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A Comparison Study between the Recently Developed Methods of Transportation Problem: A Study on the Lower-Dimensional Problems

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ABSTRACT

Resource allocation is one of the crucial challenges to the decision-makers. It has a significant impact on the profitability of any company. Freight transport is one type of resource allocation; here the decision-maker has to choose the quantity of products for delivering at a minimum cost from the several plants/factories/sources to the several destinations/ warehouses. We have conducted a comparative study based on secondary data to figure out the best technique for solving the freight transportation problems. Here we have selected 40 balanced and unbalanced problems randomly with dimensions 3×3 to 7×7. We have selected 23 existing methods, some of them are popular and some are recently developed. We compare these 23 methods regarding firstly the optimal solution criterion, and secondly which one can give us the solution in the least step or short time. We have checked the solution at first manually, then by GNU Octave to figure out if there is any inconsistency. Here, the GNU octave is chosen for its easy acceptance and easy input procedure. On our selected problems, the findings show us that the Faster STrongly Polynomial method (FSTP) is best if we consider the least step but concerning the short time MOdifiedDIstribution method worked on Vogel's Approximation Method, well known as VAM-MODI is performing the best.

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1. Introduction

The Transportation Problem (TP) is a distribution problem where the products are transported from several sources (factories) to several destinations (warehouses). Its main objective is to cut the least cost in transporting the products. There are two restrictions, first, one is the total demand of warehouses and the second one is the total capacity of supplying the products. The transportation problem is classified into

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different groups based on their primary objective and source supply against destination requirements [10]. With the primary objective, the transportation problem is categorized in two ways, the minimization case and the maximization case. The minimization transportation problem is the case of shipment of commodities where the main goal is to minimize the transportation cost. If a company wants to maximize its earnings by the delivery of the products from sources to destinations, is the maximization case.Considering the source's fixed capacity and the warehouse's fixed demand, there will be two types of TP. Where the supplied quantity and the demanded quantity coincide with each other is called the balanced transportation problem. The opposite in this case is the unbalanced transportation problem. Hence two instances can arise. The number of supply quantities of sources is more than the number of demands of destinations, or vice versa can happen [15].

The extensions of the transportation problem model help to obtain an optimal solution in the other sectors of the business problem, specifically in employee scheduling, inventory control, personnel assignments, and multi-objective optimization as goal programming. We have passed more than two centuries in finding the algorithm that can provide an optimal solution without the initial basic feasible solution. Hence Vogel's Approximation Method (VAM) and MOdifiedDIstribution Method (MODI) or Stepping Stone Method (SSM) is known as the most efficient method in finding the optimal solution that can satisfy all the constraints and minimize the transportation cost. But in the real world, any transportation problem needs an algorithm that can help the decision-maker to know the optimized result without any computational complexity and in a short computational time. Kleinschmidt's and Schannath (1995) developed a model named STrongly Polynomial (STP) which can give the optimum result without any Initial Basic Feasible Solution (IBFS) [33], but its limitations are that it cuts a considerable run time. In 2018, an algorithm called Faster STrongly Polynomial (FSTP) claimed that it could overcome all the flaws mentioned above. This Faster Strongly Polynomial method motivates us to check whether it works better than other existing methods or not. The study aims to seek the best one from the distinguished existing methods including Faster Strongly Polynomial method. Here our focus is on the effective and popular methods that help us to choose these 23 methods. Now, if we think about the dimension, here we want to notice the lower dimensional transportation problems vary from 3x3 to 7x7. We choose the lower-dimensional problems to check the validity of the methods manually also. We make a comparison of different development methods and try to find out which can give us an accurate result by using the performance evaluation tool Average Relative Deviation (ARD) [31].

2. Literature Review:

Approximately two centuries ago, we began to find the optimal solution for the problem of transport. In 1781, a French mathematician and physicist, Gaspard Monge, developed a hypothesis of soil transport at a minimum cost [39]. In 1930, the Russian mathematician A. N. Tolstoi proposed a solution to the planning of cargo transport [53]. Subsequently, the Russian mathematician and economist Leonid Vital'evichKantrovich used transportation problems to establish the idea of duality (1940) [24]. He developed a method for addressing a linear transport problem, the potential method with M. K. Gavurin [25]. In 1941, American mathematician Frank Lauren Hitchcock [22] established the method was very close to a later established simplex method.

According to the literature, the first person who mainly developed the transportation problem was F.L. Hitchcock. He presented his study entitled "The Distribution of a product from several sources to numerous localities''. In 1949, T. C. Koopmans [34] introduced "Optimizing Utilization of the Transportation System". Then the transportation problem was converted into a linear programming problem and resolved using the simplex method by the renowned researcher G.B Dantzig [12] in 1951. He proposed a MOdifiedDIstribution method known as MODI to find an initial basic feasible solution in 1963 [13]. Charnes and Cooper (1954) [11] developed another method named the Stepping Stone Method (SSM) that provides an alternative way of determining the simplex method information. Gleyzal designed an alternative approach as in 1955 [19] by Ford and Fulkerson (1955, 1956) [16], and Munkres (1957) [40]. It is required to find an initial basic feasible solution to obtain an optimal solution to a transportation problem. In research, many methods are available to achieve an initial basic feasible solution such as North-West corner rule, Row Minima Method, Column Minima Method, Least Cost Method, Vogel's Approximation method, etc. Reinfeld and Vogel developed the Vogel's Approximation Method [47], which is usually named VAM or Unit Penalty Method. Some well-known transportation methods include the Stepping Stone Method (Charles and Copper-1954), MOdifiedDIstribution method (Dantzig, 1963), Modified Stepping Stone method (Shih, 1987) [48], and simplex type algorithm (Arsham and Khan, 1989) [5] are used in finding the optimal solution. Further then, many ways were improved by many researchers. Edward J. Russell (1969) [47] proposed Russel's Approximation method where the penalties are calculated by the difference of the corresponding row and column highest entry of every cell from the corresponding element. Then he makes his allocation to having the lowest penalty.

Shimshaket. al. (1981) [50] suggested a modification of VAM for solving the unbalanced problem. Here they followed the VAM as usual by ignoring all the penalties included in the dummy row or column. In 1984 Goyal [20] proposed a method for solving the unbalanced problem where he set the high cost as the dummy cost instead of zero and followed the same procedure as VAM. Ramakrishnan [43]suggested subtracting the smallest element from every row or column and then replacing the dummy cost with the highest unit transportation cost. And VAM is used here for finding the initial basic feasible solution. He developed the GVAM in 1988. Kirca and Satir (1990) [32] concerted the transportation cost matrix. For the Row Opportunity Cost Matrix (ROCM), they subtracted all the lowest values from every element row-wise. For the Column Opportunity Cost Matrix (COCM), they follow the subtraction of all the lowest values from every element's column-wise. Then adding the ROCM and COCM got the Total Opportunity Cost Matrix (TOCM) and used the least cost method to generate a feasible solution. NagrajBalakrishnan (1990) [9] computed all the column penalties as before, except for the dummy column and the rows, hence the penalties are the difference of the lowest, and the next lowest cost ignoring the dummy column and used as usual VAM. It was discussed in his research "Modified Vogel's Approximation Method for the Unbalanced Transportation Problem". Kore and Thakur (2000) [35] solved the unbalanced transportation problem without converting it to a balanced one.

Ping and Chu (2002) improved the Dual Matrix approach as an alternative to the Stepping Stone by converting the problem into a corresponding dual one using sequence matrix operations [41]. Mathirajan and Meenakshi (2004) [38] modified the procedure followed by Kirca and Satir and defined the penalty of the lowest and 2nd lowest in every row and column and allocation is preferred to the highest penalty cost with a minimum cost cell. Kasana and Kumar (2005) [27] imposed the Extreme Difference Method where VAM is applied to the penalty of the highest and lowest unit transportation cost. Kulkarni and Dattar (2010) [36] converted an unbalanced problem to a balanced one by increasing the demand /supply and proposed a new algorithm to solve it. Abdur Rashid (2011) applied an effective way of finding the initial feasible solution by finding the highest penalty where the penalty is the difference between the extreme and 2nd extreme of each row and column [44]. Mansi (2011) investigates the two alternative methods for solving transportation problems. MansiSuryakandGaglani (2011) allocated in the single cell that is the minimum cost point of every row of the cost matrix. If the minimum cost is the same, she breaks the tie by calculating the difference between the minimum, and the next to the minimum unit cost for all those sources where destinations are identical [17]. Aminur Rahman Khan (2011, 2012) calculated the highest cost difference as the penalty of the two highest costs and allocated this way in the most upper penalty with the lowest cost [28,29]. Sudhakar (2012) [51] developed a new direction in searching for the optimal solution by assigning one zero in each row or column by subtracting the least one from each column and row. Got a suffix value for each zero and considered the greatest one for the allocation. Quddooset. al. (2012) mentioned in their paper "A New Method for Finding an Optimal Solution for Transportation Problem" that the allocation is preferred to the cell containing the zero and for that make the zeros in every row and column and count the total number of zero [42]. N. M Deshmukh [14] mentioned in his work named "An Innovative Method for Solving Transportation Problem" in 2012 that allocation will be started by subtracting the minimum odd cost for making the cell zero. And all the elements make unit by dividing by the number itself and subtracting it again. Then the same procedure is to be followed.

Md. AshrafulBabu et al. (2013) [7] applied the method named Lowest Allocation Method as LAM where allocation started with the lowest cost and lowest-demand/supply. Jumanet. al. (2013) checked the sensitivity of VAM and observed the effect of balancing and unbalancing issues [23]. Abdur Rashid (2013, 2015) also proposed a heuristic and named it as an Average Cost Method (ACM) where the penalty is calculated from the average of each row and column [46]. NigusGirmay and Tripty Sharma (2013) proposed to reduce the extra demand/supply and follow the conventional approach of VAM [18]. Aramuthakannan&Kandasamy (2013) presented a new approach to the transportation problem, namely, the Revised DIstribution method (RDI), for solving an extensive range of such problems. The new method is based on the allocation of units to cells in the transport matrix starting with the least supply or demand to the cell with the lowest cost in the transport matrix and trying to find an optimal solution to the transmission given [4]. Babuet. al. (2014) [8]

developed an idea to allocate zero quantity supply and demand for VAM and other transportation algorithms.

Soomroet. al. (2014) modified the VAM. The proposed Minimum Transportation Cost Method (MTCM) by calculating the difference between the two most massive transportation costs for row penalty and the two lowest costs for column penalty [52]. Ahmed et. al. (2014) modified an effective method in finding the minimum cost where the allocation is made in the lower indicator, and the indicator is calculated by the subtraction of the most extensive entry of each row and each column from every element [1]. A. R. Khan et. al. (2015) preferred the cell containing the highest indicator of the Total Opportunity Cost Matrix (TOCM) in their work “Determination of Initial Basic Feasible Solution of a Transportation Problem; A TOCM-SUM Approach” [30]. MuwafaqAlkubaisi (2015) used the median cost as an indicator and then used the VAM in finding the transportation solution [3]. Mesbahuddin Ahmed et. al. discussed a new method in 2016 in their paper titled “A New Approach to Solve Transportation Problem”. They selected the cell containing Minimum Odd Cost (MOC i.e. 1) and if it doesn’t exist, make the elemental unit dividing by two and make the allocation with the lowest cost satisfying the demand and supply [2]. The Faster Strongly Polynomial Method (FSTP) was developed by AshrafulBabu [6] in the year 2018. The run time is faster than Kleinschmidt’s STP. A comparative study has shown that FSTP provides the optimal solution without the Initial Basic Feasible Solution.

3.Objectives of the study:

The core objective of this study is to compare the existing developed methods in solving the balanced and unbalanced transportation problems. The performances will be evaluated based on some criteria that cover the specific objectives. The specific objectives are extended as:

i) To focus on a sufficiently large number of lower-dimensional transportation problems. The reason for choosing the lower dimension is to check whether any inconsistency of the result got manually and the software is or not. Here we aim to find the result manually and by software. As the higher dimension than 7x7 is more complicated to solve manually, that's why we need to select the problems with the limited dimensions. We want to cover at least 40 problems from 3x3 to 7x7.

ii) To select a number of the most effective methods and compare between them. The selected 23 methods are chosen based on their popularity that measures their effectiveness. In the related literature review, we found these methods provided the optimal solution mostly. Hence some are prevalent and some are recently developed.

iii) To run the selected algorithms by software. Here all the selected methods are fit for the solver GNU Octave, included code of these methods generated on GNU octave and the problems with lower dimension solved by it.

iv) To measure the performances on account of

a) Frequency of the Optimal Solution

b) Average Relative Deviation (ARD)

c) Execution time

The performances will be measured on some queries, are the selected methods can provide the optimal solution, if so, how many times they will be able to provide the optimal solution, which method is best in comparison of the Average Relative Deviation (ARD) and if there is any tie occurred, our target to break the tie by their runtime. The best method will give the optimal solution at least runtime.

v) To record the performance of all the methods. A comparison table can show the optimal solution obtained by the distinguished methods; another one can be made based on the Average Relative Deviation (ARD). If there will be two or more two effective methods, recorded runtime will be helpful to break the tie.

4. Methodology:

In completing the research work, we have gone through related literature, and we have achieved knowledge in solving the Transportation Problems by various existing algorithms. Hence we have used 40 transportation problems with different dimensions from 3×3 to 5×7. There are both types of balanced and unbalanced problems. Therefore, the most popular twenty-two methods (that can solve any type of TP for the minimization case) are tested named North West Corner Method (NWC), Least Cost Method (LCM), Row Minima Method (RMM), Column Minima Method (CMM), Vogel's Approximation Method (VAM), Extreme Difference Method (EDM), ASM method (ASM), Revised Distribution Method (RDM), Average Cost Method (ACM), Zero Assignment Method (ZAM), Highest Cost Difference Method (HCDM), Russel's Approximation Method (RAM), Least Cost Position Method (LCPM), Cost Minimization Approach(CMA), Improved NMD Method (INMD), MTCM-HCDM, TOCM-LCM Approach, TOCM-VAM Approach, TOCM-EDM Approach, TOCM-HCDM Approach, TOCM-SUM Approach and Faster Strongly Polynomial method (FSTP). The optimality test describes the feasible allocations to convert to the optimal allocations. We test the optimality by using one of the most popular methods, the MODI or u-v method where the loop of distribution is restructured. After completing the data collection, we have calculated the total minimum cost by using these methods manually and we recheck these with the help of GNU Octave. These computer programs are coded by C Programming Language and run on a laptop with Intel Core i3 8GB of RAM. For solving the problem by the methods, we need the software's required input. There are three .dat files included here; these are c.dat, demand.dat, and supply.dat. The c.dat file shows the transportation cost unit matrix, the demand.dat file represents the demand of the destinations by a column vector where the 1d shows the total demand of the item of the 1st destination, 2d represents the total demand of the 2nd destination, and so on. The supply.dat file presents the capacity of each source or factory. This file inputs the data in a column vector also, here 1s is the capacity of the 1st source, 2s represents the capacity of the 2nd source, and so on. For a 2x2 dimensional problem, the c.dat file will be as C_{11}, C_{12} in the 1st row, and $C_{21,}$ C_{22} in the 2nd row, another two separate files named the demand.dat and supply.dat will include d_1, $d_{2,}$ andS_1, S_2 in column respectively.

By comparing 23 methods we have found the most effective method which can provide the least cost among these by the performance evaluation tool Average Relative Deviation (ARD).

$$RD(H,i) = \frac{IBFSCost - OBFSCost}{OBFSCost}, i = 1,2,....N$$

And

$$ARD(H) = \frac{1}{N}\sum_{i=1}^{N} RD(H,i)$$

IBFS=Initial Basic Feasible Solution, OBFS = Obtained Basic Feasible Solution

And ARD (H) = Average Relative Deviation of the given heuristic method H

RD (H, i) = Relative Deviation of the i^{th} problem for the given heuristic method

Now we have compared the effective method to MODI, the most popular method in the optimality test. We consider the criterion of the best method in obtaining the optimal solution.

5. Statement of the problem:

The transportation problem mainly evaluated the quantity of distributed products from the different sources to the different destinations. There will be at least 2 sources and 2 destinations.

A manufacturing company has m^{th} plants to produce their product. They have n^{th} warehouses to distribute their product. The unit cost of delivering the products from the plant P_1, P_2,..….and P_m to the warehouses $D_{1,}$ $D_{2,}$ $D_{3,}$.......andD_n are taka c_{11}, c_{12},c_{13},......c_{1n}; c_{21}, c_{22}, c_{23},......c_{2n};..........; c_{m1}, c_{m2}, c_{m3},......c_{mn}respectively. The demands of the warehouses are b_1, b_2, b_3, …...andb_n units respectively. The capacity of producing the products are a_1, a_2, a_3, …...and a_m units respectively. The company should know the optimal quantity of delivering the products from the plants to the warehouses that will be helpful to cut a minimum cost [21]. The transportation problem is given in the tabular form:

Table 1: General Transportation Problem

	D_1	D_2	D_3	….	D_4	Supply
P_1	c_{11}	c_{12}	c_{13}	….	c_{1n}	a_1
P_2	c_{21}	c_{22}	c_{23}	….	c_{2n}	a_2
P_3	c_{31}	c_{32}	c_{33}	….	c_{3n}	a_3
…..	….	….	….	….	….	….

P_m	c_{m1}	c_{m2}	c_{m3}	….	c_{mn}	a_m
Demand	b_1	b_2	b_3	….	b_n	

A transportation problem is balanced if the total supply (a_i) from all sources is equal to the total demand (b_j) in the destinations i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

A transportation problem is said to be unbalanced if the total supply (a_i) from all sources is not equal to the total demand (b_j) in the destinations i.e.,

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

The mathematical formulation of the above general transportation problem is [26]

$$Z_{min} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to,

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,3,...,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,3,...,n$$

$$where, x_{ij} \geq 0$$

6. Model Development:

Different indicators are used in these 23 methods by the researchers. Some methods are based on the minimum cost containing cell (LCM), some starting the allocation from the upper left corner (NWC), or by a penalty of row-wise entries (RMM) or column-wise entries (CMM), or measuring the penalties of each row and column (VAM). Sometimes the penalties are calculated as the difference of the highest and the lowest (EDM), sometimes between the highest and the next to the highest (HCDM), it may be the difference of the two lowest values (LCPM). In some methods, the calculation is based on making zero in each cell (ASM). For each, the $(i,j)^{th}$ zero cells, calculate the quantities Δ_{ij} by adding the reduced unit costs of the corresponding i^{th} row and j^{th} column (ZAM), or every element are subtracted from the sum of the highest component of the existing row and column. Then choose the smallest penalty to make an allocation (RAM). Or, the allocation starts with the minimum odd cost cell (INMD). If not, they are doing it by dividing the number itself by comparing the figure of available supply in the row and demand in the column. And allocation of the units' equals capacity or demand, whichever is less by using the average cost calculations. Somewhere the total opportunity cost table is made. And they are using the usual methods (TOCM-LCPM, TOCM-VAM, TOCM-EDM, TOCM-HCDM, TOCM-SUM) or by making a modified transportation matrix where the deduction is made a row and column-wise individually and followed by the LCPM algorithm (MTCM-LCPM). Every method wants to make the optimal allocations to find the basic feasible solution. By these methods, the unbalanced problem can be solved by introducing a dummy row or column as it needs. Some techniques used the VAM in modifying ways to solve the unbalanced problem. The dummy is not under consideration on some methods; some used the highest cost for the dummy one, some deduct the extra demand or supply that does not exist.

7. Findings:

The performance evaluation tool Average Relative Deviation (ARD) is

$$RD(H,i) = \frac{IBFSCost - OBFSCost}{OBFSCost}, i = 1,2,....N$$

And

$$ARD(H) = \frac{1}{N}\sum_{i=1}^{N} RD(H,i)$$

IBFS=Initial Basic Feasible Solution, OBFS= Obtained Basic Feasible Solution

And, ARD (H) = Average Relative Deviation of the given heuristic method H,

RD (H,i) = Relative Deviation of the i^{th} problem for the given heuristic method.

ARD is measured by the average of the relative deviation of the problems. It specifies the average performance of numerous techniques relating to the optimal solution is compared over the number of case instances [37]. The least ARD-providing method is most preferable. In measuring ARD, No. of optimal Solution (Shown in Appendix B: Comparative Study) obtained by using these different methods and also the percentage of obtaining an optimal solution on these 40 randomly selected studied cases are shown in the following table and hence also made a list on the base of the performance measuring by ARD:

Table 2: ARD, No. and percentage of the optimal solution in several methods

No	Name of Methods	Average Relative Deviation (ARD)	No. of Optimal solution	Percentage of No. of Optimal Solution
01.	FSTP	0	40	100
02.	MODI	0	40	100
03.	TOCM-VAM	0.01	26	65
04.	VAM	0.02	21	52.5
05.	EDM	0.05	17	42.5
06.	TOCM-EDM	0.05	17	42.5
07.	TOCM-SUM	0.06	16	40
08.	ZAM	0.08	20	50
09.	HCDM	0.09	12	30
10.	CMA	0.10	20	50
11.	TOCM-HCDM	0.10	10	25
12.	RAM	0.11	16	40
13.	ASM	0.13	16	40
14.	LCPM	0.14	11	27.5
15.	TOCM-LCM	0.14	8	20
16.	ACM	0.15	10	25
17.	LCM	0.16	10	25
18.	CMM	0.17	4	10
19.	RDM	0.18	9	22.5
20.	RMM	0.18	4	10
21.	INMD	0.21	4	10
22.	NWC	0.69	1	2.5
23.	MTCM-HCDM	2.51	0	0

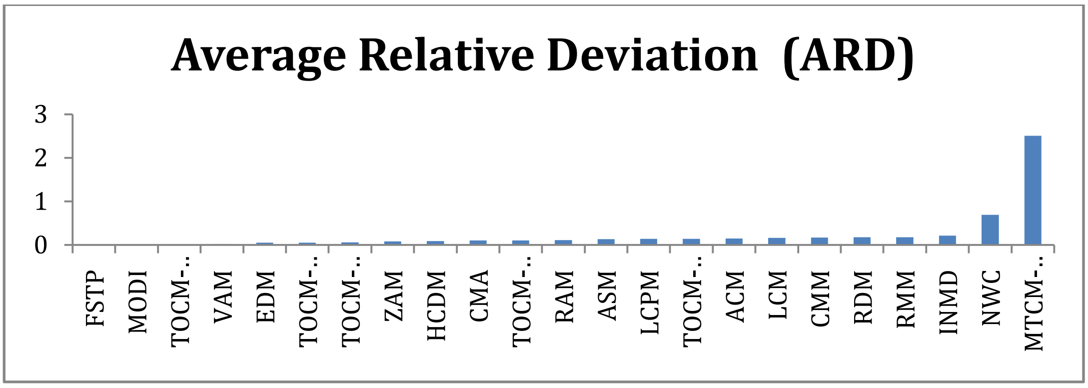

Figure 1: Average Relative Deviation (ARD) of several methods on the lower dimension

Source: Table 2(Table of the ARD in several methods)

Less ARD gives us the best method. Hence the value closest to zero represents the least deviation. The increasing value 0 to positive means, the obtained solution is going far from the initial basic feasible solution. By the ARD, we have chosen the most effective way from the considered problems in this study. We observed here the least ARD is 0, and this least ARD is for both MODI and FSTP.

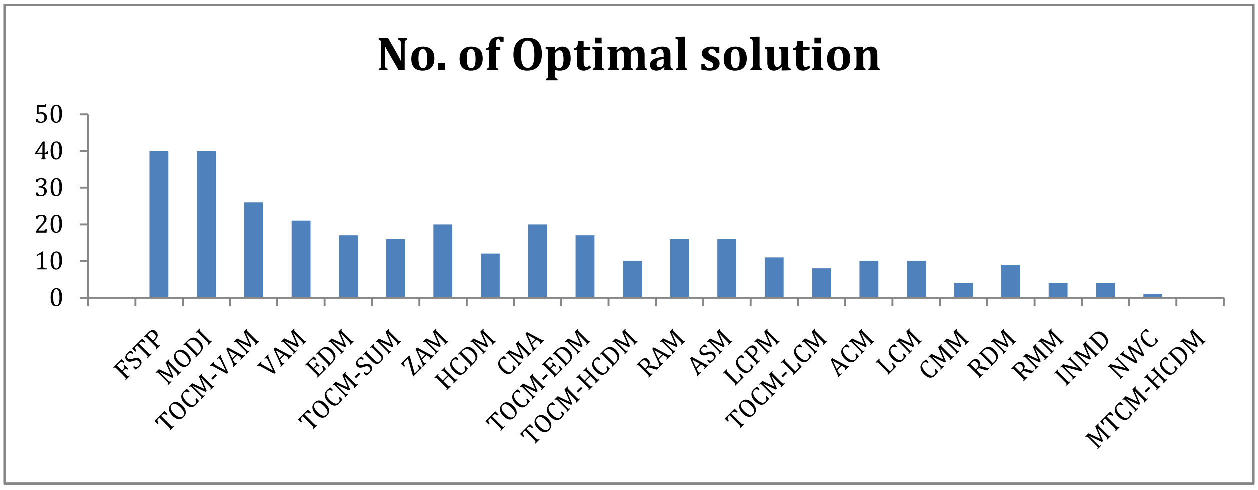

Figure 2: The frequency of no. of the optimal solution by several methods on the lower dimension

Source: Table 2 (Table of the frequency of no. of the optimal solution in several methods)

From the above table and graph, we can conclude that the best method between the existing processes within the study period is FSTP (Faster Strong Polynomial algorithm for Transportation Problem). Hence the number of the optimal solution obtained by FSTP is 40 (out of 40), which means 100% of the optimal solution can be obtained by this method. And also, it has the value of the ARD=0, which implies it is the most effective method comparing the others; hence MODI also provides all optimal solutions. MODI gives the optimal result with the initial basic feasible solution but FSTP can provide the same without it. The last comparison criteria, i.e. the execution time of solving these problems are given below:

Table 3 (a): Execution Time (Seconds) byVAM-MODI and FSTP in lower-dimensional TP (cont.)

Problem Number	Dimension	Execution Time (in Seconds)		The least execution time is shown by the method
		VAM-MODI	FSTP	
1	4×5	2.391	0.017	FSTP
2	3×5	1.443	0.026	FSTP
3	3×3	1.176	0.014	FSTP
4	4×5	1.119	0.083	FSTP
5	4×3	1.107	0.018	FSTP

6	3×5	1.012	0.070	FSTP
7	3×3	1.09	0.048	FSTP
8	4×3	1.364	0.215	FSTP
9	4×5	1.424	0.080	FSTP
10	2×3	1.636	0.045	FSTP
11	4×4	1.404	0.058	FSTP
12	3×4	1.443	0.061	FSTP
13	3×4	1.262	0.058	FSTP
14	*6×6*	1.122	0.064	FSTP
15	3×4	1.359	0.061	FSTP
16	4×6	1.483	0.078	FSTP
17	5×5	1.331	0.089	FSTP
18	3×4	1.227	0.050	FSTP
19	3×3	1.163	0.046	FSTP
20	3×4	2.182	0.066	FSTP
21	3×4	1.389	0.059	FSTP
22	3×3	1.996	0.057	FSTP
23	3×5	2.538	0.061	FSTP

Table 3 (b): Execution Time (Seconds)byVAM-MODI and FSTP in lower-dimensional TP.

Problem Number	Dimension	Execution Time (in Seconds)		The least execution time is shown by the method
		VAM-MODI	FSTP	
24	4×4	1.603	0.071	FSTP
25	4×5	1.393	0.086	FSTP
26	5×5	2.817	0.087	FSTP
27	5×5	1.32	0.086	FSTP
28	3×5	1.156	0.065	FSTP
29	3×4	1.28	0.061	FSTP
30	3×3	1.357	0.056	FSTP
31	4×5	1.25	0.071	FSTP
32	4×5	1.19	0.091	FSTP
33	5×6	1	0.086	FSTP
34	4×6	1.201	0.085	FSTP
35	3×4	0.967	0.066	FSTP
36	5×7	0.894	0.062	FSTP
37	4×3	1.293	0.060	FSTP
38	3×3	1.328	0.047	FSTP
39	3×4	1.851	0.050	FSTP
40	3×5	1.41	0.020	FSTP

By observing the least execution time shown by the method, it is clear that FSTP gives a faster solution than VAM-MODI on the randomly selected lower-dimensional transportation problem of this survey.

8. Conclusions:

We selected some recently developed methods to test on some randomly chosen lower-dimensional transportation problems. We set up our focus based on the number of the optimal solution, average relative deviation, and when there made a tie between FSTP and VAM-MODI, we broke it by the computational or execution time. On our survey, FSTP gives us the best solution with the best performance. It may be an amazing method in our working area. There are some limitations in our research also; we focused on only 40 problems. Definitely the size and the dimension of samples are important factors here. Another research can be done on the more problems with higher dimensions. Execution time can be considered as a comparison criterion if a tie happens between the solutions. A research question also arises "Can the method FSTP be able to solve the assignment problem as assignment problem is a special type of a Transportation problem?"

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Appendix A: Comparative Study

To figure out the methods easily we level the methods as M 1 to M 23

Table 1(a): Comparison Table of the obtained solution in several methods (cont.)

Method No.		**Optimal Solution**	M 1 NWC	M 2 RMM	M 3 CMM	M 4 LCM	M 5 VAM	M 6 ASM	M 7 ZAM	M 8 RDM
Case	Size									
1	4×5	***510***	635	525	525	***510***	***510***	540	***510***	555
2	3×5	***290***	363	320	321	313	***290***	305	***290***	***290***
3	3×3	***750***	1430	770	770	***750***	***750***	***750***	***750***	***750***
4	4×5	***870***	1250	910	900	880	900	880	880	880
5	4×3	***90***	130	91	99	96	***90***	107	96	***90***
6	3×5	***290***	363	295	295	295	305	305	280	335
7	3×3	***425***	545	***425***	433	433	***425***	***425***	***425***	447
8	4×3	***76***	102	80	111	83	80	***76***	***76***	***76***
9	4×5	***2280***	2610	2320	2290	2320	2290	***2280***	***2280***	***2280***
10	2×3	***3450***	4650	4650	4650	4650	***3450***	***3450***	***3450***	4650
11	4×4	***285***	425	385	***285***	310	***285***	***285***	***285***	***285***
12	3×4	***435***	520	505	475	475	475	425	***435***	500
13	3×4	***7350***	7700	7700	8525	8525	7425	7500	7700	8150
14	*6×6*	***2170***	4285	2275	2915	2455	2310	3290	2390	2630
15	3×4	***2550***	2690	2640	2670	***2550***	***2550***	***2550***	***2550***	***2550***
16	4×6	***68***	95	99	76	***68***	***68***	***68***	75	72
17	5×5	***1102***	1994	1123	1123	1123	1104	1238	1127	1496
18	3×4	***799***	975	1064	859	894	859	***799***	***799***	***799***
19	3×3	***1390***	1500	1450	1500	1450	1500	1450	***1390***	1660
20	3×4	***796***	1095	922	1037	922	***796***	1037	832	867
21	3×4	***200***	273	231	231	231	204	***200***	***200***	***200***
22	3×3	***3430***	3650	***3430***	***3430***	***3430***	***3430***	3560	***3430***	3560
23	3×5	***9240***	11120	9360	10060	10240	9360	9480	9360	9360
24	4×4	***410***	540	470	435	435	470	***410***	440	515
25	4×5	***316***	560	364	420	408	322	356	408	420
26	5×5	***1200***	18450	4650	4650	4650	***1200***	4650	3450	4650
27	5×5	***1475***	1870	1475	1545	1685	1505	1515	1595	1595
28	3×5	***745***	835	795	810	775	***745***	***745***	765	830
29	3×4	***39500***	55500	42000	***39500***	48000	42000	***39500***	42700	52500
30	3×3	***9696***	14112	11872	10848	10848	***9696***	10832	***9696***	11568
31	4×5	***420***	670	450	450	***420***	***420***	480	***420***	540
32	4×5	***1610***	2430	1770	1940	1860	1640	1670	1650	2180
33	5×6	***116***	129	124	132	134	***116***	142	118	136
34	4×6	***112***	139	143	120	***112***	***112***	114	114	116
35	3×4	***1160***	1265	1165	1220	1165	1220	1165	1165	1165
36	5×7	***1900***	3180	1970	1940	***1900***	1930	1910	2380	2010
37	4×3	***238***	248	251	248	242	***238***	***238***	***238***	254
38	3×3	***131***	***131***	***131***	***131***	***131***	***131***	136	***131***	136
39	3×4	***75500***	97500	105000	79500	115000	***75500***	***75500***	***75500***	81000
40	3×5	***920***	1260	980	1100	***920***	***920***	1010	980	1040
ARD			0.69	0.18	0.17	0.16	0.02	0.13	0.08	0.18

Table 1(b): Comparison Table of the obtained solution in several methods (cont.)

Method No.		Optimal	M 9	M 10	M 11	M 12	M 13	M 14	M 15	M 16
Case	Size	Sol.	ACM	INMD	LCPM	RAM	EDM	HCDM	MTCM HCDM	CMA
1	4×5	***510***	515	585	***510***	***510***	***510***	520	655	***510***
2	3×5	***290***	318	***290***	***290***	295	295	321	525	***290***
3	3×3	***750***	***750***	***750***	770	***750***	***750***	***750***	1730	***750***
4	4×5	***870***	900	1000	1155	880	880	***870***	1180	880
5	4×3	***90***	102	115	100	***90***	***90***	100	117	***90***
6	3×5	***290***	305	310	320	295	295	321	525	292
7	3×3	***425***	439	439	***425***	***425***	439	***425***	593	***425***
8	4×3	***76***	83	129	80	82	80	111	122	***76***
9	4×5	***2280***	***2280***	2400	***2280***	***2280***	***2280***	***2280***	2770	***2280***
10	2×3	***3450***	***3450***	4650	4650	***3450***	***3450***	***3450***	4650	***3450***
11	4×4	***285***	***285***	310	325	***285***	295	310	395	***285***
12	3×4	***435***	460	460	475	475	475	475	760	475
13	3×4	***7350***	7700	9325	7700	7700	7975	7700	10875	7700
14	6×6	***2170***	2570	2930	2310	2700	2580	2630	4895	2495
15	3×4	***2550***	***2550***	2590	***2550***	***2550***	***2550***	***2550***	2670	***2550***
16	4×6	***68***	109	81	78	71	***68***	74	139	71
17	5×5	***1102***	1363	1208	1154	1103	***1102***	1215	1986	1103
18	3×4	***799***	1028	975	975	855	859	864	1190	***799***
19	3×3	***1390***	1500	1500	1450	***1390***	***1390***	***1390***	1900	***1390***
20	3×4	***796***	922	832	832	***796***	***796***	***796***	1246	***796***
21	3×4	***200***	***200***	218	***200***	***200***	218	242	394	***200***
22	3×3	***3430***	***3430***	3560	3450	***3430***	***3430***	***3430***	4170	***3430***
23	3×5	***9240***	9480	**9240**	9360	9360	9360	9360	14080	9480
24	4×4	***410***	455	430	435	420	415	415	570	440
25	4×5	***316***	326	368	322	318	318	322	688	342
26	5×5	***1200***	3450	4650	4650	4950	2100	2100	96000	4950
27	5×5	***1475***	1555	2235	1550	1730	1685	1850	1945	1730
28	3×5	***745***	755	870	795	***745***	775	795	1015	775
29	3×4	***39500***	**39500**	45500	42000	45500	***39500***	***39500***	50500	***39500***
30	3×3	***9696***	11968	10848	11456	10336	***9696***	10848	14496	10336
31	4×5	***420***	***420***	520	***420***	***420***	***420***	440	710	***420***
32	4×5	***1610***	1690	1980	1740	1740	1870	1870	2420	1650
33	5×6	***116***	118	122	118	118	121	131	159	118
34	4×6	***112***	153	125	122	115	***112***	118	183	115
35	3×4	***1160***	1280	1165	1165	1165	1165	1165	2010	1165
36	5×7	***1900***	1940	2650	***1900***	1930	2070	1960	3430	1930
37	4×3	***238***	246	246	***238***	***238***	***238***	***238***	277	***238***
38	3×3	***131***	***131***	***131***	***131***	***131***	***131***	***131***	231	***131***
39	3×4	***75500***	103900	87000	79700	***75500***	79500	81000	119000	***75500***
40	3×5	***920***	1040	1040	**920**	930	980	980	1260	***920***
ARD			0.14	0.21	0.14	0.11	0.05	0.09	2.51	***0.1***

Table 1 (c): Comparison Table of the obtained solution in several methods

Method No.		Optimal solution	M17	M18	M 19	M 20	M 21	M 22	M 23
Case	Size		TOCM					FSTP	MODI
			LCM	VAM	EDM	HCDM	SUM		
1	4×5	***510***	***510***	***510***	***510***	520	***510***	***510***	***510***
2	3×5	***290***	295	***290***	295	295	***290***	***290***	***290***
3	3×3	***750***	***750***	***750***	***750***	***750***	770	***750***	***750***
4	4×5	***870***	880	900	880	***870***	900	***870***	***870***
5	4×3	***90***	96	98	***90***	98	99	***90***	***90***
6	3×5	***290***	313	***290***	295	321	305	***290***	***290***
7	3×3	***425***	433	***425***	439	439	439	***425***	***425***
8	4×3	***76***	83	***76***	80	81	***76***	***76***	***76***
9	4×5	***2280***	2320	2290	***2280***	***2280***	2490	***2280***	***2280***
10	2×3	***3450***	4650	***3450***	***3450***	***3450***	***3450***	***3450***	***3450***
11	4×4	***285***	305	***285***	***285***	335	***285***	***285***	***285***
12	3×4	***435***	475	***435***	475	475	520	***435***	***435***
13	3×4	***7350***	8525	7700	7975	8425	7700	***7350***	***7350***
14	6×6	***2170***	2470	***2170***	2470	2470	***2170***	***2170***	***2170***
15	3×4	***2550***	2610	***2550***	***2550***	***2550***	***2550***	***2550***	***2550***
16	4×6	***68***	70	***68***	72	100	79	***68***	***68***
17	5×5	***1102***	1123	1104	**1102**	1433	1127	***1102***	***1102***
18	3×4	***799***	874	***799***	859	864	***799***	***799***	***799***
19	3×3	***1390***	1450	1500	***1390***	***1390***	1440	***1390***	***1390***
20	3×4	***796***	***796***	***796***	***796***	***796***	***796***	***796***	***796***
21	3×4	***200***	204	204	231	255	***200***	***200***	***200***
22	3×3	***3430***	***3430***	***3430***	***3430***	***3430***	***3430***	***3430***	***3430***
23	3×5	***9240***	9360	9360	9480	9360	9400	***9240***	***9240***
24	4×4	***410***	435	430	415	415	455	***410***	***410***
25	4×5	***316***	408	322	322	372	364	***316***	***316***
26	5×5	***1200***	4650	***1200***	2100	2100	2100	***1200***	***1200***
27	5×5	***1475***	1760	1515	1685	1850	1545	***1475***	***1475***
28	3×5	***745***	795	***745***	775	795	880	***745***	***745***
29	3×4	***39500***	42000	42000	***39500***	***39500***	42000	***39500***	***39500***
30	3×3	***9696***	11488	***9696***	***9696***	10336	***9696***	***9696***	***9696***
31	4×5	***420***	***420***	***420***	***420***	440	***420***	***420***	***420***
32	4×5	***1610***	1860	1640	1930	1930	1620	***1610***	***1610***
33	5×6	***116***	134	116	125	128	123	***116***	***116***
34	4×6	***112***	114	112	116	144	123	***112***	***112***
35	3×4	***1160***	1165	1165	1165	1165	1280	***1160***	***1160***
36	5×7	***1900***	***1900***	***1900***	2070	1930	2100	***1900***	***1900***
37	4×3	***238***	***238***	***238***	***238***	246	246	***238***	***238***
38	3×3	***131***	***131***	***131***	***131***	***131***	***131***	***131***	***131***
39	3×4	***75500***	85000	***75500***	***75500***	81000	***75500***	***75500***	***75500***
40	3×5	***920***	1160	***920***	980	980	***920***	***920***	***920***
ARD			0.14	0.01	0.05	0.10	0.06	0	0