GANIT *J. Bangladesh Math. Soc.* (ISSN 1606-3694) Vol. 33 (2013) 33 -39

GENERALIZED DERIVATIONS OF PRIME GAMMA RINGS

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Received 28.08.12 Accepted 31.07.13

ABSTRACT

Let M be a prime Γ-ring satisfying a certain assumption $a\alpha b\beta c = a\beta b\alpha c$ for all a, b, $c\in M$ and α, $\beta\in\Gamma$, and let I be an ideal of M. Assume that (D, d) is a generalized derivation of M and $a\in M$. If $D([x, a]_\alpha) = 0$ or $[D(x), a]_\alpha = 0$ for all $x\in I$, $\alpha\in\Gamma$, then we prove that $d(x) = p\beta[x, a]_\alpha$ for all $x\in I$, α, $\beta\in\Gamma$ or $a\in Z(M)$ (the centre of M), where p belongs $C(M)$ (the extended centroid of M).

Mathematics Subject Classification: 16N60, 16W25.

Keywords: Prime Γ-ring, derivation, generalized derivation, commuting mapping.

1. Introduction

The notion of a Γ-ring was first introduced by Nobusawa [9]. Barnes [5] weakened slightly the conditions in the definition of Γ-ring in the sense of Nabosawa [9]. Ceven and Ozturk [6] studied on Jordan generalized derivations in Γ-rings and they proved that every Jordan generalized derivation on some Γ-rings is a generalized derivation and an example of a generalized derivation and a Jordan generalized derivation for Γ-rings are given. Hvala [8] first introduced the generalized derivations in rings and obtained some remarkable results in classical rings. Generalized derivations of semiprime rings has been worked by Ali and Chaudhry [1]. They proved that $d(x)[y, z] = 0$ for all x, y, $z\in R$ and the associate derivation d is central. They characterized a decomposition of R relative to the generalized derivations. Atteya [4] obtained some results on generalized derivations of semiprime rings. He proved that the ring R contains a nonzero central ideal. Rehman [12] studied on generalized derivations acting as homomorphisms and anti-homomorphisms. He investigated the commutativity of R by means if generalized derivations acting as homomorphisms and anti-homomorphisms. Aydin [3] studied on generalized derivations of prime rings. Assuming $F([x, a]) = 0$ or $[F(x), a] = 0$ for all $x\in I$, he proved that $d(x) = \lambda[x, a]$ for all $x\in I$ or $a\in Z$, (F, d) is a generalized derivation of R, I is an ideal of R, $a\in R$ and $\lambda\in C(R)$ (the extended centroid of R).

In this paper, we obtain the analogous results of Aydin [3] in Γ-rings. If M is a prime Γ-ring satisfying a certain assumption (*) $a\alpha b\beta c = a\beta b\alpha c$ for all a, b, c$\in M$ and α, $\beta\in\Gamma$, and I is an ideal of M, then we prove that $d(x) = p\beta[x, a]_\alpha$ for all $x\in I$, α, $\beta\in\Gamma$ or $a\in Z(M)$ (the centre of M), $p\in C(M)$ (the extended centroid of M) by assuming that $D([x, a]_\alpha) = 0$ or $[D(x), a]_\alpha = 0$ for all $x\in I$, $\alpha\in\Gamma$, where $a\in M$.

2. Preliminaries

Let M and Γ be additive abelian groups. M is called a Γ-ring if for all $a, b, c\in M$, $\alpha, \beta\in\Gamma$, the following conditions are satisfied:

(i) $a\alpha b\in M$,

(ii) $(a + b)\alpha c = a\alpha c + b\alpha c$, $a(\alpha + \beta)b = a\alpha b + a\beta b$,

$a\alpha(b + c) = a\alpha b + a\alpha c$,

(iii) $(a\alpha b)\beta c = a\alpha(b\beta c)$.

This definition of a Γ-ring is given by Barnes [5]. We represent $Z(M)$ as the centre of a Γ-ring M. Let M be a Γ-ring. A subring I of M is an additive subgroup which is also a Γ-ring. *A* right ideal of M is a subring I such that $I\Gamma M \subset I$. Similarly a left ideal can be defined. If I is both a right and a left ideal then we say that I is an ideal.

The commutator $x\alpha y - y\alpha x$ will be denoted by $[x, y]_\alpha$. We know that $[x\beta y, z]_\alpha = [x, z]_\alpha\beta y + x\beta[y, z]_\alpha + x[\beta, \alpha]_Z y$

and $[x, y\beta z]_\alpha = y\beta[x, z]_\alpha + [x, y]_\alpha\beta z + y[\beta, \alpha]_x z$, for all $x, y, z\in M$ and $\alpha, \beta\in\Gamma$.

We take an assumption (*) $x\beta y\alpha z = x\alpha y\beta z$ for all $x, y, z\in M$ and $\alpha, \beta\in\Gamma$. Using the assumption the basic commutator identities reduce to

$[x\beta y, z]_\alpha = [x, z]_\alpha\beta y + x\beta[y, z]_\alpha$

and $[x, y\beta z]_\alpha = y\beta[x, z]_\alpha + [x, y]_\alpha\beta z$, for all $x, y, z\in M$ and $\alpha, \beta\in\Gamma$.

Recall that a ring M is semiprime if $a\Gamma M\Gamma a = 0$ implies $a = 0$ and is prime if $a\Gamma M\Gamma b = 0$ implies $a = 0$ or $b = 0$. An additive mapping $d : M \to M$ is called a derivation on M if $d(x\alpha y) = d(x)\alpha y + x\alpha d(y)$ for all $x, y\in M$, $\alpha\in\Gamma$. An additive mapping $f : M \to M$ is called commuting if $[f(x), x]_\alpha = 0$ for all $x\in M$, $\alpha\in\Gamma$. It is called central if $f(x)\in Z(M)$ for all $x\in M$. Let $a\in M$, then the mapping $d : M \to M$ given by $d(x) = [a, x]_\alpha$ is a derivation on M. It is called inner derivation on M.

An additive mapping D of M into itself is called a generalized derivation of M, with associated derivation d, if there is a derivation d of M such that $D(x\alpha y) = D(x)\alpha y + x\alpha d(y)$ for all $x, y\in M$, $\alpha\in\Gamma$. Obviously this notion covers the notion of a derivation (in case $D = d$) and a left centralizer (in case $d = 0$). An additive mapping $D : M \to M$ is called a left centralizer if $D(x\alpha y) = D(x)\alpha y$ for all $x, y\in M$, $\alpha\in\Gamma$.

We refer to [10, 11] for the definitions of the centroid and of the extended centroid of Γ-rings.

3. Generalized Derivations of Prime Γ-rings

In this section, we prove our main results. Before proving our results, we need the following three lemmas which are given below.

Lemma 3.1. Let d be a derivation of a prime Γ-ring M and a be an element of M. If $a\Gamma d(x) = 0$ for all $x \in M$ then either $a = 0$ or $d = 0$.

Proof. Let $a \in M$, and $\alpha \in \Gamma$, then $a\alpha d(x) = 0$. Replacing $x\beta y$ for x, $(y \in M, \beta \in \Gamma)$ we get $a\alpha d(x\beta y) = a\alpha d(x)\beta y + a\alpha x\beta d(y) = a\alpha x\beta d(y) = 0$. By the primeness of M, we obtain either $a = 0$ or $d = 0$.

Lemma 3.2 Let M be a Γ-ring satisfying the condition (*), I be an ideal of M and (D, d) be a generalized derivation of M and $a \in M$. If $a \notin Z(M)$ and

$[D(x), a]_\alpha = 0$ for all $x \in I$, $\alpha \in \Gamma$, then $D([x, a]_\alpha) = 0$ for all $x \in I$, $\alpha \in \Gamma$.

Proof. We replace x by $x\delta r$, $r \in M$, $\delta \in \Gamma$, in the defining equation

$[D(x), a]_\alpha = 0$ for all $x \in I$, $\alpha \in \Gamma$ (1)

and hence we obtain,

$$0 = [D(x\delta r), a]_\alpha = [D(x)\delta r + x\delta d(r), a]_\alpha$$
$$= [D(x)\delta r, a]_\alpha + [x\delta d(r), a]_\alpha.$$

By using the condition (*) we obtain

$$[D(x)\delta r, a]_\alpha + [x\delta d(r), a]_\alpha.$$
$$= D(x)\delta[r, a]_\alpha + [D(x), a]_\alpha\delta r + x\delta[d(r), a]_\alpha + [x, a]_\alpha\delta d(r)$$

for all $x \in I$, $r \in M$, $\alpha, \delta \in \Gamma$, which implies that

$D(x)\delta[r, a]_\alpha + x\delta[d(r), a]_\alpha + [x, a]_\alpha\delta d(r) = 0$ for all $x \in I$, $r \in M$, $\alpha, \delta \in \Gamma$ (2)

In (2), replacing x by $x\beta y$, $(y \in I, \beta \in \Gamma)$ and using (2), we obtain

$$0 = D(x\beta y)\delta[r, a]_\alpha + x\beta y\delta[d(r), a]_\alpha + x\beta[y, a]_\alpha\delta d(r) + [x, a]_\alpha\beta y\delta d(r)$$
$$= D(x)\beta y\delta[r, a]_\alpha + x\beta d(y)\delta[r, a]_\alpha + x\beta y\delta[d(r), a]_\alpha + x\beta[y, a]_\alpha\delta d(r)$$
$$+ [x, a]_\alpha\beta y\delta d(r)$$
$$= D(x)\beta y\delta[r, a]_\alpha + x\beta d(y)\delta[r, a]_\alpha + x\beta(y\delta[d(r), a]_\alpha + [y, a]_\alpha\delta d(r))$$
$$+ [x, a]_\alpha\beta y\delta d(r)$$
$$= D(x)\beta y\delta[r, a]_\alpha + x\beta d(y)\delta[r, a]_\alpha - x\beta D(y)\delta[r, a]_\alpha + [x, a]_\alpha\beta y\delta d(r)$$
$$= (D(x)\beta y + x\beta d(y) - x\beta D(y))\delta[r, a]_\alpha + [x, a]_\alpha\beta y\delta d(r)$$

so we get

$(D(x)\beta y + x\beta d(y) - x\beta D(y))\delta[r, a]_\alpha + [x, a]_\alpha\beta y\delta d(r) = 0$,

for all $x, y \in I$, $r \in M$, $\alpha, \beta, \delta \in \Gamma$. (3)

Replace r by a in (3), we have $[x, a]_\alpha\beta y\delta d(a) = 0$, $x, y \in I$, $\alpha, \beta, \delta \in \Gamma$.

Since $a \notin Z(M)$ and the primeness of I, yields $d(a) = 0$

If we substitute $s\lambda x$, $(s \in M, \lambda \in \Gamma)$, for x in (3), then we get

$$0 = (D(s\lambda x)\beta y + s\lambda x\beta d(y) - s\lambda x\beta D(y))\delta[r, a]_\alpha + [s\lambda x, a]_\alpha\beta y\delta d(r)$$
$$= ((D(s)\lambda x + s\lambda d(x))\beta y + s\lambda x\beta d(y) - s\lambda x\beta D(y))\delta[r, a]_\alpha$$

$+ s\lambda[x, a]_\alpha\beta y\delta d(r) + [s, a]_\alpha\lambda x\beta y\delta d(r)$

$= (D(s)\lambda x\beta y + s\lambda d(x)\beta y + s\lambda x\beta d(y) - s\lambda x\beta D(y))\delta[r, a]_\alpha$

$+ s\lambda[x, a]_\alpha\beta y\delta d(r) + [s, a]_\alpha\lambda x\beta y\delta d(r)$

$= D(s)\lambda x\beta y\delta[r, a]_\alpha + s\lambda d(x)\beta y\delta[r, a]_\alpha + s\lambda x\beta d(y)\delta[r, a]_\alpha$

$- s\lambda x\beta D(y)\delta[r, a]_\alpha + s\lambda[x, a]_\alpha\beta y\delta d(r) + [s, a]_\alpha\lambda x\beta y\delta d(r)$

$= (D(s)\lambda x\beta y + s\lambda d(x)\beta y)\delta[r, a]_\alpha + s\lambda((x\beta d(y) - x\beta D(y))\delta[r, a]_\alpha$

$+ [x, a]_\alpha\beta y\delta d(r)) + [s, a]_\alpha\lambda x\beta y\delta d(r)$

$= (D(s)\lambda x\beta y + s\lambda d(x)\beta y)\delta[r, a]_\alpha + s\lambda(-D(x)\beta y\delta[r, a]_\alpha) + [s, a]_\alpha\lambda x\beta y\delta d(r)$

$= (D(s)\lambda x\beta y + s\lambda d(x)\beta y - s\lambda D(x)\beta y)\delta[r, a]_\alpha + [s, a]_\alpha\lambda x\beta y\delta d(r)$

and so

$(D(s)\lambda x + s\lambda d(x) - s\lambda D(x))\delta y\beta[r, a]_\alpha + [s, a]_\alpha\lambda x\beta y\delta d(r) = 0,$

for all $x, y\in I$, $r, s\in M$, $\alpha, \beta, \delta, \lambda\in\Gamma$. (4)

In (4) replacing s by a,

$(D(a)\lambda x + a\lambda d(x) - a\lambda D(x))\beta y\delta[r, a]_\alpha = 0,$

for all $x, y\in I$, $r\in M$, $\alpha, \beta, \delta\in\Gamma$. (5)

Using $a\notin Z(M)$ and the primeness of I, we obtain

$$D(a)\lambda x + a\lambda d(x) - a\lambda D(x) = 0.$$

Then we have

$D(a\lambda x) = a\lambda D(x)$, for all $x\in I$, $\lambda\in\Gamma$, (6)

On the other hand, since $d(a) = 0$, we see that the relation

$$D(x\lambda a) = D(x)\lambda a + x\lambda d(a) = D(x)\lambda a$$

is reduced to $D(x\lambda a) = D(x)\lambda a$, for all $x\in I$, $\lambda\in\Gamma$.

$\Leftrightarrow D(x\alpha a) = D(x)\alpha a$, for all $x\in I$, $\alpha\in\Gamma$. (7)

Combining (6) and (7), we arrive at

$D([x, a]_\alpha) = D(x\alpha a) - D(a\alpha x) = D(x)\alpha a - a\alpha D(x) = [D(x), a]_\alpha$ for all $x\in I$, $\alpha\in\Gamma$.

By using the hypothesis, we have

$$D([x, a]_\alpha) = [D(x), a]_\alpha = 0, \text{ for all } x\in I, \alpha\in\Gamma.$$

This completes the proof.

Lemma 3.3 Let M be a prime Γ- ring satisfying the condition (*), I be an ideal of M, (D, d) be a generalized derivation of M and $a\in M$. If $a\notin Z(M)$ and $D([x, a]_\alpha) = 0$ for all $x\in I$, $\alpha\in\Gamma$, then $[D(x), a]_\alpha = 0$ for all $x\in I$, $\alpha\in\Gamma$.

Proof. We replace x by $x\beta a$ $(\beta\in\Gamma)$ in the defining equation $D([x, a]_\alpha) = 0$ to obtain $0 = D([x\beta a, a]_\alpha) = D([x, a]_\alpha\beta a) = D([x, a]_\alpha)\beta a + [x, a]_\alpha\beta d(a)$

and so

$[x, a]_\alpha\beta d(a) = 0$, for all $x\in I$, $\alpha, \beta\in\Gamma$. (8)

Taking $x\delta y$, $y\in I$, $\delta\in\Gamma$, instead of x in (8),

$0 = [x\delta y, a]_\alpha\beta d(a) = x\delta[y, a]_\alpha\beta d(a) + [x, a]_\alpha\delta y\beta d(a)$

and using (8) we obtain

$[x, a]_\alpha\delta y\beta d(a) = 0$, for all $x\in I$, $\alpha, \beta, \delta\in\Gamma$, (9)

By the primeness of I and $a\notin Z(M)$, (9) implies that $d(a) = 0$.

Now we replace x by $x\lambda y$, ($y\in I$, $\lambda\in\Gamma$) in the defining equation

$D([x, a]_\alpha) = 0$ to obtain

$$0 = D([x\lambda y, a]_\alpha) = D(x\lambda[y, a]_\alpha + [x, a]_\alpha\lambda y)$$
$$= D([x, a]_\alpha\lambda y) + D(x\lambda[y, a]_\alpha)$$
$$= D([x, a]_\alpha)\lambda y + [x, a]_\alpha\lambda d(y) + D(x)\lambda[y, a]_\alpha + x\lambda d([y, a]_\alpha)$$
$$= [x, a]_\alpha\lambda d(y) + D(x)\lambda[y, a]_\alpha + x\lambda([d(y), a]_\alpha + [y, d(a)]_\alpha)$$

Since $d(a) = 0$, we have

$D(x)\lambda[y, a]_\alpha + [x, a]_\alpha\lambda d(y) + x\lambda[d(y), a]_\alpha = 0$,

for all $x, y\in I$, $\alpha, \lambda\in\Gamma$, (10)

Substitute $y\delta z$, ($z\in I$, $\delta\in\Gamma$), instead of y in equation (10) and use the equation (10), we obtain,

$$0 = D(x)\lambda[y\delta z, a]_\alpha + [x, a]_\alpha\lambda d(y\delta z) + x\lambda[d(y\delta z), a]_\alpha$$
$$= D(x)\lambda y\delta[z, a]_\alpha + D(x)\lambda[y, a]_\alpha\delta z + [x, a]_\alpha\lambda d(y)\delta z$$
$$+ [x, a]_\alpha\lambda y\delta d(z) + x\lambda[d(y)\delta z, a]_\alpha + x\lambda[y\delta d(z), a]_\alpha$$
$$= D(x)\lambda y\delta[z, a]_\alpha + (D(x)\lambda[y, a]_\alpha + [x, a]_\alpha\lambda d(y))\delta z + [x, a]_\alpha\lambda y\delta d(z)$$
$$+ x\lambda d(y)\delta[z, a]_\alpha + x\lambda[d(y), a]_\alpha\delta z + x\lambda y\delta[d(z), a]_\alpha + x\lambda[y, a]_\alpha\delta d(z)$$
$$= D(x)\lambda y\delta[z, a]_\alpha + (D(x)\lambda[y, a]_\alpha + [x, a]_\alpha\lambda d(y) + x\lambda[d(y), a]_\alpha)\delta z$$
$$+ [x, a]_\alpha\lambda y\delta d(z) + x\lambda d(y)\delta[z, a]_\alpha + x\lambda y\delta[d(z), a]_\alpha + x\lambda[y, a]_\alpha\delta d(z)$$
$$= D(x)\lambda y\delta[z, a]_\alpha + [x, a]_\alpha\lambda y\delta d(z) + x\lambda d(y)\delta[z, a]_\alpha + x\lambda y\delta[d(z), a]_\alpha$$
$$+ x\lambda[y, a]_\alpha\delta d(z)$$
$$= (D(x)\lambda y + x\lambda d(y))\delta[z, a]_\alpha + [x, a]_\alpha\lambda y\delta d(z)$$
$$+ x\lambda(y\delta[d(z), a]_\alpha + [y, a]_\alpha\delta d(z))$$
$$= (D(x)\lambda y + x\lambda d(y))\delta[z, a]_\alpha + [x, a]_\alpha\lambda y\delta d(z) - x\lambda D(y)\delta[z, a]_\alpha$$

and so

$$(D(x)\lambda y + x\lambda d(y) - x\lambda D(y))\delta[z, a]_\alpha + [x, a]_\alpha\lambda y\delta d(z) = 0,$$

for all $x, y, z\in I$, $\alpha, \lambda, \delta\in\Gamma$, (11)

Replace x by $a\alpha x$, ($\alpha\in\Gamma$) in equation (11), we obtain,

$$0 = (D(a\alpha x)\lambda y + a\alpha x\lambda d(y) - a\alpha x\lambda D(y))\delta[z, a]_\alpha + a\alpha[x, a]_\alpha\lambda y\delta d(z)$$
$$= D(a\alpha x)\lambda y\delta[z, a]_\alpha + a\alpha(x\lambda d(y)\delta[z, a]_\alpha - x\lambda D(y)\delta[z, a]_\alpha + [x, a]_\alpha\lambda y\delta d(z))$$
$$= D(a\alpha x)\lambda y\delta[z, a]_\alpha - a\alpha D(x)\lambda y\delta[z, a]_\alpha$$

Hence we get

$(D(a\alpha x) - a\alpha D(x))\lambda y\delta[z, a]_\alpha = 0$, for all $x, y, z\in I$, $\alpha, \lambda, \delta\in\Gamma$. (12)

Since $a\notin Z(M)$ and the primeness of M, we have

$D(a\alpha x) = a\alpha D(x)$, for all $x\in I$, $\alpha\in\Gamma$. (13)

On the other hand, since $d(a) = 0$,

$D(x\alpha a) = D(x)\alpha a + x\alpha d(a) = D(x)\alpha a$ (14)

Combining (13) and (14) we arrive at

$$[D(x), a]_\alpha = D(x)\alpha a - a\alpha D(x)$$
$$= D(x\alpha a) - D(a\alpha x) = D([x, a]_\alpha) = 0$$

and so

$$[D(x), a]_\alpha = 0, \text{ for all } x\in M, \alpha\in\Gamma.$$

Thus the proof is complete.

Theorem 3.4 Let M be a Γ- prime ring satisfying the condition (*), I be an ideal of M, (D, d) a generalized derivation of D and $a\in M$. If $a\notin Z(M)$ and $D([x, a]_\alpha) = 0$ or $[D(x), a]_\alpha = 0$ for all $x\in I$, $\alpha\in\Gamma$, then $d(x) = p\beta[x, a]_\alpha$, where $p\in C(M)$, the extended centroid of M, for all $x\in I$, $\alpha, \beta\in\Gamma$.

Proof. Since $a\notin Z(M)$ and $[D(x), a]_\alpha = 0$ for all $x\in I$, $\alpha\in\Gamma$, then by Lemma 2.2 we have $D([x, a]_\alpha) = 0$ and $d(a) = 0$

By the proof of the Lemma 2.2, we have the equation (3), in the equation (3), replace y by $[a, y]_\alpha$ then we get

$$0 = (D(x)\beta[a, y]_\alpha + x\beta d([a, y]_\alpha) - x\beta D([a, y]_\alpha))\delta[r, a]_\alpha + [x, a]_\alpha\beta[a, y]_\alpha\delta d(r)$$
$$= (D(x)\beta[a, y]_\alpha + x\beta[a, d(y)]_\alpha\delta[r, a]_\alpha + [x, a]_\alpha\beta[a, y]_\alpha\delta d(r)$$
$$= -(D(x)\beta[y, a]_\alpha + x\beta[d(y), a]_\alpha)\delta[r, a]_\alpha + [x, a]_\alpha\beta[a, y]_\alpha\delta d(r)$$

In the above equation, using the equation (10)

$$[a, x]_\alpha\beta d(y) = D(x)\beta[y, a]_\alpha + x\beta[d(y), a]_\alpha$$

in the proof of the Lemma 2.2, we obtain

$$[a, x]_\alpha\beta(d(y)\delta[r, a]_\alpha - [y, a]_\alpha\delta d(r)) = 0$$

Define $h : M \to M$ by $h(x) = [a, x]_\alpha$, then the above equation yields

$h(x)\beta(d(y)\delta[r, a]_\alpha - [y, a]_\alpha\delta d(r)) = 0$. Since $a\notin Z(M)$, by Lemma 2.2, we get

$d(y)\delta[r, a]_\alpha = [y, a]_\alpha\delta d(r)$, for all $y\in I$, $r\in M$, $\alpha, \beta, \lambda\in\Gamma$. (15)

Replace r by $r\lambda s$, $(s\in M, \lambda\in\Gamma)$, in (15) and use (15), we obtain

$d(y)\delta r\lambda[s, a]_\alpha = [y, a]_\alpha\lambda r\delta d(s)$, for all $r, s\in M, y\in I, \alpha, \delta, \lambda\in\Gamma$, (16)

Substitute $y\gamma z$, $(z\in M, \lambda\in\Gamma)$ instead of y in (16) and use (16) it gives us

$d(z)\delta r\lambda[s, a]_\alpha = [z, a]_\alpha\lambda r\delta d(s)$ for all $r, s, z\in M, \alpha, \delta, \lambda\in\Gamma$, (17)

Now, define $g : M \rightarrow M$ by $g(x) = [x, a]_\alpha$, then from (17) we have

$d(z)\delta r\lambda g(s) = g(z)\lambda r\delta d(s)$, for all $r, s, z\in M, \delta, \lambda\in\Gamma$.

Since $g \neq 0$, we get, for some $p\in C(M)$, $d(x) = p\beta[x, a]_\alpha$, for all $x\in I$, $\alpha, \beta\in\Gamma$. Thus, the proof is complete.

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