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Application of Stochastic Programming in Agricultural and Newsvendor Problems and It's Application in Real Life

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Abstract

The study of making the best decision under risk management in a variety of areas of our lives is known as Stochastic Programming. We will go through two-stage Stochastic Linear Programming approaches for a variety of real-world choice issues, as well as how to solve them. We will achieve this by constructing stochastic linear programming models based on real-world situations like the well-known Farmer's situation and News Vendors problems. The influence of pricing, Stochastic Integer Linear Programming problem, second stage Stochastic Integer Linear Programming problem, first stage Stochastic Binary Linear Programming problem, risk aversion problem, and continuous function for random variables based on two-stage SLP with the aid of Farmer's problem will all be examined. We will address the Newsvendor's problem with Deterministic Equivalent Stochastic Linear Programming, an extension of Deterministic Stochastic Linear Programming for risk aversion with a high number of decision variables and restrictions, utilizing the two-stage Stochastic Linear Programming approach once more. Hand calculation is a challenging way to acquire the solution to the problems. As a result, we will use the programming language AMPL to design computer solutions for tackling both farmer and newsvendor difficulties. We will also utilize MATLAB to create graphs for the farmer's problem's continuous function.

Keywords: Deterministic Linear Programming (DLP), Stochastic Linear Programming (SLP), Two-Stage Stochastic Linear Programming, Deterministic Equivalent Stochastic Linear Programming, Stochastic Integer Linear Programming (SILP), Stochastic Binary Linear Programming (SBLP), Continuous Function.

I. Introduction

Stochastic Operations Research deals with complex systems that operate randomly and uncertainly and aims to develop mathematical models and techniques for analyzing and optimizing such systems. This process has been particularly successful in two areas of decision analysis Deterministic Optimization which involves parameters with certainty and Stochastic Optimization which involves some or all parameters with uncertainty. Stochastic Linear Programming (SLP), Stochastic Integer Linear Programming (SILP), Stochastic Binary Linear Programming (SBLP), and Nonlinear Stochastic Programming can be extremely useful in stochastic optimization applications such as cost-volume-profit analysis, performance-based budgeting, production, selling and buying products, and so on. In either the first or second stage of a stochastic problem, stochastic integer programming is a stochastic program in which some of the variables are constrained to be integers[1]. Mixed integer SIP and binary SIP are two types of SIP. The adoption of the two-stage SLP technique can be particularly advantageous for an agribusinessman who produces or sells various items in his firm to effectively and efficiently utilize restricted resources such as budget and purchasing limits. Assume that all cost-per-unit selling prices for the producing goods remain constant. The newsvendor, too, faces many uncertainties in his job; he must decide how much newspaper to buy for each situation in order to make a living. As a result, he can benefit from this technique as well. The majority of the time, these issues are expressed as deterministic models, with either predicted or expected values of scenarios. Scenarios can be discrete or continuous in practice, and scenario analysis, as well as operation management methods, can aid businesspeople in planning. Each situation may be expressed as a basic operation research issue that can be addressed with the best possible answer. In actuality, though, a farmer may face a variety of conditions. Implementing all of these numerous circumstances when it comes to decision-making can be nearly impossible. Using anticipated values of input variables and solving simple deterministic stochastic linear programming (SLP) is one technique to evaluate all situations. In the long run, however, there is a distinct lack of attention dedicated to hedging all uncertainties with potential. The rest of the paper is organized as follows. In Section 2, we have presented a review of some relevant articles. In Section 3 we have presented the formulation process of SP. In Section 4 we have explored farmers' problems. In Section 5 we have explored the newsvendor problem and In Section 6 we conclude the paper.

II. Literature Review

Hasan & Chakroborty[7]

Mohammad Babul Hasan and Sajal Chakroborty have mentioned the solution method of stochastic programming and formed an easier way to resolve the SP trouble. The

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authors have used decomposition techniques and decomposed the hassle into sub-issues and their grasp troubles which have been received for the Dantzig-wolf decomposition set of rules. They have used the concept of DBP (decomposition-based pricing) to increase their approach to fixing SP. The nested decomposition technique is a solution technique for a multi-stage stochastic programming problem. Their purpose has needed to follow this approach to an actual-lifestyles-oriented version. And that they efficiently applied this approach to solve a farmer's problem. They have accrued information from a Bangladeshi farmer and taken into consideration three scenarios proper, horrific, and ordinary. To find maximum make the most of planting one-of-a-kind forms of plants on farmer's land they've applied the method. To find most earnings they have evolved the AMPL language.

Zhengyang Hu &Guiping Hu[8]

Zhengyang Hu and Guiping Hu have discussed two-stage stochastic programming versions under requirement and uncooked material proficiency uncertainties for the production of lot-sizing and scheduling manner. To minimize the cost they have shaped a model for multi-duration, multi-product, lot sizing, and scheduling hassle considering uncertainties in requirement and manufacturing. They have covered their manufacturing plan along with the quantities of merchandise inside the first stage which is upgraded inside the second degree with recourse on extra time manufacturing. Blended integer and binary programming models are used to remedy this problem within the placing of manufacture which series depends on the fee and times. For locating correct viable answers they have used Lagrange decomposition emerged on the heuristic. In this version, all of the normal manufacturing assets are used starting beyond regular time production because the unit production is ordinary period manufacturing. This model can be beneficial for manufacturing agencies

Arno et al[9]

Arno Maatman et al have discussed Farmers' issues with uncertain rainfall and their techniques. Their purpose needs to be enlarged to which farmers' techniques confirm sufficient food for their families and word which adjustments will be built to confirm better, sustainable ranges of meals surety. They've developed a linear stochastic version that targets the farm domestic and used this model for risk analysis. In this paper, they've implemented the famous strategies from stochastic programming which is reputed as a resource version with three stages. They have used number one and secondary sources for growing their models. In this paper, they set the harvest length at three months. After formulating a linear programming hassle in GAMS they've solved it with MINOS.

Tirupathi Rao Padi et al[10]

Tirupathi Rao Padi et al have mentioned the most reliable stochastic programming for finding the best manpower recruitment from graded manpower systems. For manpower recruitment of an employer, they have formed a version based totally on stochastic programming issues of manpower where they have got observed the graded manpower process. For forming the most advantageous decisions on recruitment rules this model has a like-minded tool to serve the selection for ascertaining the most reliable recruitment to destine how tons of employees should be recruited via numerous grades to allow the entire wide variety of employees for promotion and retirement. The authors have found all of the regulations and developed a graded populace shape through the usage of linear programming where the recruitment and remodel ratio are depending on several grades, which are regulated by control. The authors have formed stochastic programming trouble to optimize the value of recruitment by concerning the limitations that depend upon salaries and funding in several chiefs. The goal feature and constraints are formed by presuming a linear method. Non-linear programming trouble by using growing a goal function of minimizing the cost in non-linear that is influenced through the limitations which might be relying on several expenses in the course of the shortage of factors is needed.

Hashnayne Ahmed[11]

Hashnayne Ahmed September has discussed stochastic programming issues with modeling and implemented this in actual life. He has mentioned stochastic programming, stochastic process, Uncertainty and Stochasticity, the distinction between stochastic and deterministic worlds, branches of utility, and different types of programming consisting of Linear Programming. He has mentioned an instance that is based on an actual-existence farmers' hassle and the solution to this hassle is obtained by using the use of two-stage stochastic programming. For finding this SP trouble answer a mathematical language Lindo has been used. He has also mentioned the newsvendor hassle and solved this procedure using the usage of stochastic programming that's -degrees. In which he has described the anticipated 2nd stage ultimate cost feature which becomes an everyday Non-Linear programming, then he has discussed the L-shaped technique. He has given an example and built up a model of the usage of the mathematical language Lindo. Additionally, he has discussed Regularized decomposition, fundamental Factorization andinterior factor approach, the internal Linearization method, and the Lagrang technique. He has discussed the necessity of stochastic programming and built up fashions for the empty field allocation hassle. And he has mentioned finding a manner for subduing to save resource allocation like time and energy which can also decrease the cost.

In this section, we have discussed some most relevant research papers. In the next section, we will discuss real-life farmers' problems.

III. Two-Stage SLP Formulation

In two-stage stochastic programming, the decisions should be based on data available at the time the decisions are composed and cannot depend on oncoming observations. Stochastic Programming can be solved by two methods. which are Two Stage Method and Multi-Stage Method. The basic idea of two-stage stochastic programming is that (optimal) decisions should be based on data available at the time the decisions are made and cannot depend on future observations. The set of decisions is divided into two groups:

Table1. Difference between 1st and 2nd stage

1st Stage	2nd Stage
1. Making Decisions Before Experiment	1 Making Decisions after Experiment
2. Decision Vector x	2. Decision Vector y
3. Without full information of the random events	3. With full information of the random events

General Form of Two Stage Method[5,11]

We have to make some decisions before the experiment on some random cases. These are the first-stage decisions signified by a vector x_l. Afterward, we get the results from the experiment pursuing some random variables x_i. Then we take the second stage decisions $y_l(\xi)$. Using the concept of Mathematical Programming, this ascertains the alleged two-stage stochastic program with the recourse of the following form ;

First Stage

$\min c^T x_l + E_{\xi n} [Q(x_l,\xi_n)]$ min cost: 1^{st}stage + expected 2^{nd} stage

s.t $Ax_l <= B$ first Stage Constraints

$x_l >= 0$ first-stage decision variables

Where the first term of the objective function characterizes the first-stage decisions and the second term characterizes the second-stage decisions. Here,

Second Stage

$Q(x_l,\xi_n)=\min\ q_n (\xi_n)^T y_l(\xi_n)$ 2^{nd}stage recourse action

s.t. $T_n(\xi_n)x_l+W_n*y_l(\xi_n)<=H_n(\xi_n)$, 2^{nd} Stage constraints

$y_l(\omega)>=0$,y_l second stage decision variables

Where (q_n,T_n,W_n,H_n) signifies the vector form of the second stage parameters. The ideal value of the second stage is defined by $Q(x_l,\xi_n)$ If at least one $n\epsilon 1,2,\ldots N$ The second stage system has no solution, the associated second stage issue is infeasible, and hence the ideal value $+\infty$. If at least one $Q(x,\xi)=\infty$, the total of the right-hand side equals $+\infty$. In the next section, we will discuss real-life farmers' problems.

IV. Exploration of Farmer's Problem

In Bangladesh, a farmer's family has lived for generations, and he owns 600 acres of land. On his farm, he has focused on cultivating rice, wheat, corn, and sugar beets, among other crops. He has determined that his livestock requires 300 tons of rice, 250 tons of wheat, and 240 tons of corn. These various crops might either be grown on his farm or purchased from a supplier. Due to the wholesaler's margin and shipping costs, he must offer 40 % more than the

selling price for acquired crops. Any crops that are left over after meeting the minimum feeding requirements can be sold. The average selling prices for rice, wheat, and corn in the previous decade were \$160, \$170, and \$150, respectively. Sugar beets are another profitable expense; he anticipates a selling price of \$36 for sugar beets. However, his sugar beet production is restricted by the research institute. The selling price of sugar beets is just \$ 10 for any sugar beets that exceed the quota. The farmer's quota for this year is 6000. He knew that the typical production on his property is

3.5 T (tons) per acre for rice, 2.5 T per acre for wheat, 3 T per acre for corn, and 20 T per acre for sugar beet based on his past perspective.

Table 2. Information of farmer's problem

Crops Name	Rice	Wheat	Corn	Sugar beet
Planting cost of land (/acre)	150	180	200	260
Selling price of crops (/T)	160	170	150	36 under 6000T or10 above 6000
Purchasing price of crops (/T)	224	238	210	------
Amount of Minimum requirement (T)	300	250	240	------

To put it another way, allocating land per acre to discover the best answers to the farmer's dilemma is a heuristic method that reduces profit per acre. However, if labor requirements and crop production are neglected, this heuristic rule will not last long. After learning all of this, he has grown concerned. He has experienced that the yield values of different years are varied due to the shifting weather. After sowing or planting the crops, the majority of them require rain. Sunlight is beneficial to crops throughout their growth season. It harms the development of crops if it become drip. Dry weather is not ideal for harvesting. Now, in scenario 1:Yield values of these crops vary 20-25% below the mean yield.

scenario 2: Crop yield values are averaged in this scenario.

scenario 3: Yield values of these crops 20-25 % above the mean yield

The yield of each scenario in 2019;

Table 3. Yield value T/acre

Yield (T/acre)	Rice	Wheat	Corn	Sugar beet
Scenario 1	2.8	2	2.4	16
Scenario 2	3.5	2.5	3	20
Scenario 3	4.2	3	3	24

Deterministic Linear Programming (DLP) for each scenario

Notation

First stage variable:

x_1= allocated land for Rice

x_2= allocated land for wheat

x_3= allocated land for corn

x_4= allocated land for sugar beets

Second stage variable:

w_1= amount of selling Rice

w_2= amount of selling wheat

w_3= amount of selling corn

w_4= amount of selling sugar beets at $36/T

w_5= amount of selling sugar beets at $10/T

y_1= amount of purchasing Rice

y_2= amount of purchasing Wheat

y_3= amount of purchasing Corn

other notations

Υ_1=mean yield per acre of rice

Υ_2= mean yield per acre of wheat

Υ_3= mean yield per acre of corn

Υ_4= mean yield per acre of Sugar Beets

β_1= purchasing cost of rice

β_2=purchasing cost of wheat

β_3= purchasing cost of corn

α_1= selling price of rice

α_2= selling price of wheat

α_3= selling price of corn

α_4= selling price of sugar

α_5= selling price of sugar forthe above quota

c_1= Planting cost for rice

c_2= Planting cost for wheat

c_3=Planting cost for corn

c_4= Planting cost for sugar beet

b_1=Minimum Requirement for rice

b_2= Minimum Requirement for wheat

b_3=Minimum Requirement for corn

b_4= Quota for sugar beet

d=total land of farmer

Suppose the probability of each scenario is 1/3. So, the stochastic decision problem is

Minimum $Z=c_1*x_1+c_2*x_2+c_3*x_3+c_4*x_4+(1/3)\sum_{i=1}^{3} Q_i(x,\omega)$

where $Q_j(x,\omega)$ is the optimal value of the second stage problem after the scenario has been ascertained, given that the first stage variables x have been picked. Where,

$Q_i(x,\omega)$=Minimum $\beta_1*y_1- \alpha_1*w_1+\beta_2*y_2 - \alpha_2*w_2 + \beta_3*y_3 - \alpha_3 * w_3 - \alpha_4 * w_4 - \alpha_5 *w_5$

First Stage Constraints

s.t. $x_1+x_2+x_3+x_4<=d$, $x_j>= 0$,j=1,2,3,4

For the individual scenarios

2nd Stage Constraints

Rice constraint: $\Upsilon_1[i] * x_1+ y_1-w_1>=b_1$

Wheat constraint: $\Upsilon_2[i] * x_2 +y_2-w_2>=b_2$

Corn constraint: $\Upsilon_3[i] * x_3+y_3 -w_3>=b_3$

sugar beet constraint: $w_4 +w_5<=\Upsilon_4[i] * x_4$

Quota constraint: $w_4<=b_4$

$x_j,y_i,w_k>=0$, where i=1,2,3;j=1,2,3,4;k=1,2,3,4,5;Other parameters stay the same in 2020 and 2021 while planting costs, selling prices, and minimum requirements fluctuate somewhat, and yield value varies between 25% and 20% in 2020 and 2021

Table 4. Information of farmer's problem

	2020			2021		
Crops	Planting Cost	Selling price	Min requirement	Planting Cost	Selling price	Min requirement
Rice	160	165	320	165	165	340
Wheat	170	160	260	175	160	270
Corn	200	155	220	220	155	260
Sugar	260	36		260	40	
		10 above quota			15 above quota	
		6000			7000	

Profit of Deterministic Model for each scenario in the farmers' concern in 2019,2020,2021

Table 5. Individual profit for 3 scenarios

Scenario	Profit in 2019 $	Profit 2020 $	Profit 2021 $
1	28250	9188.57	30671.4
2	105200	106927	116713
3	179067	200138	194943

DLP problem focuses on one scenario. While SLP problems focus on a set of scenarios that may randomly happen. In business strategy, we have to determine decisions focusing on all uncertainties.

Model of Deterministic Equivalent SLP

Scenario analysis is often applied to deal with an unexpected situation. However, in practice, the agriculture business often has uncertainties and risks. All scenario constraints are in the following form

$\text{Min}150*x_1+180*x_2+200*x_3+260*x_4+ \sum_{i=1}^{3}(1/3)(\beta_1*y_{1,I}-\alpha_1*w_{1,i}+\beta_2*y_{2,i}-\alpha_2 w_{2,i}+\beta_3*y_{3,i}-\alpha_3*w_{3,i}-\alpha_4*w_{4,i}-\alpha_5*w_{5,i})$

Equation of the first stage

subject to

s.t. $x_1+x_2+x_3+x_4<=d$, $x_j>= 0$,j=1,2,3,4

Equation of the second stage for scenarios

Rice constraint: $\Upsilon_1 [i] * x_1+ y_{1,i}-w_{1,i}>=b_1$

Wheat constraint: $\Upsilon_2 [i] * x_2 +y_{2,i}-w_{2,i}>=b_2$

Corn constraint: $\Upsilon_3 [i] * x_3+y_{3,i} -w_{3,i}>=b_3$

sugar beet constraint: $w_{4,i}+w_{5,i}<=\Upsilon_4[i] * x_4$

$x_j,y_{m.i},w_{k,i}>=0$,where i,m=1,2,3;j=1,2,3,4;k=1,2,3,4,5;

Now,,the solutions to the problem are;

Table 6. Solutions of deterministic equivalent SP problem

Crops Name		Rice	Wheat	Corn	Sugar beets
First Stage	Area(acres)	200	83.3333	66.6667	250
Scenario 1	Yield(T)	560	166.6666	160.0008	4000
low	Sell(T)	260	0	0	4000 above quota 0
	Purchase (T)	0	83.888	80	0
Scenario 2	Yield(T)	700	208.33325	200.001	5000
average	Sell(T)	400	0	0	5000 above quota 0
	Purchase (T)	0	41.6667	40	0
Scenario 3	Yield(T)	840	249.999	240.00012	6000
above	Sell(T)	540	0	0	6000 above quota 0
	Purchase (T)	0	0	0	0
					Overall profit 102350$

We have arrived at an optimal value of -102350 and a corresponding optimal solution for allocating land x*= (200,83.3333,66.6667,250). All uncertainties are taken into account while utilizing this method. As a result, this model is superior at making more accurate decisions.

Extension of Deterministic Equivalent SLP for Price Effect Problem

The farmer insists that he is incapable of making the ideal selection in every circumstance. As a result, he wants to prolepsis the profit and losses of each x_i,j=1..4, but the number of sales and purchases $w_{k,i}y_{m,i}$ are dependent on the yields. It's a good idea to use a scenario index i = 1,2,3 that represents below-average yields, average or, above-average respectively. Assuming the farmer's goal is to maximize long-term profit, finding a solution that maximizes his net gain is the primary motivation.

Table 7. Price effect

Scenario	**Price**	Rice	Wheat	Corn
1	Selling price ($/T)	192	204	180
	Purchase price ($/T)	268.8	285.6	252
2	Selling price ($/T)	160	170	150
	Purchase price ($/T)	224	238	210
3	Selling price ($/T)	128	136	120
	Purchase price ($/T)	179.2	190.4	168

$$\text{Min:}z=150*x_1+180*x_2+200*x_3+260*x_4+\sum_{i=1}^{3}(1/3)(\beta_1[i]*y_{1,i}-\alpha_1[i]*w_{1,i}+\beta_2[i]*y_{2,i}-\alpha_2[i]*w_{2,i}+\beta_3[i]*y_{3,i}-\alpha_3[i]*w_{3,i}-\alpha_4[i]*w_{4,i}-\alpha_5[i]*w_{5,i})$$

Equation of the first stage

subject to

s.t. $x_1+x_2+x_3+x_4<=d$, $x_j>=0$,j=1,2,3,4

Equation of the second stage for scenarios

Rice constraint: $\Upsilon_1[i] * x_1+ y_{1,i}-w_{1,i}>=b_1$

Wheat constraint: $\Upsilon_2[i] * x_2 +y_{2,i}-w_{2,i}>=b_2$

Corn constraint: $\Upsilon_3[i] * x_3+y_{3,i} -w_{3,i}>=b_3$

sugar beet constraint: $w_{4,i}+w_{5,i}<=\Upsilon_4[i] * x_4$

$x_j,y_{m.i},w_{k,i}>=0$,where i,m=1,2,3;j=1,2,3,4;k=1,2,3,4,5;

Table 8. Solutions of price effect problem in SP

Crops Name		Rice	Wheat	Corn	Sugar beets
First Stage	Area(acres)	200	83.3333	66.6667	250
Scenario 1	Yield(T)	560	166.6666	160.00008	4000
low	sell(T)	260	0	0	4000 above quota 0
	purchase (T)	0	83.3333	80	0
Scenario 2	yield(T)	700	208.33325	200.0001	5000
average	sell(T)	400	0	0	5000 above quota 0
	purchase (T)	0	41.6667	40	0
Scenario 3	yield(T)	840	249.9999	240.00012	6000
above	sell(T)	540	0	0	6000 above quota 0
	purchase (T)	0	0	0	0
					Overall profit 96921.1$

If the production is low then the price will go up this is a very normal and timely decision in business. So, cultivating the land according to this model will be good for the farmers.

Deterministic Equivalent SILP

This problem is an extension of deterministic SLP. In this problem, all values of variables will be in integer form. In integer SLP we have defined all the variables will be integer. This method can be used for easy calculation for selling and buying products and allocating land. The following are the model's current solutions

Table 9. Solutions of the SILP problem

		Rice	Wheat	Corn	sugar beets
First Stage	Area(acres)	205	80	65	250
scenario 1	Yield(T)	574	160	156	4000
low	sell(T)	274	0	0	4000 above quota 0
	purchase (T)	0	90	84	0
scenario 2	yield(T)	717.5	200	195	5000
average	sell(T)	417	0	0	5000 above quota 0
	purchase (T)	0	50	45	0
scenario 3	yield(T)	861	240	234	6000
above	sell(T)	561	0	0	6000 above quota 0
	purchase (T)	0	10	6	0
					Overall profit 102273$

We arrived at an ideal objective value of -102273, resulting in a profit of 102273 and a corresponding optimal solution for land allocation x* = (205,80,65,250). We can see that the best solutions to this problem are nearly identical to the deterministic comparable problem.

Extension of Deterministic Equivalent SLP for 2nd stage SILP

This model will have integer values for all variables in the second stage. Consider a case where the selling and purchasing value of maize and wheat can only be achieved if the selling and purchasing amounts of wheat and corn total hundreds of tons. In the second stage, transform the model into a stochastic program using a mixed-integer variable. If we want to make sure that wheat and corn harvests are only bought and sold in multiples of 100, we must be able to identify how many units of corn we have for sale (where a single unit is equal to 100). The insertion of two new variables, $m_{j,i}$ and $n_{j,i}$ (indexed by J ϵ C), is the best way to do this.The easiest way to achieve this is to create two new variables, $m_{j,i}$ and$n_{j,i}$(indexed by JϵCand IϵS). Let $m_{j,i}$ denote the number of wheat and corn units purchased in scenario J, and $n_{j,I}$denote the number of wheat and corn units sold in the scenario i.Now the second stage constraints are;

$w_{j,i}$= 100 x $n_{j,i}$

$y_{j,i}$= 100 $xm_{j,i}$

$n_{j,i}$: The amount of crop j bought in scenario i (in 100's).

$m_{j,i}$: The amount of crop j sold in scenario i (in 100's).

j=2,3,i=1...3

Now,the solutions to the problems are given below in table form;

Table 10. Solutions of stochastic 2nd stage SILP problem

Crops Name		Rice	Wheat	Corn	sugar beets
First Stage	Area(acres)	308.333	25	16.6667	250
scenario 1	Yield(T)	863.3324	50	40.00008	4000
low	sell(T)	563	0	0	4000 above quota 0
	purchase (T)	0	200	200	0
Scenario 2	yield(T)	1079.1655	62.5	50.00001	5000
average	sell(T)	779	0	0	5000 above quota 0
	purchase (T)	0	200	200	0
Scenario 3	yield(T)	1294.9986	75.00012	60.00012	6000
above	sell(T)	995	0	0	6000 above quota 0
	purchase (T)	0	200	200	0
					overall profit 95959.6 $

We arrived at an optimal objective value of -95959.6 and the corresponding optimal solution for allocating land x* = (308.33,25,16.6667, 250). As before, note that this optimal objective solution is less than the unrestricted stochastic problem. If a farmer wants to buy or sell crops which are multiplier of 100 tons then it can use the measure of these models. After taking these measures he can get a profit which is not bad at all.

Extension of Deterministic Equivalent SLP for first stage SBLP

Binary IP in which all the variables sum in the first stage must be equal to 0 or 1. Let us consider that the farmer possesses four fields which sizes of 150,200,120, and 130 acres, respectively. We observe that the total acre of land 600 is unchanged. Unfortunately, the fields are located in different parts of the village. The reasoning for efficiency, the farmer wants to grow on each field only one type of crop. The model is formulated as a two-stage stochastic program where all the variables of the first stage will be binary. To account for the additional constraint of 4 fields and the fact that each field must be utilized by only one crop, we need to make the first stage decision variable binary and increase its dimension. Thus x_i, which was the number of acres planted for crop i, becomes $x_{i,k}$, which is equal to 1 if crop i is planted on field k and 0 otherwise. This is achieved by adding an additional set of F = {1,2,3,4 }; which represents the 4 fields so that we may index $x_{i,k}$ by k ϵ F . We must also add an additional constraint which says that the maximum number of crops planted on a single field is 1. We can accomplish this by addingthe constraint $\sum_{i\epsilon c} x[i,k]$=1, for alliϵ Crops, for all kϵFwhich ensures that the number of crops planted on field k is 1.

Table 11. Amount of land each field

Field (Acre)	
Field 1	150
Field 2	200
Field 3	120
Field 4	130

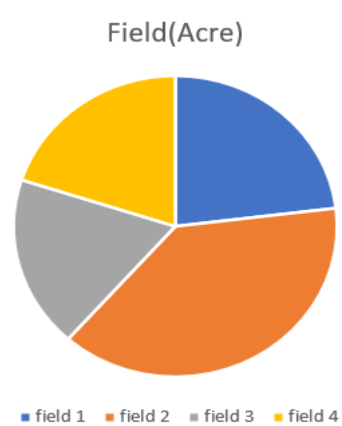

Fig. 1. Flow chart of 4 fields

The first and second-stage equations for this issue are as follows:

Min

$z=\sum_{k=1}^{4}(c_1*x[1,k]+c_2*x[2,k]+c_3*x[3,k]+c_4*x[4,k]) +(\frac{1}{3})\sum_{i=1}^{3}(\beta_1*y_{1,i}-\alpha_1*w_{1,i}+\beta_2*y_{2,i}-\alpha_2*w_{2,i}+\beta_3*y_{3,i}-\alpha_3*w_{3,i}-\alpha_4*w_{4,i}-\alpha_5*w_{5,i})$

First Stage constraint;

$\sum_{k=1}^{4} x[i,k]=1$ Where k ϵ Field(Acre)

2nd stage constraint;

Rice-constraint: $\sum_{k=1}^{4} x[1,k]*FieldAcres[k]*\Upsilon_1[j]+y_{1,j}-w_{1,j}>=b_1$

Wheat constraint:

$\sum_{k=1}^{4} x[2,k]*FieldAcres[k]*\Upsilon_2[j]+y_{2,j}-w_{2,j}>=b_2$

Corn constraint:

$\sum_{k=1}^{4} x[3,k]*FieldAcres[k]*\Upsilon_3[j]+y_{3,j}-w_{3,j}>=b_3$

Sugar Beets constraint:

$w_{4,j}+w_{5,j}<=\sum_{k=1}^{4} x[4,k]*FieldAcres[k]*\Upsilon_4[j]$

Quota constraint: $w_{4,j}<=b_4$

k=1,2,3,4;j=1,2,3;

Now, the solutions to the problem are given below in table form;

Table 12. Solutions of first stage SBLP

Crops Name		Rice	Wheat	Corn	sugar beets
	Fields	1,2	0	0	3,4
First Stage	Total Area(acres)	350	0	0	250
scenario 1	Yield(T)	980	0	0	4000
low	sell(T)	680	0	0	4000 above quota 0
	purchase (T)	0	250	240	0
scenario 2	yield(T)	1225	0	0	5000
average	sell(T)	925	0	0	5000 above quota 0
	purchase (T)	0	250	240	0
scenario 3	yield(T)	1470	0	0	6000
above	sell(T)	1170	0	0	6000 above quota 0
	purchase (T)	0	250	240	0
					overall profit 100600 $

We've come up with a -100600 ideal goal value, with rice on fields 1 and 2 and sugar beets on fields 3 and 4 as the matching answer. The farmer usually does not have all of his land in one location. This strategy may be employed for the farmer's convenience if his acreage is spread out over multiple areas. This model has the advantage of being analogous to the deterministic model.

Extension of Deterministic Equivalent SLP for Risk Aversion

A farmer is accustomed to taking risks. A risk aversion model may be built in several different ways. In the worst-case scenario, a simplistic approach for generating this model. In the worst-case situation, the purpose of this strategy is to maximize earnings. However, we must keep in mind that predicting which circumstance would yield the lowest profit is impossible. It's hard to know which situation will yield the lowest profit. In our model, the worst-case situation is similar to scenario 1. Scenario 1 generates a profit of $28250. A new constraint has been added to this problem:

Risk constraint:

$-(c_1*x_1+c_2*x_2+c_3*x_3+c_4*x_4+\sum_{i=1}^{3}(1/3)(\beta_1*y_{1,i}-\alpha_1*w_{1,i}+\beta_2*y_{2,i}-\alpha_2*w_{2,i}+\beta_3*y_{3,i}-\alpha_3*w_{3,i}-\alpha_4*w_{4,i}-\alpha_5*w_{5,i}))>=28250$

where i=1,2,3, Now, the solutions to the problem are;

We came up with a -66250 ideal goal value and x*= (125,0,100,375) optimal solution for distributing land. Risk aversion is natural, but smart planning and research may help you achieve better results. We could advise the farmer to go with a solution that has a greater multiple expected return, quite apart from the

risk of even lower performance in low yield years. In the long run, this will considerably increase the likelihood of gaining extra benefits.

Table 13. Solutions of the risk aversion problem in SLP

Crops Name		Rice	Wheat	Corn	sugar beets
First Stage	Area(acres)	125	0	100	375
scenario 1	Yield(T)	350	0	240	6000
low	sell(T)	50	0	0	6000
	purchase (T)	0	250	0	0
scenario 2	yield(T)	437.5	0	300	7500
average	sell(T)	137.5	0	60	6000 above quota 1500
	purchase (T)	0	250	0	0
scenario 3	yield(T)	525	0	360	9000
above	sell(T)	225	0	120	6000 above quota 3000
	purchase (T)	0	250	0	0
					overall profit 66250 $

Comparison profit in all cases

Table 14. The Problem's overall profits

Crops Name	Profit in 2019	2020	2021
Deterministic Equivalent SP Problem	102350	103885	109227
In price effect problem	96921.1	95043.2	96972.3
Integer Stochastic Programming Problem	102273	103885	109121
Integer Second stage Stochastic Programming Problem	95956.7	96721.7	98311.3
Binary First Stage Stochastic Programming Problem	100600	100220	104850
Risk Aversion Problem in Stochastic Programming	66250	62288.6	96972.3

Fig. 2. Flow chart for the profit of all problems

The profit for each instance in the years 2019, 2020, and 2021 is depicted in this graph. Many agriculture choices are not only based on profit. Much is contingent on the situation. Any of the reasons stated above might influence the farmer's decision. There will not be much of a difference in terms of profit. However, while the earnings are virtually the same in all scenarios, the risk aversion model's profit in 2019 and 2020 is smaller than the profit from other models. As a result, agribusinessmen may keep away of these business concepts. These models can be used if he does not want to risk earning a profit that is less than the worst case scenario.

Continuous random variables function for Farmer's Problem

In this paper, we used a different technique for two-stage SP with recourse that only depends on the first stage variable. In the second step, the anticipated values of the cost functions are replaced by continuous functions generated using a sample average approximation technique[2,6].

Min $z= c^T*x_l\ N^{-1}\sum_{k=1}^{N} Q_{cr}(x_l,\omega^k)$,

subject to

$x_l \epsilon X$ allocated land for crops where, $Q_{cr}(x_l,\omega^k)$:{ inf $y_n \epsilon$ $q^T y_n$:W* y_n>=H-T *x_l}

Where ω:=(T, W, H,q) represents the vector form of parameters of the second stage. Now we replaced the optimal value of the second stage function $Q_{cr}(x_l,\omega^k)$ by continuous function

Continuous Function For Rice

From the second stage constraints and objective function;

$Q_1(x_1,)$ Minimum $=224\ y_1-160\ w_1$

s.t $y_1-w_1>= 300-t_1 *x_1$

There $l_1=2.8$, $u_1=4.2$, $t_1=3.5$

so, $2.8\, x_1 <= 300 <= 4.2 * x_1$

or, $300/4.2 <= x_1 <= 300/2.8$

$Q_1(x_1)=$

$$\begin{cases} 224(300 - 3.5 * x1) & x1 < 300/4.2 \\ 9600 - 380.8 * x1 + (2.05714 * 10^6)/x1 & \frac{300}{4.2} \le x1 \le 300/2.8 \\ 160(300 - 3.5 * x1) & x1 > 300/2.8 \end{cases}$$

In this similar way we have found the continuous equation of other crops.

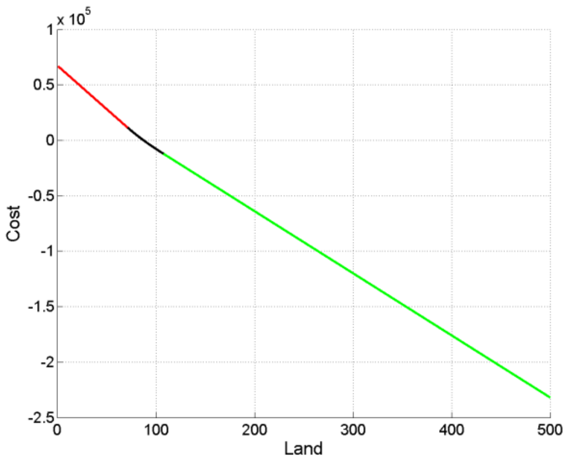

Fig. 3. Continuous function of rice

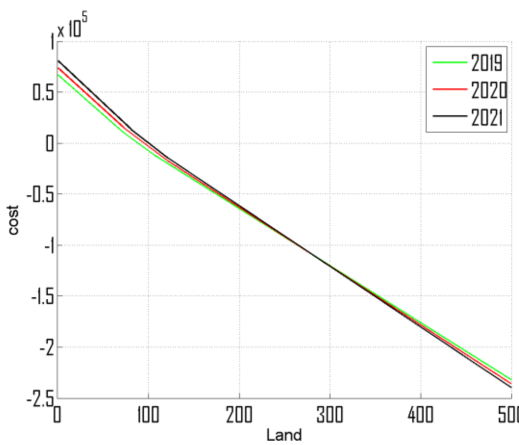

Fig. 4. Graph of 2019 2020 2021

$Q_2\,(x_2)$

$$= \begin{cases} 59500 - 595 * x2 & x2 < 250/3 \\ -289 * x2 + \frac{2125000}{x2} + 8500 & \frac{250}{3} \le x2 \le 250/2 \\ 42500 - 425 * x2 & x2 > 250/2 \end{cases}$$

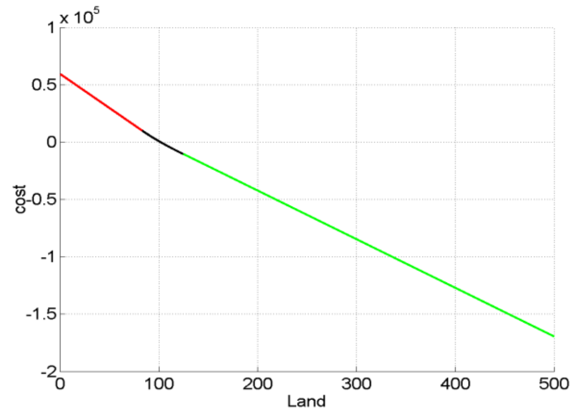

Fig. 5. Continuous function of wheat

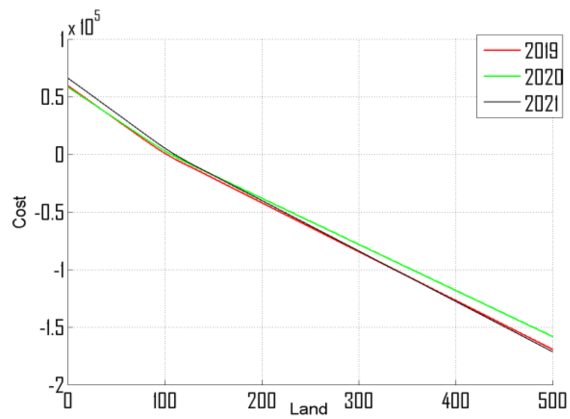

Fig. 6. Graph of 2019 2020 2021

$Q_3(x_3)=$

$$\begin{cases} 50400 - 630 * x3 & x3 < 240/3.6 \\ -306 * x3 + \frac{1.44*10^6}{x3} + 7200 & \frac{240}{3.6} \le x3 \le 240/2.4 \\ 36000 - 450 * x3 & x3 > 240/2.4 \end{cases}$$

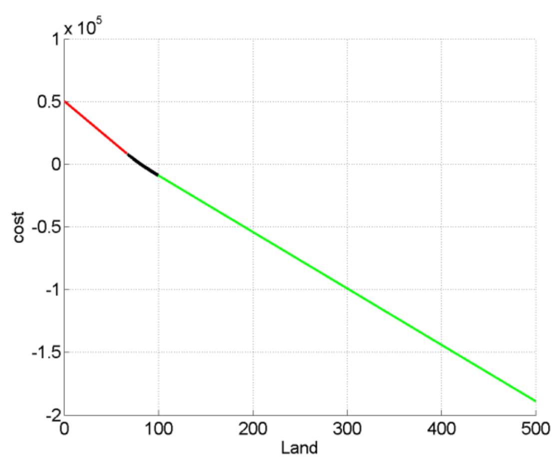

Fig. 7. Continuous function of corn

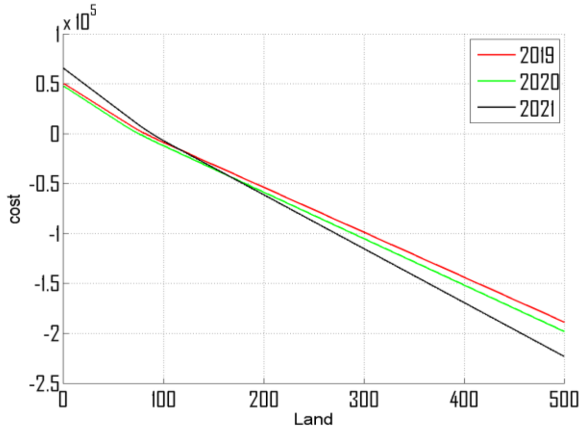

Fig. 8. Graph of 2019 2020 2021

$Q_4(x_4)$=

$$\begin{cases} -720*x4 & x4<6000/20 \\ -720*x4+\frac{13(24*x4-6000)^2}{8*x4}\ \frac{6000}{24}\le x4\le 6000/16 & \\ -156000-200*x4 & x4>6000/16 \end{cases}$$

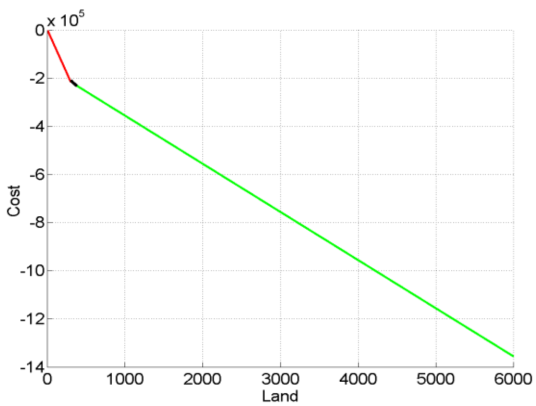

Fig. 9. Continuous function of sugar beets

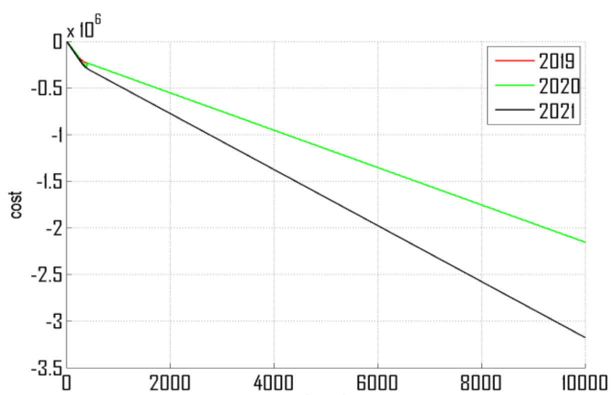

Fig. 10. Graph of 2019 2020 2021

The x-axis in each graph represents assigning land for crops, while the y-axis represents the ideal cost value of each crop in the second stage. In Fig. 10, the graphs for 2019 and 2020 are practically identical. There are four continuous, convex, and differentiable functions in $Q_i(x_i)$, and the first-stage constraints is linear. According to the graph, lowering the minimum size of farmed land lowers yield while raising cost value, resulting in a loss of profit. If the value of the land grows, so will the yield and profit margin. The largest and lowest earnings, on the other hand, would swing up and down in the intermediate stage. However, because the quantity of land available is fixed and the farmer only has a few needs, he cannot grow the cultivating area without taking into account the entire amount of land available.

KKT Method[3]

In this section we will show that the solution of this problem converge to the deterministic equivalent problem solution.From the deterministic solution we see that the solution lieon this interval $x_1>=200/2.8$, $250/3<=x_2<=250/2, 240/3.6<=x_3<=240/2.4, 6000/24<=x_4<=6000/16$.NowLagrange form is;

$f(x_1,x_2,x_3,x_4,\lambda_l)$ $=150*x_1+180*x_2+200*x_3+260*x_4+48000-560*x_1-289*x_2+(2125000/x_2)+8500-306*x_3+1.44*10^6/x_3+7200-720*x_4+(13/8)*(24*x_4-6000)^2/x_4+\lambda_l*(x_1+x_2+x_3+x_4-600)$;

The k-k-T conditions for this problem;

$\Delta f/\Delta x_i=0$ where i=1..4

$\Delta f/\Delta\lambda_l=0$,

$\lambda_l*\sum_{i=1}^{4} x_i=0$ where$\lambda_l>0$so,

$\sum_{i=1}^{4} x_I-600=0$

Simplification form of this problem is

$\lambda_l-410=0$

$\lambda_l- 2125000/x_2^2 - 109=0$

$\lambda_l- 1440000/x_3^2 - 106=0$

$\lambda_l+ (13*(1152*x_4 - 288000))/(8*x_4) - (13*(24*x_4-6000)^2)/(8*x_4^2) – 460=0$

$x_1 + x_2+ x_3 + x_4 - 600=0$

Solving this system of equations yielded $(\lambda_l, x_1, x_2, x_3, x_4)$=(410,190.195,84.023,68.825,256.957) , which meets all of the essential requirements and is thus optimum. ALL of these numbers add up to a deterministic problem. However, there are several drawbacks to this strategy. We must examine the non-linear continuous function in order to obtain an estimated value.

In this section, we have studied real-life farmers' problems. In the next section, we will study the real-life news-vendor problem.

V. Exploration of Newsvendor Problem

In this section, we'll look at the newsvendor problem and how stochastic programming may help us solve it.

A newsvendor from Dhaka, Bangladesh, must pick how many newspapers to order for the following day. Each newspaper purchasing cost α=1.300$ and she sells them for $\alpha+\beta$=1.90$. Any papers leftover can be sent back for recycling with a reimbursement Υ=0.30$ per newspaper. Suppose we have scenarios τ_i, i = 1, 2, . . . ,5, each with probability p_i . In all scenarios probability are same. The total number of selling newspaper and the unsatisfied demand of newspaper are varies in scenarios which are ;

Table 15. Demand of newspaper

τ_i ,i=	1	2	3	4	5
total demand of newspaper	30	50	80	100	120

Deterministic Model for Newsvendor Problem for each Scenario

Notations

Let, x_l= number of buying newspapers

$y_n[1]$= number of selling newspaper

$y_n[2]$= number of unsatisfying

$y_n[3]$= number of recycling newspaper

t[i]=demand of newspaper

Min $z=1.30*x_l+\sum_{i=1}^{5} p_i \, Q(x_l,t[i])$

$Q(x_l,t[i])= -1.9*y_n[1]-0.3*y_n[3]$.

For the individual scenario's

$z=1.30*x_l+ Q(x_l,t[i])$

s.t $y_n[1]+ y_n[2]= t[i]$

$y_n[1]+y_n[3]=x_l$

$y_n[1],y_n[2],y_n[3],x_l>=0$

Where all the variables will be integer.

Expected Value Finding

For expecting value and varience value the piecewise function is;

$$C(x_l,t)= \begin{cases} -\beta x, \ if \ xl < \tau i \\ (\alpha - \Upsilon)xl - (\alpha + \beta - \Upsilon) * t, \ \ otherwise \end{cases}$$

$$\text{Or, } C(x_l,t)= \begin{cases} -0.6 \text{ x, if} xl < t \\ 1 * \text{xl} - 1.6 * \text{t} \ , otherwise \end{cases}$$

t is uniformly distributed on [0,150].

$E[C(x_l,t)] =\int_{\omega} c(x_l,t)d\omega/(b-a)$

$= \int_0^{xl}(1*xl-1.6*\omega)(d\omega/150)+\int_{xl}^{150}(-0.6x_l)(d\omega/150)$

$=-0.004*(150 - x_l)*x_l+ 0.00133333*x_l^2$

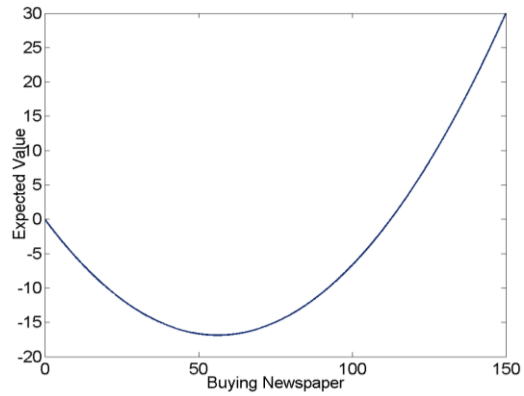

Fig. 11. Graph of expected value

Finding Variance Value

$Var [C(x_l, t)] = E[C(x_l, t)^2] - (E[C(x_l,t)]))^2$

$E[C(x_l, t)^2] = \int_0^{xl}(1*x_l-1.6*\omega)^2 (d\omega/150)+\int_{xl}^{150}(-0.6x_l)^2 (d\omega/150)$

so,var $Var [C(x_l, t)] =0.0024*(150 - x_l)*x_l^{2} + 0.00168889*x_l^3- (-0.004*(150 - x_l)*x_l + 0.00133333*x_l^2)^2$

Solution of the Deterministic Model for individual scenario's

After solving the problem then we get

Table 16. Profit of 5 scenario

Scenario	Buying Newspaper	Selling newspaper	Profit $
1	30	30	18
2	50	50	30
3	80	80	48
4	100	100	60
5	120	120	72

We can see in DLP that the newsvendor would make the most money if she sold every newspaper, but this is not realistic. SLP and scenario analysis are used to deal with unanticipated situations. As a result, she may use SLP to figure out which metric is best for newsvendor.

Deterministic Equivalent SLP for Newsvendor Problem

Minimum z=1.30*x_l+(1/5)$\sum_{i=1}^{5}$(-1.9* y_n[1,i]-0.3* y_n[3,i])

for t[i]

y_n[1,i]+y_n[2,i]=t[i]

y_n[1,i]+y_n[3,i]=x_l

i=1,2...5

The following are possible solutions to the problem:

Table 17. Solution of newsvendor problem

Expected value	-16.6667		
Variance	533.333		
First stage variable	bought newspaper 50		
Second stage variable	Sold newspaper	Recycle newspaper	Unsatisfied demand
1	30	20	0
2	50	0	0
3	50	0	30
4	50	0	50
5	50	0	70
			overall profit 23.6 $

Using this information, we can determine how many newspapers we need to purchase in order to make a profit under all situations. This problem's optimal profit is 23.6, however the predicted anticipated value is 16.6667. Using this strategy, he can figure out how many newspapers he has to buy in order to make a profit. We are unable to obtain the best value using another way. We can observe that the other method's predicted value is lower than SLP's value. We can tell from the variance that the distribution is significantly skewed.

Extension of Deterministic Equivalent SLP for Risk Aversion

A risk aversion model may be created in a variety of ways. A simple method for creating this model in the worst-case situation. It entails making profit under the most adverse conditions conceivable. However, we must keep in mind that in some models, it is impossible to predict which circumstance would result in the lowest profit. The worst-case situation in our example is connected to Situation 1. The profit in scenario 1 is 18 dollars. The problem's optimum function and restrictions are:

Minimum z=1.30*x_L +(1/5)\$\sum_{i=1}^{5}$(-1.9 y_n[1,i]-0.3 y_n[3,i])

for t[i]

y_n[1,i]+y_n[2,i]=t[i]

y_n[1,i]+y_n[3,i]=x_l

-(1.30x_l +(-1.9 y_n[1,i]-0.3 y_n[3,i]))>=18

where i=1,2...5 and all the variable will be integer.

The following are possible solutions to the problem:

Table 18. Solution of newsvendor risk aversion problem in SP

First stage variable	Bought newspaper	30	overall profit 18$
Second stage variable	Sold newspaper	Recycle newspaper	Unsatisfied demand
1	30	0	0
2	30	0	20
3	30	0	50
4	30	0	70
5	30	0	90

Risk-averse behavior is normal, but careful preparation and analysis can provide superior results. Using this strategy, he can figure out how many newspapers he has to buy in order to make the most money with the least amount of risk.

Profit comparisons

Table 19. Profit of all cases

Problem Name	Profit $
Deterministic Equivalent SP Problem	23.6
Risk Aversion Problem	18.00

The advantages of utilizing the risk aversion model include that it is equivalent to the worst-case scenario, hence it is best not to utilize it. For the newsvendor, a deterministic SLP model is more appropriate.

In this section, we looked at newsvendor challenges and how to address them with the AMPL Language.

VI Conclusion

The basic idea of this study is to use stochastic programming skills in real life. We started by going over the basics of stochastic linear, binary, and integer programming. As a result, we have gone over the two-stage SLP formulation procedure. We investigated a real-life farmer's problem using the Deterministic LP Model for each scenario, the Deterministic equivalent SLP Model for unexpected scenarios, the Extension of the Deterministic SLP Model for the price effect problem, the second stage SIP, SIP (for all stages), the first stage SBLP, and the risk aversion problem through SLP. We may claim that Deterministic Equivalent SLP models are suited for making timely judgments while taking into account all uncertainties for these models. We have shown that in the Deterministic Equivalent Stochastic Linear Programming model, all uncertainties are taken into account, but in Deterministic Linear Programming, only one-time uncertainties are taken into account, and we can not make any future decisions using this model. This SLP model, however, has limitations in that it only works with a limited number of circumstances. Then, to compare the results with the Deterministic Equivalent Stochastic Programming problem, we turned the models into continuous functions and solved the problem using the KKT approach. By adopting AMPL, we built computer algorithms for each scenario, allowing us to solve the problem flawlessly. We have also spoken about the issue of news vendors. For this computer approach, many variables and limitations are irrelevant. Drawing the graph of the piecewise and polynomial functions by hand is tough. As a consequence, for second-stage SLP issues, we utilized MATLAB to construct the resultant continuous function graph.

References

1. Winston Wayne L.and Goldberg, Jeffrey B., Operations Research: Applications and Algorithms, 4th Addition,pp.475-485.
2. John R. Birge and Francois Louveaux Introduction to Stochastic Programming, Springer Series in Operations Research and Financial Engineering, pp.1-32
3. Peter Kall and Stein Wallace, Stochastic Programming, JOHN WILEY & SONS, New York, USA,pp.1-102
4. Lecture on Stochastic Programming: Modelling and Theory, SIAM, Alexander Shapiro, Darinka Dentcheva and Andrzej Ruszczynski. Philadelphia, pp.40-72
5. Operations Research. An Introduction. Tenth Edition. Global Edition,pp., Hamdy A. Taha. University of Arkansas, Fayetteville, 257-262,11-23.
6. Shabbir Ahmed, The sample average approximation method for stochastic programs with integer recourse. Technical Report, IsyE, Georgia Tech, 2002.
7. Mohammad Babul Hasan & Sajal Chakroborty, An Impeccable Solution Procedure of Stochastic Programming Problems by Inserting Scenarios In Deterministic Case, Article in GANIT Journal of Bangladesh Mathematical Society. January 2017
8. Zhengyang Hu & Guiping Hu, A two-stage stochastic programming model for production lot-sizing and scheduling under demand and raw material quality uncertainties, Article in International Journal of Planning and Scheduling · January 2019
9. Arno Maatman,Caspar Schweigman , Arjan Ruijs& Maarten H. van der Vlerk, Modeling Farmers' Response to Uncertain Rainfall in Burkina Faso:A Stochastic Programming Approach ,Article in Operations Research June 2002
10. Tirupathi Rao Padi, Putha R S Reddy & A. V. S. Suhasini Optimal manpower recruitment by stochastic programming in graded manpower systems, Article in International Journal of Systems Assurance Engineering and Management · April 2012.
11. Hashnayne Ahmed,A comparative study on Stochastic Programming Problems: modeling and applications in real life. September 2019
12. A Stochastic Programming Model, Shinji Kataoka,Source: Econometrica, **31**, No. 1/2 (Jan. - Apr., 1963), *Justin Dupar Busili Kampempe, Monga Kalonda Luhandjula*
13. Rukav,Suvak Stra zanac , Marija, Kruno Nenad and Zoran Tomljanovi, "Markov decision processes in minimization of expected costs"
14. On Some Theorems of Stochastic Linear Programming with Applications, J. K. Sengupta, G. Tintner and C. Millham
15. Schaum's Outline of Operations Research, Richard Bronson..

16. Optimization and estimation study of manpower, KEAMOGET SESETLHARE planning models, , Setlhare, K

17. Linear Programming Solvers for Markov Decision Processes, Diego Bello and German Riano.

18. Solving Binary Cutting Stock Problems by Column Generation and Branch-and-Bound. Pamela H. Vance, Cynthia Barnhart, Ellis L. Johnson, George L.Nemhauser

19. Operations Research,S.R.Yadav, 2014

20. Hillier, Frederick S and Lieberman, Gerald J, Introduction to Operations Research, 7TH Addition

21. Mostafa Ekhtiari and Keivan Ghoseiri,"Multi-objective stochastic programming to solve manpower allocation problem"