helping to identify future demand patterns, it enables a company to utilize its resources optimally with greatest assurance to profit over the long term[13].

We point out the followings to emphasize the importance of forecasting. Forecasting provides the reliable information about the future events which is necessary for sound planning and making important decisions.It is the basis for making planning premises.

However, the factors relating to business are changing constantly. Changes in interest rates, customer preferences, suppliers etc. makes the manager of the company to change the decisions accordingly.

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Methodology

The data, we will use in our numerical example, are secondary type. We analyze our data using R (a statistical data analysis programming language), Microsoft Excel and we use MATHEMATICA to solve LP and R in analysis data pattern and suitable ARIMA model selection. Mean absolute deviation (MAD) and mean absolute percentage error (MAPE) are used as error evaluation.

The rest of the paper is organized as follows. In Section 2, we will discuss combined forecasting. In Section 3,we will present LP model for combining forecasts proposed by Zhou and Yang. In Section 4, we will devise an alternative solution technique to solve LP model which is comparatively easy and time saving to obtain the weighted coefficients of combined forecasting. In Section5, we will draw our conclusion about our research work.

II. Combining Forecasts

Literary, combining forecast is to combine several forecasts to produce a better forecast so that forecasting errors in individual forecasting technique are significantly reduced to a minimal level through the combinations. Using a combination of forecasts rather than one forecast based on a single forecasting model is increasingly advocated by many researchers and practitioners[13]. An important factor, which encouraged the introduction of methods for combining forecast and facilitated their acceptability, was the pragmatic need for forecasting based on a wide range of contexts which guarantees an average accuracy.

Procedures for Combining Forecasts

If the component forecasts contains independent information then combined forecasts provide better accuracy. For combining forecast we first analyze different data using different forecasting methods. Tocombine several forecasts, we consider the following facts[14, 16]. Use different data or different method and make more forecast. We also use formal procedures to combine forecasts using equal weights andtrimmed mean (n% trimmed mean of a sample is the mean of reduced sample discarding n% small and large data from the original sample).

III. LP Model for Combining Forecast

The general method for combining forecasts is LP. The simplest LP model is the single objective LP model for combining n different forecasts over *m* observations[14]. The objective in this model is to minimize the total deviation that means the sum of forecast errors[15]. The model may be expanded to include multiple accuracy measures. Another approach also based on LP includes a multiple objective LP model. We will function with the former technique. In practice, the most commonly used method is the simple averaging of the forecasts.

We determination the weight coefficients of our combined forecasting modelas follows:

Let $A_{it}\left(i=1,2,\ldots.,n;t=1,2,\ldots\ldots,m\right)$ be the forecast of the t^{th} period, then the combined forecasting value is as follows.

$$\sum_{i=1}^{n} W_i A_{it} \ , \quad t=1,2,\ldots.., m. \tag{1}$$

where $W_i\left(i=1,2,\ldots.,n\right)$ are weight coefficients and satisfy

$$\sum_{i=1}^{n} W_i = 1, \ \ W_i \geq 0, i=1,2,\ldots.,n. \tag{2}$$

The aim is to minimize the errors between combined forecasting value $\sum_{i=1}^{n} W_i A_{it}$ and expectation in advance Y_t. So we write the relevant LP Model to obtain the required weights is of the following form.

$$Min \sum_{t=1}^{m} \left| \sum_{i=1}^{n} W_i A_{it} - Y_t \right| \tag{3}$$

$$\text{Subject to } \sum_{i=1}^{n} W_i = 1 \text{and} W_i \geq 0 \tag{4}$$

Here, $Y_t\left(t=1,2,\ldots..,m\right)$ are expectations in advance.

Optimal combined weighted coefficients [39-40] W^* can be obtainedusing (3) and (4):

$$W^* = B - CP^{-1}\left(C^T Y - 1\right) \tag{5}$$

where $B=\left(A^T A\right)^{-1}$, $\quad C=\left(A^T A\right)^{-1} E$,

$P = E^T (\left(A^T A\right) E$

and, $A=\left(A_{it}\right)$ is a full rank $m \times n$ matrix.

The resulting optimal forecasting value is given by

$E=\left(1,1,1,\ldots\ldots.,1\right)^T \quad Y=\left(Y_1, Y_2,\ldots\ldots., Y_m\right)^T$,
$W^*=\left(W_1, W_2,\ldots\ldots., W_n\right)^T$

$$\sum_{i=1}^{n} W_i^* A_{it} = F_t^*, \ t=1,2,3,\ldots,m. \tag{6}$$

Numerical Example on Combining Forecasts

Let us consider the Table 1 in which expected value of a variable in the first column and five forecasted values in the next five columns has been given for five successive periods. We are to combine these forecasts to obtain a reliable single forecast.

Table 1. Expected and Forecasts Values

Expected	Forcast-1	Forecast-2	Forecast-3	Forecast-4	Forecast-5
40	32	47	34	33.5	46.5
47	41.5	40	38	53	39
50	40	57	59	58	42
56	44	69	43	67	48
61	50	73	49	70	53

Firstly, we have a look to the errors. The error for the five forecasts obtained from five different forecasting methods are calculated by mean absolute percentage error (MAPE) method and listed in the Table 2.

Table 2. Errors calculated by MAPE

Err./Fore.	Forecast-1	Forecast-2	Forecast-3	Forecast-4	Forecast-5
MAPE (%)	18.23	17.86	19.01	15.88	15.33
MAD	46.50	32.00	31.00	27.50	25.50

From Table 2, it is obvious that none of the five forecasts is individually produces reasonable forecasts. In such a situation we need to develop an approach for combining them.

Here

$$(A_{it}) = \begin{pmatrix} 32 & 41.5 & 40 & 44 & 50 \\ 47 & 40 & 57 & 69 & 73 \\ 34 & 59 & 38 & 43 & 49 \\ 33.5 & 53 & 58 & 67 & 70 \\ 46.5 & 39 & 42 & 48 & 53 \end{pmatrix}$$

$$Y = \begin{pmatrix} 40 \\ 47 \\ 50 \\ 56 \\ 61 \end{pmatrix}; \text{ And } \quad E = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$B = \left(A^T A\right)^{-1} A^T Y = \begin{pmatrix} 1.69741 \\ 0.716306 \\ 5.45506 \\ 2.2081 \\ -7.18805 \end{pmatrix};$$

$$C = \left(A^T A\right)^{-1} E = \begin{pmatrix} 0.00992415 \\ 0.00262672 \\ 0.0529071 \\ 0.022823 \\ -0.0716032 \end{pmatrix};$$

$$P = E^T ((A^T A) E = (306036)$$

$$W^* = B - CP^{-1}\left(C^T Y - 1\right) =$$

$$\begin{pmatrix} 1.69741 \\ 0.716306 \\ 5.45506 \\ 2.2081 \\ -7.18805 \end{pmatrix} - \begin{pmatrix} -2.99611\times10^{-8} \\ -7.9307\times10^{-9} \\ -1.5972\times10^{-7} \\ -6.8903\times10^{-8} \\ 2.16171\times10^{-7} \end{pmatrix} = \begin{pmatrix} 1.69741 \\ 0.716306 \\ 5.45506 \\ 2.2081 \\ -7.18805 \end{pmatrix}$$

But the weights W_i^* are not all non-negative. So the solution by (5) is not valid in this case. We devise an alternative process to get the weights and the next section gives a good understanding about it.

IV. Alternative Process to Weights' Calculations

In this section, we solve the LP model by our own programming technique in MATHMATICA. Here the objective function remains the same. We only change the constraints. For example, we replace $W_i \geq 0$ by

$$W_i - surplus\,variable = positive\,value\,close\,to\,zero$$

.

Here W_i are weights.

Now we see the complete LP model for this problem so that we can solve in order to get the optimal combined weighted coefficients.

Objective Function:

$$Min\left|207.5W_1 + 286W_2 + 223W_3 + 281.5W_4 + 228.5W_5 - 254\right|$$

s. t: $W_1 + W_2 + W_3 + W_4 + W_5 = 1$

$$W_1 - S_1 = 0.01$$

$$W_2 - S_2 = 0.02$$

$W_3 - S_3 = 0.03$ $W_4 - S_4 = 0.04$

$W_5 - S_5 = 0.05$

; W_i , $S_i > 0$

Computer Technique for our Combined Model

In this section, we develop a computer technique for solving the integrated model by MATHEMATICA programming.

```
In[1]:= <<LinearAlgebra`MatrixManipulation`

In[2]:= Clear[basic,sset,AA,bb]

In[3]:= basic[AA_,bb_]:=Block[{m,n,pp,ss,ns,B,v,vv,var,vplus,vzero,BB,RBB,sol,new,sset,bs},{m,n}=Dimensions[AA];pp=Permutations[Range[n]];

ss=Union[Table[Sort[Take[pp[[k]],m]],{k,1,Length[pp]}]];

ns=Length[ss];B={};

For[k=1,k<=ns,k=k+1,

v=Table[TakeColumns[AA,{ss[[k]][[j]]}],{j,1,m}];

vv=Transpose[Table[Flatten[v[[i]]],{i,1,m}]];

B=Append[B,vv]];

var=Table[x[i],{i,1,n}];

vplus[k_]:=var[[ss[[k]]]];

vzero[k_]:=Complement[var,vplus[k]];

sset={};For[k=1,k<=ns,k=k+1,BB=B[[k]];RBB=RowReduce[BB];

If[RBB==IdentityMatrix[m],sol=LinearSolve[BB,bb],sol={}];

If [Length[sol]==0||Min[sol]<0,new={},new=sol;

sset=Append[sset,{vplus[k],new}]]];

bs[k_]:=Block[{u,v,w,zf1,f2},

u=sset[[k,1]];v=sset[[k,2]];w=Complement[var,u];

z=Flatten[ZeroMatrix[Length[w],1]];

f1=Transpose[{u,v}];f2=Transpose[{w,z}];

Transpose[Union[f1,f2]][[2]]];

Table[bs[k],{k,1,Length[sset]}]]

In[4]:=

optimal [AA_, bb_, cc_]:= Block[{vertex, val, opt, pos,optsol, lfpsoln}, vertex = basic [AA, bb];

val=Abs[Table[((vertex[[k]].c)-α), {k, 1, Length[vertex]}]];

opt = Min[val]

pos = Flatten[Position[val, opt]];

optsol = vertex[[pos[[1]]]];

lfpsoln = {optsol, opt};

Print ["The optimal value of the objective function is ", lfpsoln[[2]]];

Print ["The optimal solution is ", lfpsoln[[1]]]]

In[5]:=
```

Input

```
Clear[A,b, c]

A = { { 1 , 1 , 1 , 1 , 1 , 0 , 0 , 0 , 0 , 0 } , { 1 , 0 , 0 , 0 , 0 , -1,0,0,0,0},{0,1,0,0,0,0,-1,0,0,0},{0,0,1,0,0,0,0,-1,0,0},{0,0,0,1,0,0,0,0,-1,0},{0,0,0,0,1,0,0,0,0,-1}};

b={1,.01,.02,0.03,0.04,0.05};

c= {207.5,286,223,281.5,228.5,0,0,0,0,0};

α = 254;

basic[A,b]

optimal[A, b, c]
```

Final output

{{0.86,0.02,0.03,0.04,0.05,0.85,0,0,0,0},{0.01,0.87,0.03,0.04,0.05,0,0.85,0,0,0},{0.01,0.02,0.88,0.04,0.05,0,0,0.85,0,0},{0.01,0.02,0.03,0.89,0.05,0,0,0,0.85,0},{0.01,0.02,0.03,0.04,0.9,0,0,0,0,0.85}}

The optimal value of the objective function is 22.445

The optimal solution is {0.01,0.02,0.03,0.89,0.05,0,0,0,0.85,0}sset={};For[k=1,k<=ns,k=k+1,BB=B[[k]];RBB=RowReduce[BB];

We have $W_1 = 0.01, W_2 = 0.02, W_3 = 0.03, W_4 = 0.89, W_5 = 0.05$ with optimal value 22.445.

Therefore, combined forecastsobtained by the equation (6) are presented in Table 3.

Table 3. Combined Forecasts

Expected Value (A)	Combined Forecast(B)	$\frac{\lvert A-B \rvert}{A} \times 100\%$
40	34.42	13.95
47	51.475	9.52
50	57.03	14.06
56	65.14	16.32
61	68.38	12.10
		$\sum = 65.95$

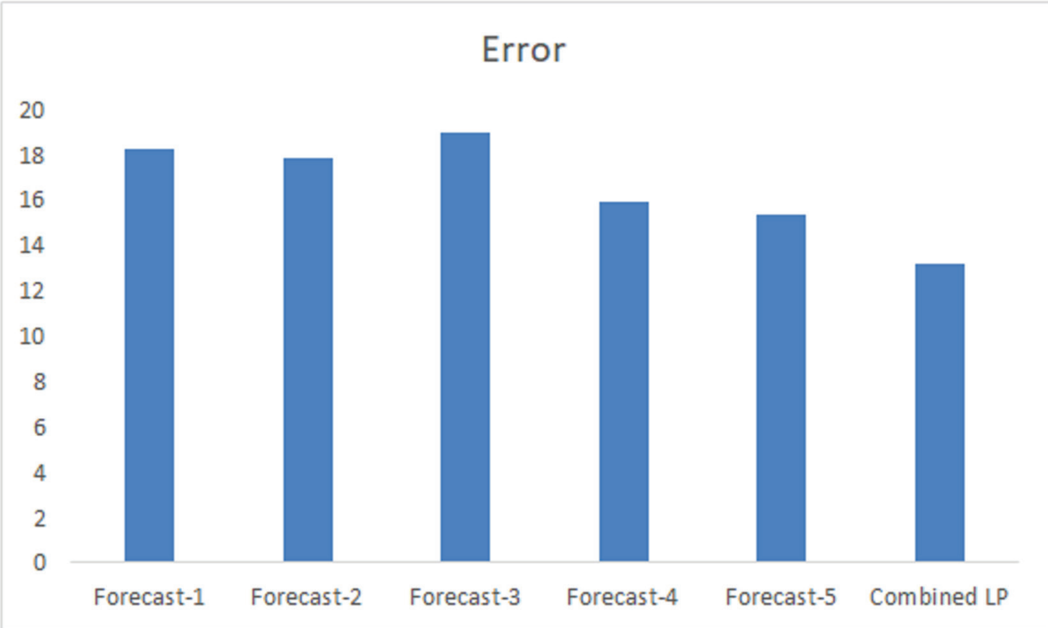

Fig. 1. Comparison of errors

Figure 1 shows that the error obtained from our combined forecasting LP method by using MAPE is less than that of any other individual forecasts.

Now we check the MAPE for combined forecast.

$$\text{MAPE} = \frac{65.95}{5} = 13.19$$

It is noted that some may apply simple averaging technique to combine several forecasts but we are not concentrating on that technique. Our focus was to combine forecasting technique with the LP model to improve the accuracy of forecasting.

V. Conclusion

If the observed data is more diverted from the existing pattern, it becomes more difficult to fit it into a suitable forecasting model.Then it requires to combine several forecasts to reach a better forecast. In this paper, our primary goal was to see how LP model can be helpful to determine the coefficients of combined forecasts. For this, we have developedan alternative technique to calculate the weights by combining the weighted average method with Linear Programming (LP) model. We carried out our analysis by using Microsoft Excel, statistical data analysis tool R and MATHEMATICA. We showed that our devised solution technique produced relatively low error. We think that it will be very helpful in such a situation whether the MAD or MAPE is very high for single forecast.

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