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Application of Singular Value Decomposition (SVD) in Seismic Imaging: Theory, Methods, and a Synthetic Case Study

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Abstract

Singular Value Decomposition (SVD) is a powerful linear algebra technique widely applied in geophysical imaging, especially seismic data processing. Its utility in solving ill-posed inverse problems, denoising, and data compression makes it indispensable for interpreting large and noisy datasets. This study explores the theoretical foundations of SVD and its application in seismic imaging, with emphasis on inversion and noise attenuation. A synthetic seismic dataset is used to demonstrate how truncated SVD improves the quality of subsurface models. The results highlight the effectiveness of SVD in stabilizing inversion processes and enhancing resolution in seismic sections. We also discuss the implications of singular value truncation on data fidelity and inversion accuracy, setting the stage for future work combining SVD with machine learning and real-time imaging systems.

Keywords: Singular Value Decomposition (SVD), Seismic Imaging, Noise Reduction, Seismic Inversion, Synthetic Seismic Data.

I. Introduction

Geophysical imaging, particularly seismic imaging, is central to subsurface exploration in fields ranging from hydrocarbon prospecting to earthquake hazard analysis. These techniques rely on interpreting seismic wave responses to subsurface structures, often recorded as large and noisy datasets. A fundamental challenge in this process is the ill-posed nature of the inverse problem: recovering the subsurface model from seismic observations is often unstable and sensitive to noise[1,2].

To mitigate these issues, mathematical techniques such as **Singular Value Decomposition (SVD)** are employed. SVD offers a way to analyze, compress, and regularize data and operators, providing numerical stability and improving interpretability. It is especially effective in separating signal from noise and enabling truncated or damped solutions to inverse problems.

The objective of this research is to present a comprehensive overview of how SVD contributes to seismic imaging, both theoretically and in practice. We provide:

- A review of the mathematics behind SVD
- Practical uses in seismic data processing
- A case study with a synthetic seismic section demonstrating SVD-based inversion and denoising

The purpose of this research is to demonstrate the practical effectiveness of Singular Value Decomposition in seismic imaging tasks, particularly in data denoising and inversion.

While SVD has been widely studied in theory, this work provides a reproducible case study showing:

how SVD can be applied to synthetic seismic data, how truncating small singular values reduces noise and stabilizes inversionand How 2D and 3D visualizations enhance understanding of seismic reflection patterns before and after filtering. This paper is intended as a self-contained reference for geophysicists, researchers, and students looking to apply SVD to real-world seismic problems using a computational and visual approach.

The remainder of the paper is structured as follows: Section 3 discusses the mathematical foundation of SVD, Section 4 explains its role in seismic imaging, Section 5 presents the case study, Section 6 provides a discussion of the results, and Section 7 concludes the paper.

II. Background and Related Work

Seismic imaging is a cornerstone of geophysical exploration, widely used in resource discovery, environmental monitoring, and earthquake hazard analysis. At its core, it involves recovering an estimate of the Earth's interior from surface-recorded seismic waveforms. This recovery process is an archetypal ill-posed inverse problem: data is incomplete, noisy, and resolution decreases with depth and wave attenuation[3,4].

To tackle these challenges, linear algebra-based regularization techniques have been widely adopted. Among them, Singular Value Decomposition (SVD) has emerged as a powerful tool for stabilizing inversion and filtering seismic

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data. The classical work of Aster et al.[1] introduced SVD in the context of geophysical inverse problems, highlighting its utility in data compression, noise reduction, and spectral filtering.

Early applications of SVD in seismic data processing focused on denoising and multichannel filtering, such as in the *Multichannel Singular Spectrum Analysis (MSSA)* framework[2], where signal components are reconstructed by isolating dominant singular values. Similarly, Golyandina et al.[3] demonstrated how low-rank SVD approximations could separate coherent signal from noise in time-series data.

In seismic inversion, SVD has been used to regularize the inversion of large, poorly conditioned Jacobian or Hessian matrices[4]. Truncating small singular values helps to avoid amplifying measurement noise and ensures that the estimated subsurface model captures only the most stable and interpretable features.

More recent advancements explore combinations of SVD with machine learning and compressed sensing. For instance, Wen et al.[5] integrate sparse SVD approximations in seismic migration workflows, while Zhang et al.[6] incorporate SVD layers into deep networks for seismic data recovery.

Despite these developments, there remains a need for accessible workflows and reproducible examples of SVD-based filtering and inversion. This study contributes to that effort by presenting a complete pipeline for seismic denoising using SVD, supported by synthetic data and interpretable 2D/3D visualizations.

III. Mathematical Background of Singular Value Decomposition (SVD)

Singular Value Decomposition (SVD) is a foundational tool in linear algebra, widely used in solving large-scale, ill-posed problems such as those encountered in seismic data processing[7,14]. For any real matrix $A \in R^{m \times n}$, the SVD is defined as:

$$A = U\Sigma V^T$$

Where

(i) $U \in R^{m \times m}$ is an orthogonal matrix containing the left singular vectors of A,
(ii) $\Sigma \in R^{m \times n}$ is a diagonal matrix who's non-zero elements $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$ are the singular values of A,
(iii) $V \in R^{n \times n}$ is an orthogonal matrix containing the right singular vectors of A,
(iv) $r = \text{rank}(A)$.

This decomposition allows A to be expressed as a sum of rank-one matrices:

$$A = \sum_{i=1}^{r} \sigma_i u_i v_i^T$$

Each term in the sum represents a mode of the data, capturing the essential structure of the matrix.

Geometric Interpretation

Geometrically, SVD can be viewed as a sequence of linear transformations: a rotation or reflection of the input space (via V^T), followed by scaling along principalaxes (via Σ , and a final rotation or reflection of the output space (via U). This is particularly insightful in seismic imaging, where the structure of the earth affects wave propagation in a directional and scale-dependent manner[7,15].

Moore-Penrose Pseudoinverse

The SVD also provides a basis for computing the Moore-Penrose pseudoinverse of a matrix A [7]

$$A^+ = V\Sigma^+ U^T$$

where Σ^+ is obtained by taking the reciprocal of the non-zero singular values in Σ and transposing the result. In geophysical inverse problems, where a model x is inferred from observed data d , the pseudoinverse provides a least-squares solution:

$$x = A^+ d$$

Truncated SVD

Due to noise and instability in inverse problems, truncated SVD is commonly used[10]. Here, only the largest k singular values and corresponding singular vectors are retained to form an approximation A_k of A:

$$A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

This approximation acts as a filter, eliminating small singular values that may amplify noise and degrade inversion quality.

Computational Considerations

The computation of full SVD for large matrices is computationally intensive, with time complexity $O(mn^2)$ for dense matrices[10]. In seismic applications involving massive datasets, efficient algorithms such as:

- Randomized SVD,
- Lanczos bidiagonalization,
- Sparse SVD methods

are often used to compute only the dominant singular values and vectors needed for inversion and filtering.

IV. SVD in Seismic Imaging

Seismic imaging aims to reconstruct subsurface structures by analyzing the propagation of seismic waves. These waves are affected by the physical properties of the Earth, and their recorded reflections provide indirect information about subsurface heterogeneities. However, the process of converting seismic data into images involves solving an inverse problem that is often ill-posed, sensitive to noise, and computationally demanding[13,16].

Singular Value Decomposition (SVD) plays a crucial role in addressing these challenges by enabling stable inversion, noise attenuation, and data compression. This section outlines the key applications of SVD in seismic imaging.

Denoising and Data Filtering

Seismic data is typically contaminated by noise from environmental sources, instrumentation, and acquisition artifacts. Since noise often manifests in the lower-energy components of the data matrix, SVD provides a natural mechanism for separation of signal and noise. By retaining only, the dominant singular values in a truncated SVD, one can reconstruct a denoised version of the data matrix[11].

$$A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

Here, A_k represents a rank- k approximation of the original data matrix A , preserving the most significant features while suppressing low-energy noise components.

This method is widely used in:

(i) Common Midpoint (CMP) gather processing,
(ii) Shot gather enhancement,
(iii) Multichannel singular spectrum analysis (MSSA).

Stabilizing Seismic Inversion

In seismic inversion, the goal is to estimate a model of subsurface parameters (e.g., velocity, impedance) from observed wavefields. The relationship is commonly linearized as:

$$d = G\,m + \epsilon$$

where d is the observed data, G is the forward modeling operator, m is the model to be recovered, and ϵ represents noise[6].

When G is ill-conditioned or rank-deficient, direct inversion is unstable. Applying SVD to G , we obtain:

$$G = U\Sigma V^T$$

The solution to the least-squares problem is then given by

$$m = V\Sigma^{+}U^T d$$

To reduce sensitivity to small singular values that amplify noise, a truncated SVD can be used:

$$m_k = \sum_{i=1}^{k} \frac{u_i^T d}{\sigma_i} v_i$$

This regularized solution captures the most stable and meaningful components of the model, improving resolution and robustness.

Low-Rank Representations in Migration and Full Waveform Inversion

Advanced imaging techniques such as reverse-time migration (RTM) and full waveform inversion (FWI) involve manipulating large-scale wavefields. These wavefields often exhibit a low-rank structure in specific domains (e.g., frequency, wavenumber, or source-receiver space), making SVD an attractive tool for:

(i) Reducing data dimensionality,
(ii) Compressing Green's function operators,
(iii) Enhancing computational efficiency.

Hybrid approaches combining SVD with curvelets, wavelets, or deep learning are also emerging for high-resolution and real-time imaging[12].

V.Synthetic Data Generation and SVD-Based Denoising

Seismic data is collected by measuring the reflection of seismic waves off subsurface layers, which allows geologists to map structures such as oil reservoirs or fault zones. However, seismic data often includes background noise. By applying SVD, we can identify and preserve coherent reflections, isolating them from noise. For example, in processing a seismic reflection dataset, retaining only the leading singular values associated with primary reflections enhances the clarity of the resulting image. This approach is particularly useful in identifying fault lines and stratigraphic boundaries with higher accuracy. Here's an example of a practical demonstration using Singular Value Decomposition (SVD) to process seismic imaging data.

Synthetic Data Generation

We begin by generating synthetic seismic data to simulate subsurface reflections in a controlled and noise-free environment. The underlying data is constructed using sinusoidal wave patterns to emulate primary seismic reflections across a spatial grid.

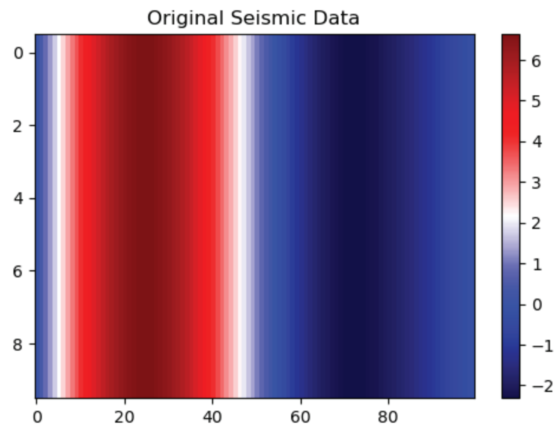


Fig. 1A. Heatmap of the original seismic data

Figure 1A presents a heatmap of this clean seismic dataset. Each row corresponds to a different spatial trace or scan, while each column represents a time or spatial sample. The color scale (e.g., red-to-blue using a seismic colormap) indicates reflection amplitude and polarity. Positive amplitudes (typically shown in red) represent upward-traveling wavefronts, and negative amplitudes (blue) represent downward-traveling reflections. This idealized dataset serves as a baseline for assessing the impact of noise and the effectiveness of denoising methods.

Noisy Seismic Data

To simulate realistic acquisition conditions, we add zero-mean Gaussian white noise to the clean dataset. This mimics environmental interference, acquisition imperfections, and sensor noise that typically corrupt field-recorded seismic data.

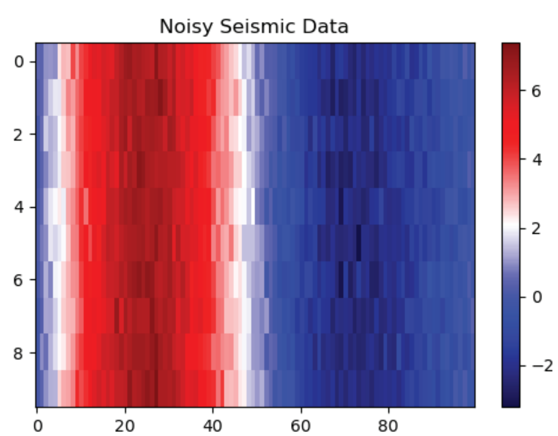


Fig. 1B. Heatmap of the noisy seismic data

Figure 1B shows the resulting noisy seismic section. Although the primary wave patterns remain, they are visibly distorted by random fluctuations. Reflections appear blurred or scattered, making interpretation difficult and masking important subsurface structures. This visual degradation motivates the need for advanced denoising strategies such as Singular Value Decomposition (SVD).

Denoising with SVD

We apply SVD to the noisy data matrix and reconstruct it using only the top 15 singular values, effectively filtering out low-energy components likely associated with noise.**Figure 1C** displays the denoised seismic data reconstructed from the truncated SVD. Compared to Figure 1B, this version recovers the essential wavefront structure while suppressing much of the random noise. The denoised reflections are smoother and more coherent, and transitions between positive and negative amplitudes are clearer. The result is a more interpretable seismic section that better highlights subsurface features such as fault planes and stratigraphic interfaces.

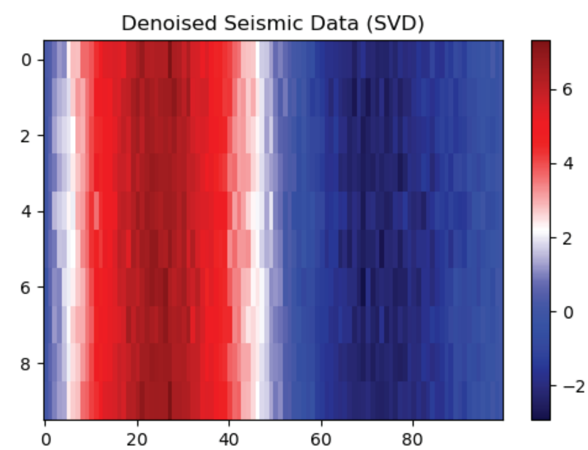


Fig. 1C. Heatmap of the denoised seismic data

3D Surface Visualization

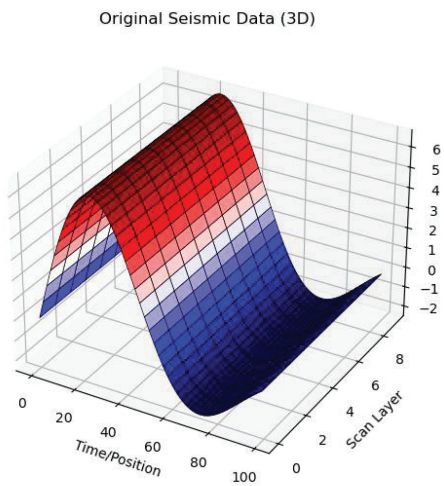
To further analyze data quality and structure, we generate 3D surface plots of the seismic sections, treating amplitude as elevation.

Original Seismic Data (3D): The clean dataset forms a structured landscape of peaks and troughs, visually representing coherent reflections across space and time.

i. **Noisy Seismic Data (3D)**: Gaussian noise disrupts the regular surface, introducing roughness and obscuring reflection continuity. This highlights the difficulty of interpreting raw data in practice.

ii. **Denoised Seismic Data (3D)**: SVD-filtered data shows a much smoother surface, where reflection geometries are more easily identified. The primary reflectors are restored, and high-frequency noise is significantly attenuated.

iii. **Figure 1D** summarizes the 3D comparison across the clean, noisy, and denoised seismic volumes. These visualizations underscore the effectiveness of SVD in preserving critical geological features while eliminating incoherent noise.



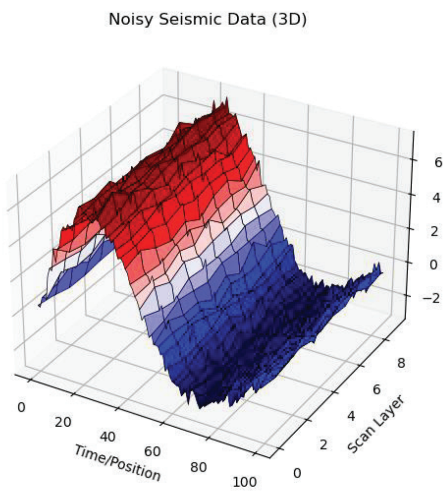


Fig. 1D. 3D surface plot of the seismic data

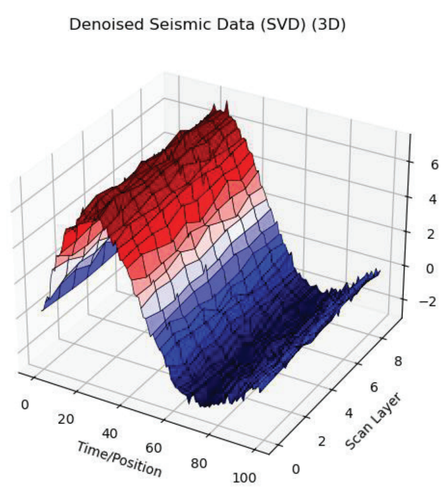


Fig. 1D. 3D surface plot of the seismic data

VI. Conclusion

Singular Value Decomposition (SVD) provides a robust and mathematically sound framework for enhancing the quality and interpretability of seismic imaging data. In this study, we demonstrated how SVD can be effectively applied for noise attenuation, data compression, and the stabilization of seismic inversion processes. The theoretical foundation of SVD was reviewed, emphasizing its ability to regularize ill-posed inverse problems by truncating noise-sensitive singular values.

Using a synthetic seismic dataset, we simulated the degradation caused by Gaussian noise and showed how truncated SVD can recover the dominant reflection signals. The resulting denoised sections closely resembled the original structure, as confirmed through both 2D heatmaps and 3D surface visualizations. These results affirm that SVD not only improves the signal-to-noise ratio but also preserves critical geological features necessary for subsurface interpretation.

The study underscores the practical value of SVD as a preprocessing tool in seismic workflows—enhancing data quality before more complex operations such as migration, inversion, or interpretation. Looking ahead, future research may incorporate adaptive rank-selection techniques, integrate SVD within machine learning architectures, and explore real-time denoising applications for large-scale 3D seismic volumes.

In conclusion, SVD continues to be a fundamental component of the geophysical imaging toolkit—offering a balance between computational efficiency, mathematical rigor, and interpretability in the ongoing quest for accurate subsurface models.

Declarations

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