

AN IMPROVED DECOMPOSITION APPROACH AND ITS COMPUTATIONAL TECHNIQUE FOR ANALYZING PRIMAL-DUAL RELATIONSHIP IN LP & LFP PROBLEMS

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Received 04-11-2012

Accepted 30-10-2013

ABSTRACT

In this paper, we study the methodology of primal dual solutions in Linear Programming (LP) & Linear Fractional Programming (LFP) problems. A comparative study is also made on different duals of LP & LFP. We then develop an improved decomposition approach for showing the relationship of primal and dual approach of LP & LFP problems by giving algorithm. Numerical examples are given to demonstrate our method. A computer programming code is also developed for showing primal and dual decomposition approach of LP & LFP with proper instructions using AMPL. Finally, we have drawn a conclusion stating the privilege of our method of computation.

Keywords: LP, LFP, Primal-Dual, Decomposition and Computer Algebra

1. Introduction

In 1960's and 1970's several authors, Swarup [17], Chadha [5], Kaska[12], Gol'estein [9,10], Sharma and Swarup [16], Seshan [16] and Joshi, Singh and Gupta[11] and many other authors proposed different types of methods for dual problems related to the primal LFP problems consisting of optimizing linear fractional objective function subject to a system of linear constraints. Most of the authors proposed a dual form in which the objective function is linear. Some of them are based on the well-known Charnes and Coopers transformation [4] leads to the duality theory of LP. Only Sharma and Swarup [16] and Seshan [17] have defined a dual form in which the objective function is fractional, that is, ratio of two linear functions. All these duals proposed by different authors are actually equivalent to one another. Most of the authors proved all the duality theorems and did not apply the decomposition primal dual approach to solve LFP problems. Walker [18], Kornbluth and Salkin [13] stated the decomposition approach for LFP. Also Dantzig & Wolfe [7], Dantzig [6], Wolfe [19], Baumol and Fabian [1] are silent about the LFP decomposition but in this paper we show their LP decomposition is sophisticated not only for solving LFP but also in primal dual approach of LFP. Every

mathematical programming problem has an associated dual problem. For any primal problem, a dual problem can be constructed which is very closely connected with the original problem. In this paper, we study the methodology of optimal dual solution in LP & LFP and improve a decomposition approach to furnish the complete primal dual solutions which depends on DWD. We also develop a computational technique for showing optimal dual decomposition approach of LP & LFP using programming language AMPL [8].

The rest of the paper is organized as follows. Section 2 is based on the discussion of primal dual LP & LFP problems. Section 3 and 4 are based on the formulation and algorithms of Kornbluth & Salkin and our improved optimal dual decompose of LP & LFP problems respectively. In Section 5, we demonstrate of our improved decomposed algorithm by a numerical example. In Section 6, we develop a computer programming code using the programming language AMPL. In section 7, we present the comparison of different methods.

2. Discussions of Primal Dual LP & LFP

In this section, we briefly discuss about the primal dual LP & LFP model and its complexity in the real world problems. We also precisely discuss about the formulation of LP & LFP problems.

2.1 Primal Dual Approach of Linear Programming (LP) [18]

The linear decomposition, the final tableaux of the master program and subprograms provide not only the primal solution but also the full dual evaluation. Consider the following LP problem

$$\begin{aligned} \text{primal Maximize } & c_1 \lambda_1 + c_2 \lambda_2, \quad \text{subject to: } A_1 \lambda_1 \leq b_1, \quad A_2 \lambda_2 \leq b_2, & (1) \\ & M_1 \lambda_1 + M_2 \lambda_2 \leq b & (*) \\ & \lambda_i \geq 0 \end{aligned}$$

Here, the equation (*) is called complex constraints and M_1, M_2 are the coefficients of the equation (*). The dual solutions of this primal are given as follows.

$$\begin{aligned} \text{Minimize } & \pi_\alpha b_\alpha = \pi_1 b_1 + \pi_2 b_2 + \pi_b b \\ \text{subject to: } & \pi_\alpha A_\alpha + \pi_b M_\alpha \geq c_\alpha, \quad \pi_\alpha \geq 0, \quad \alpha=1,2 & (2) \end{aligned}$$

The dual variables π^*_α for (i) and (ii) according as $\alpha=1, 2$. Walker's [18] proof relies on the linear duality theorems equality the optimal primal and dual objective functions. So it is clear that: optimal objective function=dual objective functions i.e. $c_1 \lambda_1^* + c_2 \lambda_2^* = \pi_1 b_1 + \pi_2 b_2 + \pi_b b$

2.2 Primal Dual Approach of Linear Fractional Programming (LFP) [13]

The LFP decomposition problem involving two divisions or sub problems can be defined as in the following ways:

$$\begin{aligned}
 \text{Maximize } : f &= \frac{c_1 \lambda_1 + c_2 \lambda_2}{d_1 \lambda_1 + d_2 \lambda_2 + \beta} \\
 \text{subject to: } & A_1 \lambda_1 \leq b_1 & (i) \\
 & A_2 \lambda_2 \leq b_2 & (ii) \\
 & M_1 \lambda_1 + M_2 \lambda_2 \leq b & (iii) \\
 & \lambda_i \geq 0
 \end{aligned} \tag{3}$$

Here, the equation (iii) is called complex constraints and M_1, M_2 are the coefficients of the complex constraints. Hence π_1, π_2 and π_b are the duals of (i), (ii) and (iii) respectively. Hence primarily,

$$\frac{c_1 \lambda_1^* + c_2 \lambda_2^*}{d_1 \lambda_1^* + d_2 \lambda_2^* + \beta} = \pi_1^* b_1 + \pi_2^* b_2 + \pi_b^* b \tag{4}$$

Hence λ_1^*, λ_2^* are primal solutions and π_1^*, π_2^* and π_b^* are the dual solutions.

3. Formulation of Kornbluth & Salkin Optimal Dual Solution [13]

In this section, we present the Kornbluth & Salkin's optimal dual solution in LFP problems. A dual problem to (3) is find $\theta = (\pi_1^*, \pi_2^*, \pi_b^*, \lambda_1^*, \lambda_2^*)$ where θ being the solution to the dual problem.

$$\text{dual: } \pi_1 b_1 + \pi_2 b_2 + \pi_b b$$

$$\text{subjectto: } \pi_1 A_1 + \pi_b M_1 \geq \left[\frac{\partial f}{\partial \lambda_1} \right]_{\lambda=\lambda^*}, \pi_2 A_2 + \pi_b M_2 \geq \left[\frac{\partial f}{\partial \lambda_2} \right]_{\lambda=\lambda^*}, \pi_1, \pi_2, \pi_b \geq 0 \tag{5}$$

$$\text{Hence for } \lambda_1^*, \lambda_2^* \text{ and } \bar{C}_{\alpha i} = \left[\frac{\partial f}{\partial \lambda_i} \right]_{\lambda=\lambda^*} \tag{6}$$

where λ^* is optimal for (3) and from the primal (3) we get the new LP in equation (7).

$ \begin{aligned} \text{Maximize } & \bar{c}_1 x_1 + \bar{c}_2 x_2 \\ \text{subjectto: } & A_1 x_1 \leq b_1 & (i) \\ & A_2 x_2 \leq b_2 & (ii) \\ & M_1 x_1 + M_2 x_2 \leq b, x_i \geq 0 & (iii) \end{aligned} $	$ \begin{aligned} \text{Maximize } & \sum \mu_1^i \bar{c}_1 \lambda_1^i + \mu_2^i \bar{c}_2 \lambda_2^i \\ \text{subjectto: } & \sum \mu_1^i M_1 \lambda_1^i + \mu_2^i M_2 \lambda_2^i \leq b & (i) \\ & \sum \mu_1^i = 1 & (i') \\ & \sum \mu_2^i = 1 & (i'') \end{aligned} \tag{8} $
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Then (7) is a linear decomposition problem. At this point sub-program for (7) are given as follows.

$$\text{Maximize } \sum_i (\bar{C}_{\alpha i} - \hat{\pi} \cdot M_{\alpha i}) \lambda_i, \quad \text{subject to: } A_{\alpha i} \lambda_i \leq b_{\alpha}, \quad \lambda_i \geq 0 \quad \alpha=1,2 \quad (9)$$

$$\begin{aligned} \text{Minimize } \pi_{\alpha} b_{\alpha}, \quad \text{subject to } \pi_{\alpha} A_{\alpha} \geq \bar{C}_{\alpha} - \hat{\pi} \cdot M_{\alpha} \\ \pi_{\alpha} \geq 0 \quad \alpha=1,2 \end{aligned} \quad (10)$$

The dual to these two sub-problems are as follows.

3.1 Kornbluth & Salkin Optimal Dual Decomposition Algorithm [13]

In this section, we present the Kornbluth & Salkin optimal dual decompose algorithm for LFP.

Step (a): Assume k_{α} solutions from division $\alpha = 1, 2$.

Step (b): From the master problem (M) and optimize using, say, the CC method. Let, $x = x_1, x_2$ be the ‘optimal’ program derived.

Step (c): Derive the marginal values of production for x_1, x_2 at the present solution

levels; i.e., the vector (ξ_1, ξ_2) whose components are $\xi_{\alpha i} = [\bar{C}_{\alpha i} - \pi \cdot M_{\alpha i}]$ where

$$[\bar{C}_{\alpha i} - \pi \cdot M_{\alpha i}]_{\lambda_i} = \frac{\partial f}{\partial \lambda_i} \quad \alpha=1,2 \quad (11)$$

Step (d): Present each division with these new marginal figures and request optimization of each of the sub-problems with respect to the new (linear) objective: $\max \xi_{\alpha} \cdot x_{\alpha} = f_{\alpha}$ subject to $A_{\alpha} \cdot x_{\alpha} \leq b_{\alpha}$ and $x_{\alpha} \geq 0, \alpha = 1, 2$.

Step (e): Test for optimality with the new division solutions. Let, \bar{f}_{α} be the optimal

value of f_{α} ; then, if $\sigma_{\alpha} \geq \bar{f}_{\alpha}$ ($\alpha = 1, 2$) the present solution is optimal, but, if

$\sigma_{\alpha} \leq \bar{f}_{\alpha}$ ($\alpha = 1, 2$) update k_{α} and go to Step (b). The details are discussed in [14].

Since primal dual LP & LFP relation is strongly visible. So our aim is to show the effectiveness. In the next section, we present an improve decomposition approach for optimal dual LP & LFP.

4. Formulation of Our Improved Primal Dual Solution

In this section, we present our improved decomposition technique. For this, the original LFP problem make into LP using (6). The whole process will be carried out by the following mathematical formulations. Let, the original LP problem be

$$\begin{aligned}
 & \text{Maximize: } \bar{c}_1 x_1 + \bar{c}_2 x_2 \\
 & \text{subject to: } A_1 x_1 \leq b_1 \quad (i) \\
 & \quad \quad \quad A_2 x_2 \leq b_2 \quad (ii) \\
 & \quad \quad \quad M_1 x_1 + M_2 x_2 \leq b \quad (iii) \\
 & \quad \quad \quad x_i \geq 0
 \end{aligned} \tag{12}$$

The improved decomposition principles composed of the following sub problems with the help of Lagrangian relaxation.

Sub-problem	Master problem
<p><i>primal</i></p> <p>Maximize: $\bar{c}_1 x_1 + \bar{c}_2 x_2 - \lambda_k (M_1 x_1 + M_2 x_2 - b)$</p> <p>subjectto: $A_1 x_1 \leq b_1, A_2 x_2 \leq b_2, x_i \geq 0$ (13)</p>	<p>Maximize : $(\bar{c}_1 x_1 + \bar{c}_2 x_2) \eta_k$</p> <p>subjectto : $(M_1 x_1 + M_2 x_2) \eta_k \leq b$ (14)</p> <p>$\eta_k = 1, \eta_k \geq 0$</p>

Iteration number K is continuously increased until the optimality condition is satisfied.

Optimality condition

The value of the sub-problem will be equal to the value of the master problem i.e. $V(S) = V(M)$.

Getting the primal solution

The master problem $M(\eta)$ contains the final solution in η variables. To convert the solution to the x variables, we have to calculate the optimal solution x^* by the formula

$$x^* = \sum_{l=1}^k \eta_l x_l'$$

4.1 Improved Primal Dual Decomposition Algorithm

In this section, we briefly discuss our improved optimal dual decompose algorithm for solving LP & LFP problems.

Step (a): Check that problem is LP or LFP. If LP go to Step (e) Otherwise go to Step (b).

Step (b): Assume k_α solutions from division $\alpha = 1, 2$.

Step (c): From the master problem (M) and optimize using, say, the Charces & Copper method. Let, $x = x_1, x_2$ be the ‘optimal’ program derived.

Step (d): Derive the new marginal values $[C_{\alpha i}^- = \frac{\partial f}{\partial \lambda_i}]_{\lambda=\alpha}$ $\alpha = 1, 2$ for creating LP

(12) at the present solution labels x_{1_i}, x_{2_i} from LFP (3).

Step (e): Decompose the LP problem (12) into sub problem (13) and master problem (14). First, solve the sub problem using arbitrary dual λ_1 . Then follow the following Steps.

Step (f): Using sub problem value solves the master problem.

Step (g): From master problem, we are able to obtain dual prices for each of the constraints.

Step (h): This information is then utilized in the objective function of the sub problem.

Step (i): The sub problem is solved.

Step (j): Check that $V(S) = V(M)$. If the optimality condition fails go to the following Step.

Step (k): A new variable is then added to the master problem and the master problem is re-solved. Re-solving the master problem will generate new dual values. Then go to the Step (h) and the process is repeated until the optimality condition is satisfied.

5. Numerical Experimental Results

In this section, we present a numerical example to demonstrate of and our improve decomposition technique to show the optimal dual algorithm of LP & LFP Problems.

Numerical Example: 5.1 This numerical example is taken from Kornbluth & Salkin[13].

$$\begin{aligned} & \text{Maximize } \frac{4x_1 + 2x_2 + 4y_1 + 3y_2}{x_1 + x_2 + y_1 + y_2 + 12.5} \\ & \text{subject to: } x_1 + 2x_2 + 2y_1 + y_2 \leq 15, \quad x_1 + 3x_2 \leq 30, \quad 2x_1 + x_2 \leq 20 \\ & \quad \quad \quad y_1 \leq 10, \quad y_2 \leq 10, \quad y_1 + y_2 \leq 20, \quad x_i \geq 0, \quad y_i \geq 0 \end{aligned} \quad (15)$$

Solution: The optimal solution which can be found by any method such as Charnes and Copper (CC) method is $f^* = 2$, $x_1 = 10$, $x_2 = 0$, $y_1 = 2.5$, $y_2 = 0$.

The dual variables are $\pi_b = \frac{1}{25}$, $\pi_1 = (0, \frac{1}{25})$, $\pi_2 = (0, 0, 0)$ For the first set of

decompose we have tendered the first basic plans $\underline{x} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, the second basic

plans $\underline{y} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$c_{11}^- = \frac{\partial f}{\partial x_1} = \frac{(x_1 + x_2 + y_1 + y_2 + 12.5).4 - (4x_1 + 2x_2 + 4y_1 + 3y_2).1}{(x_1 + x_2 + y_1 + y_2 + 12.5)^2} \quad c_{12}^- = 0$$

$c_{21} = \frac{2}{25}$ $c_{22} = \frac{1}{25}$. Now the converted linear fractional programming to linear programming objective function is given as

$$\begin{aligned} \text{Maximize } \bar{c}_1 x_1 + \bar{c}_2 x_2 &= \bar{c}_{11} x_1 + \bar{c}_{12} x_2 + \bar{c}_{11} y_1 + \bar{c}_{12} y_2 = \frac{2}{25} x_1 + \frac{2}{25} y_1 + \frac{1}{25} y_2 \\ \text{subject to: } x_1 + 2x_2 + 2y_1 + y_2 &\leq 15, \quad x_1 + 3x_2 \leq 30, \quad 2x_1 + x_2 \leq 20 \quad (*) \\ y_1 \leq 10, \quad y_2 \leq 10, \quad y_1 + y_2 &\leq 20, \quad x_i, y_i \geq 0. \end{aligned} \quad (16)$$

5.1 Solution using Kornbluth & Salkin

In this section, we present a numerical example to demonstrate Kornbluth & Salkin decomposition technique. This is clearly shown in the reference.

Kornbluth & Salkin decomposition method, terminates if $\sigma_\alpha \geq f_\alpha$ otherwise go to step

(b). Since the 1st division dual $\sigma_1 = \frac{10}{25}$ and optimal $\hat{f}_1 = \frac{10}{25}$ and satisfies $\sigma_1 = f_1$.

Also the 2nd division dual $\sigma_2 = 0$ and optimal $\hat{f}_1 = 0$ and satisfies $\sigma_2 = f_2$. So this solution is optimal. Applying the algorithm in Section 3.1, we get optimal & dual solution

from (15) is $\sigma_b = \frac{1}{25}$, $\pi_1^* = (10, 0)$, $\pi_2^* = (0, 0, 0)$.

5.2 Solution Using Our Improved Decomposition Method

In this section, we present an example to demonstrate our improved decomposition technique.

Sub-problem for k – th iteration

We apply the Lagrangian relaxation by relaxing constraint (*), we get the general sub-problem and the last iteration is given in the following way.

$$\begin{aligned} \text{Maximize } \frac{2}{25} x_1 + \frac{2}{25} y_1 + \frac{1}{25} y_2 - \lambda_k (x_1 + 2x_2 + 2y_1 + y_2 - 15) \\ \text{subject to } x_1 + 3x_2 \leq 30, \quad 2x_1 + x_2 \leq 20, \quad y_1 \leq 10, \quad y_2 \leq 10, \quad y_1 + y_2 \leq 20, \quad x_i, y_i \geq 0 \end{aligned}$$

Master problem for k – th iteration

$$\begin{aligned} \text{Maximize } \sum_{i=1}^k \eta_i \left(\frac{2}{25} x_1^k + \frac{2}{25} y_1^k + \frac{1}{25} y_2^k - 15 \right) \\ \text{subject to } \sum_{i=1}^k \eta_i (x_1^k + 2x_2^k + 2y_1^k + y_2^k) \leq 15, \quad \sum_{i=1}^k \eta_i = 1, \quad \eta_i \geq 0 \end{aligned}$$

Iteration-4 (For k=4): Sub-problem

$$\text{Maximize} \quad \frac{2}{25}x_1 + \frac{2}{25}y_1 + \frac{1}{25}y_2 - \frac{1}{25}(x_1 + 2x_2 + 2y_1 + y_2 - 15)$$

$$\text{subject to} \quad x_1 + 3x_2 \leq 30, \quad 2x_1 + x_2 \leq 20, \quad y_1 \leq 10, \quad y_2 \leq 10, \quad y_1 + y_2 \leq 20, \quad x_i, y_i \geq 0$$

Solving by simplex method we get $x_1 = 10, x_2 = 0, y_1 = \frac{5}{2}, y_2 = 0$ and sub-problem

values $z = S_4(v) = 1$. Also the dual solution is $\sigma_b = \frac{1}{25} = \lambda_3, \pi_1^* = (10, 0),$

$$\pi_2^* = (0, 0, 0).$$

Master problem

$$\text{Maximize} \quad 0 \cdot \eta_1 + \frac{9}{5}\eta_2 + \frac{20}{25}\eta_3 + \frac{20}{25}\eta_4$$

$$\text{subject to:} \quad 0 \cdot \eta_1 + 35\eta_2 + 10\eta_3 + 10\eta_4 \leq 15, \quad \eta_1 + \eta_2 + \eta_3 + \eta_4 = 1, \quad \eta_i \geq 0$$

Solving by Dual Simplex Method we get $\eta_1 = 0, \eta_2 = 0, \eta_3 = 0, \eta_4 = 1$ and master problem value $z = M_4(v) = 1$. The first dual value $\lambda_4 = \frac{1}{25}$ and the second dual value is $\frac{10}{25}$. Since $S_4(v) = 1 = M_4(v)$.

Thus the current solution is optimal with the solution $x_i = \sum_{k=1}^K x(k, i) \eta(1, k),$ ($i = 1, 4$)

$$x_1 = 0 \cdot 0 + 10 \cdot 0 + 10 \cdot 0 + 10 \cdot 1 = 10, \quad x_2 = 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 = 0$$

$$y_1 = 0 \cdot 0 + 0 \cdot 0 + 10 \cdot 0 + \frac{5}{2} \cdot 1 = \frac{5}{2}, \quad y_2 = 0 \cdot 0 + 0 \cdot 0 + 5 \cdot 0 + 0 \cdot 1 = 0$$

which gives $x_1 = 10, x_2 = 0, y_1 = \frac{5}{2}, y_2 = 0$ with maximum of $z = 1$.

On the other hand the dual objective function value is $15 \cdot \frac{1}{25} + \frac{10}{25} = 1$.

Solving the total problem using Charnes and Cooper method or Kornluth & Salkin decomposition method, one can observe that our primal & dual objective function values will coincide with their results.

6. Programming Code in AMPL

In this section, we developed a computational technique corresponding to our improved decomposition technique for primal dual LP and LFP using the programming language AMPL.

7. Result & Discussion

In this section, we present the solution comparison between manual output and the programming output. Then we present the convergence of the master problem & Sub problem.

7.1 Solution Comparison

In the current section, the values we calculated manually earlier and the values we have now from our programming output are tabulated.

Table1: Comparison of solutions

Iteration	Manual Output	Program Output
1	<p><i>Solution:</i> $x_1 = x_2 = y_1 = y_2 = 0$ <i>Subproblem Value</i> $S_1(v) = 15$ <i>Master solution:</i> $\theta_1 = 1$ <i>Master value</i> $M_1(v) = 0$, <i>Dualvalue</i> $\lambda_1 = 1$</p>	<p><i>Solution:</i> $x_1 = x_2 = y_1 = y_2 = 0$ <i>Subproblem value</i> $S_1(v) = 15$ <i>Master solution:</i> $\theta_1 = 1$ <i>Master value</i> $M_1(v) = 0$ <i>Dual value</i> $\lambda_1 = 1$</p>
2	<p><i>Solution:</i> $x_1 = 10, x_2 = 0, y_1 = 10, y_2 = 5$ and sub-problem value $z = S_2(v) = \frac{9}{5}$. <i>Master problem solution:</i> $\eta_1 = 4/7, \eta_2 = 3/7$ and master value $z = M_1(v) = 27/35$ and dual $\lambda_2 = 0$</p>	<p><i>Solution:</i> $x_1 = 10, x_2 = 0, y_1 = 10, y_2 = 10$ and sub-problem value $z = S_2(v) = 2$ <i>Master problem solution:</i> $\eta_1 = 0.625, \eta_2 = 0.375$ and master problem value $z = M_1(v) = 0.75$ the dual value $\lambda_2 = 0$</p>
3	<p><i>Solution:</i> $x_1 = 10, x_2 = 0, y_1 = 0, y_2 = 0$ and sub-problem value $z = S_2(v) = \frac{37}{35}$. <i>Master:</i> $\eta_1 = 0, \eta_2 = 1/5, \eta_3 = 4/5$ And master value $z = M_1(v) = 1$, <i>Dual value</i> $\lambda_3 = 9/175$</p>	<p><i>Solution:</i> $x_1 = 10, x_2 = 0, y_1 = 0, y_2 = 0$ and sub-problem value $z = S_2(v) = 1.05$. <i>Master:</i> $\eta_1 = 0, \eta_2 = 1/5, \eta_3 = 4/5$ and master value $z = M_1(v) = 1$, <i>Dual value</i> $\lambda_3 = 1/25$</p>
4	<p><i>Solution:</i> $x_1 = 10, x_2 = 0, y_1 = \frac{5}{2}, y_2 = 0$ and Subproblem $S_4(v) = 1$. <i>Master problem solution:</i> $\eta_1 = 0, \eta_2 = 0, \eta_3 = 0, \eta_4 = 1$ master value $z = M_4(v) = 1$ and dual value $\lambda_4 = 1/25$.</p>	<p><i>Solution:</i> $x_1 = 10, x_2 = 0, y_1 = 0, y_2 = 0$ and subproblem value $z = S_4(v) = 1$. <i>Master Solution:</i> $\eta_1 = 0, \eta_2 = 0.166666, \eta_3 = 0.8333333, \eta_4 = 0$ master value $z = M_4(v) = 1$ and dual value $\lambda_4 = 1/25$.</p>

Form the Table 1, we can say that our computer technique gives the identical optimal value of the objective function and the optimal solutions. Since the original problem has alternative solution so it gives the alternative solutions.

7.2 Convergence of Master and Sub Problem Value

In this section, we present the graphical representation of the convergence of master and sub problem values using manual and coding figure. We have used few commands in MATHEMATICA which gives the following figure.

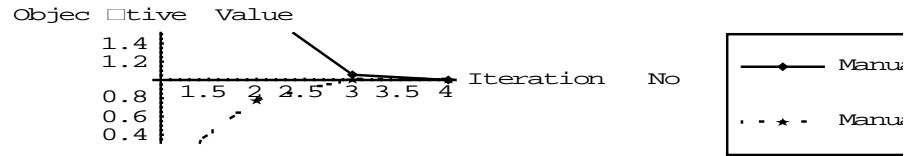


Fig. 1 Convergence of Manual Sub-problem and Master-problem value of Example 5.1

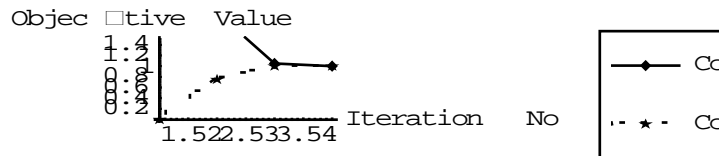


Fig. 2: Convergence of Coding Sub-problem and Master-problem value of Example 5.1

8. Conclusion

In this paper, we developed a sophisticated decomposition algorithm for showing the relationship of primal and dual approach of LP & LFP which was demonstrated with a numerical example. A computational technique is also developed using AMPL. A comparative study is also given between Kornbluth & Salkin and our improve decomposition LFP method. We conclude that our technique is more efficient for using primal dual LP & LFP problems than the others.

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