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COMMUTATIVELY OF PRIME AND SEMIPRIME Γ-RINGS WITH SYMMETRIC BI-DERIVATIONS

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ABSTRACT

Let M be a Γ-ring and let D: $M \times M \to M$ be a symmetric bi-derivation with the trace d: $M \to M$ denoted by $d(x) = D(x, x)$ for all $x \in M$. The objective of this paper is to prove some results concerning symmetric bi-derivation on prime and semiprime Γ-rings. If M is a 2-torsion free prime Γ-ring and $D \neq 0$ be a symmetric bi-derivation with the trace d having the property $d(x)\alpha x - x\alpha d(x) = 0$ for all $x \in M$ and $\alpha \in \Gamma$, then M is commutative. We also prove another result in Γ-rings setting analogous to that of Posner for prime rings.

Keywords: Γ-ring, derivation, bi-derivation, commutativity

1. Introduction and Preliminaries

The concept of a Γ-ring was first introduced by Nobuswa [5], and afterwards it was generalized by Barnes [1] in more natural sense. Maksa [14] worked on the trace of symmetric bi-derivation on classical rings theories and developed some fruitful results concerning bi-derivations. Vukman [10] proved some results relating symmetric bi-derivations on prime and semiprime rings. Ozturk, Sapanci, Soyturk and Kim [7] worked on the trace of symmetric bi-derivations in Γ-rings and extended some results of Vukman [10] to ideals of prime and semipime Γ-rings.

In this paper, we extend some results of Vukman [10] to prime and semiprime Γ-rings. Our results are quiet different from the results obtained in [9].

Let M and Γ be additive abelian groups. If there exists a mapping $(x, \alpha, y) \to x\alpha y$ of $M \times \Gamma \times M \to M$ which satisfies the conditions:

(i) $x\alpha y \in M$,
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha + \beta)y = x\alpha y + x\beta y$, $x\alpha(y + z) = x\alpha y + x\alpha z$,
(iii) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$,

then M is called a Γ-ring in the sense of Barnes [1]. Throughout this paper M denotes a Γ-ring with center $Z(M)$. For any $x, y \in M$, $\alpha \in \Gamma$, the symbol $[x, y]_\alpha$ (resp. $\langle x, y\rangle_\alpha$) will denote the commutator $x\alpha y - y\alpha x$ (resp. the anti-commutator $x\alpha y + y\alpha x$). A Γ-ring M is called commutative if $[x, y]_\alpha = 0$ for all $x, y \in M$, $\alpha \in \Gamma$. We know that

$$[x\beta y, z]_\alpha = [x, z]_\alpha\beta y + x\beta[y, z]_\alpha + x[\beta, \alpha]_z y$$

and

$$[x, y\beta z]_\alpha = y\beta[x, z]_\alpha + [x, y]_\alpha\beta z + y[\beta, \alpha]_x z.$$

We make the assumption (*) $x\beta z\alpha y = x\alpha z\beta y$ for all $x, y, z \in M$, $\alpha, \beta\in\Gamma$. Using this assumption the basic commutator identities reduce to

$$[x\beta y, z]_\alpha = [x, z]_\alpha\beta y + x\beta[y, z]_\alpha$$
$$[x, y\beta z]_\alpha = y\beta[x, z]_\alpha + [x, y]_\alpha\beta z.$$

Recall that a Γ-ring M is prime if $x\Gamma M\Gamma y = 0$ implies that $x = 0$ or $y = 0$, and is semiprime if $x\Gamma M\Gamma x = 0$ implies $x = 0$. An additive mapping d: $M \to M$ is called a derivation if $d(x\alpha y) = d(x)\alpha y + x\alpha d(y)$ holds for all $x, y\in M$, $\alpha\in\Gamma$. A derivation d is inner if there exists $a\in M$ such that $d(x) = [a, x]_\alpha$ holds for all $x\in M$, $\alpha\in\Gamma$. The mapping B: $M \times M \to M$ is said to by symmetric if $B(x, y) = B(y, x)$ holds for all $x, y\in M$. A mapping $f : M \to M$ defined by $f(x) = B(x, x)$, where B: $M \times M \to M$ is a symmetric mapping, is called the trace of B. In case B: $M \times M \to M$ is a symmetric mapping which is also bi-additive (i.e. additive in both arguments), the trace of B satisfies the relation $f(x + y) = f(x) + f(y) + 2B(x, y)$, for all $x, y\in M$. We shall use also the fact that the trace of a symmetric bi-additive mapping is an even function. A symmetric bi-additive mapping D: $M \times M \to M$ is called a symmetric bi-derivation if $D(x\alpha y, z) = D(x, z)\alpha y + x\alpha D(y, z)$ is fulfilled for all $x, y, z \in M$, $\alpha \in \Gamma$. Obviously, in this case also the relation $D(x, y\alpha z) = D(x, y)\alpha z + y\alpha D(x, z)$ for all $x, y, z\in M$, $\alpha\in\Gamma$, holds. A mapping $f : M \to M$ is said to be commuting on M if $[f(x), x]_\alpha = 0$ holds for all $x\in M$, $\alpha \in \Gamma$. A mapping $f : M \to M$ is centralizing on M if $[f(x), x]_\alpha \in Z(M)$ holds for all $x\in M$, $\alpha \in \Gamma$.

2. Bi-derivations on d-rings

We shall need the following well-known and frequently used lemmas.

Lemma 2.1. ([2, Lemma 3.2]) Let d: $M \to M$ be a derivation, where M is a prime Γ-ring. Suppose that either (i) $a\Gamma d(x) = 0$, for all $x \in M$ or (ii) $d(x)\Gamma a = 0$, for all $x \in M$ holds. Then we have (i) $a = 0$ or (ii) $d = 0$.

Lemma 2.2. ([7, Lemma 3]) Let M be a 2-torsion free prime Γ-ring and let $a, b\in M$ be fixed elements. If $a\alpha x\beta b + b\alpha x\beta a = 0$ is fulfilled for all $x \in M$, $\alpha, \beta \in \Gamma$, then $a = 0$ or $b= 0$.

We start our investigation of symmetric bi-derivations with the following results.

Theorem 2.3. Let M be a 2-torsion free prime Γ-ring satisfying the condition (*). Let D: $M \times M \to M$ and d be a symmetric bi-derivation and the trace of D, respectively. Suppose that d is commuting on M, then M is commutative or $D = 0$.

Proof. We have

$$[d(x), x]_\alpha = 0, \text{ for all } x \in M, \alpha \in \Gamma. \qquad (l)$$

The linearization of (1) gives us $[d(x) + d(y) + 2D(x, y), x + y]_\sim = 0$,

which leads to

$$[d(x), y]_\alpha + [d(y), x]_\alpha + 2[D(x, y), x]_\alpha + 2[D(x, y), y]_\alpha = 0 \text{ for all } x, y \in M, \alpha \in \Gamma. \qquad (2)$$

Substituting $-x$ for x in the relation above, we arrive at

$$[d(x), y]_\alpha - [d(y), x]_\alpha + 2[D(x, y), x]_\alpha - 2[D(x, y), y]_\alpha = 0 \text{ for all } x, y \in M, \alpha \in \Gamma. \quad (3)$$

From (2) and (3) we obtain

$$[d(x), y]_\alpha + 2[D(x, y), x]_\alpha = 0 \text{ for all } x, y \in M, \alpha \in \Gamma. \quad (4)$$

Replacing y in (4) by $x\beta y$. Then by using the condition (*),

$$0 = [d(x), x\beta y]_\alpha + 2[d(x)\beta y + x\beta D(x, y), x]_\alpha$$

$$= x\beta[d(x), y]_\alpha + 2d(x)\beta[y, x]_\alpha + 2x\beta[D(x, y), x]_\alpha$$

which, according to (4), implies

$$d(x)\beta[x, y]_\alpha = 0 \text{ for all } x, y \in M, \alpha, \beta \in \Gamma. \quad (5)$$

From the relation (5) and Lemma 2.1 one can conclude that $d(x) = 0$ or $[x, y]_\alpha = 0$ for all $x, y \in M$, $\alpha \in \Gamma$. If $[x, y]_\alpha = 0$, then M is commutative. On the other hand, for any $x \notin Z(M)$, we have $[x, y]_\alpha \neq 0$. Therefore $d(x) = 0$ (note that for any fixed $x \in M$, $\alpha \in \Gamma$, a mapping $y \to [x, y]_\alpha$ is a derivation). Let $x \in Z(M)$, $y \notin Z(M)$. Then $x + y \notin Z(M)$ and $x - y \notin Z(M)$. Thus $0 = d(x + y) = d(x) + 2D(x, y)$ and $0 = d(x) - 2D(x, y)$. From these two relations, we have $4D(x, y) = 0$. By the 2-torsion freeness of M, we have

$D(x, y) = 0$ for all $x, y \in M$. The proof of the theorem is complete.

Theorem 2.4. Let M be a 2 and 3-torsion free prime Γ-ring satisfying the condition (*). Let D: $M \times M \to M$ and d be a symmetric bi-derivation and the trace of D, respectively. Suppose that d is centralizing on M, then M is commutative or $D = 0$.

Proof We have

$$[d(x), x]_\alpha \in Z(M) \text{ for all } x \in M, \alpha \in \Gamma. \quad (6)$$

By linearization we obtain

$$[d(x) + d(y) + 2D(x, y), x + y]_\alpha \in Z(M)$$

$$\Rightarrow [d(y), x]_\alpha + [d(x), y]_\alpha + 2[D(x, y), y]_\alpha + 2[D(x, y), x]_\alpha \in Z(M) \text{ for all } x, y \in M, \alpha \in \Gamma. \quad (7)$$

since (6) holds. Replacing x in the relation (7) by $-x$, we obtain

$$-[d(y), x]_\alpha + [d(x), y]_\alpha - 2[D(x, y), y]_\alpha + 2[D(x, y), x]_\alpha \in Z(M) \text{ for all } x, y \in M, \alpha \in \Gamma. \quad (8)$$

Now (7) and (8) give us

$$[d(x), y]_\alpha + 2[D(x, y), x]_\alpha \in Z(M) \text{ for all } x, y \in M, \alpha \in \Gamma. \quad (9)$$

Replacing y in (9) by $x\beta x$, we get

$$[d(x), x\beta x]_\alpha + 2[d(x)\beta x + x\beta d(x), x]_\alpha \in Z(M)$$

$$\Rightarrow [d(x), x]_\alpha\beta x + x\beta[d(x), x]_\alpha + 2[d(x), x]_\alpha\beta x + 2x\beta[d(x), x]_\alpha \in Z(M)$$

$$\Rightarrow 6[d(x), x]_\alpha\beta x \in Z(M) \text{ for all } x, y \in M, \alpha, \beta \in \Gamma. \quad (10)$$

Using (10), (6) and the assumptions that M is 2 and 3-torsion free, we obtain

$$[d(x), x]_\alpha\beta[x, y]_\alpha = 0 \text{ for all } x, y \in M, \alpha, \beta \in \Gamma.$$

Now, the relation above makes it possible to conclude, using the same arguments as in the proof of Theorem 2.3, that for any $x \notin Z(M)$ we have $[d(x), x]_\alpha = 0$.

In view of Theorem 2.3, the proof is complete.

Theorem 2.5. Let M be a 2-torsion free prime Γ-ring satisfying the condition (*). Suppose there exist symmetric bi-derivations D_1: $M \times M \to M$ and D_2: $M \times M \to M$, such that $D_1(d_2(x), x) = 0$ holds for all $x \in M$, where d_2 denotes the trace of D_2.

Then $D_1 = 0$ or $D_2 = 0$.

Proof. By linearization of the relation

$$D_1(d_2(x), x) = 0 \text{ for all } x \in M. \tag{11}$$

we obtain according to (11),

$$D_1(d_2(x) + d_2(y) + 2D_2(x, y), x + y) = 0$$

$$\Rightarrow D_1(d_2(y), x) + 2D_1(D_2(x, y), x) + D_1(d_2(x), y) + 2D_1(D_2(x, y), y) = 0 \text{ for all } x, y \in M.$$

Replacing x by $-x$ and comparing this new equation with the preceding equation we get

$$D_1(d_2(x), y) + 2D_1(D_2(x, y), x) = 0 \text{ for all } x, y \in M. \tag{12}$$

Let us replace y by $x\alpha y$ in (12). Then

$$\begin{aligned}
0 &= D_1(d_2(x), x\alpha y) + 2D_1(D_2(x , x\alpha y), x)\\
&= D_1(d_2(x), x)\alpha y + x\alpha D_1(d_2(x), y) + 2D_1(d_2(x)\alpha y + x\alpha D_2(x, y), x)\\
&= x\alpha D_1(d_2(x), y) + 2D_1(d_2(x), x)\alpha y + 2d_2(x)\alpha D_1(y, x) + 2d_1(x)\alpha D_2(x, y) + 2x\alpha D_1(D_2(x, y), x)\\
&= x\alpha D_1(d_2(x), y) + 2x\alpha D_1(D_2(x, y), x) + 2d_2(x)\alpha D_1(x, y) + 2d_1(x)\alpha D_2(x, y)\\
&= 2d_2(x)\alpha D_1(x, y) + 2d_1(x)\alpha D_2(x, y).
\end{aligned}$$

In the above calculation we used (11) and (12). Thus we have

$$d_2(x)\alpha D_1(x, y) + d_1(x)\alpha D_2(x, y) = 0 \text{ for all } x, y \in M, \alpha \in \Gamma. \tag{13}$$

Let us replace y in (13) by $y\beta x$. We get

$$\begin{aligned}
0 &= d_2(x)\alpha D_1(y\beta x, x) + d_1(x)\alpha D_2(y\beta x, x)\\
&= d_2(x)\alpha(D_1(y, x)\beta x + y\beta d_1(x)) + d_1(x)\alpha(D_2(y, x)\beta x + y\beta d_2(x))\\
&= (d_2(x)\alpha D_1(x, y) + d_1(x)\alpha D_2(x, y))\beta x + d_1(x)\alpha y\beta d_2(x) + d_2(x)\alpha y\beta d_1(x)\\
&= d_1(x)\alpha y\beta d_2(x) + d_2(x)\alpha y\beta d_1(x).
\end{aligned}$$

Thus, we have

$$d_1(x)\alpha y\beta d_2(x) + d_2(x)\alpha y\beta d_2(x) = 0 \text{ for all } x, y \in M, \alpha, \beta \in \Gamma. \tag{14}$$

Let us assume that d_1 and d_2 are both different from zero. In other words there exist elements x_1, $x_2 \in M$ such that $d_1(x_1) \neq 0$ and $d_2(x_2) \neq 0$. From (14) and Lemma 2.2, it follows that $d_l(x_2) = d_2(x_2) = 0$. Since $d_1(x_2) = 0$, the relation (13) reduces to $d_2(x_2)\alpha D_1(x_2, y) = 0$. Using this relation and Lemma 2.1, we obtain that $D_1(x_2, y) = 0$ holds for all $y \in M$ since $d_2(x_2) \neq 0$ (recall that a mapping $y \to D_l(x_2, y)$ is a derivation). In particular we have $D_1(x_2, x_l) = 0$. Similarly, we obtain $D_2(x_l, x_2) = 0$ holds as well. Let us write y for $x_1 + x_2$. Then $d_1(y) = d_l(x_l + x_2) = d_l(x_l) + d_l(x_2) + 2D_l(x_l, x_2) = d_l(x_l) \neq 0$. Similarly, we obtain $d_2(y) \neq 0$. But $d_l(y)$ and $d_2(y)$ cannot be both different from zero according to (14) and Lemma 2.2. Therefore we have proved that $d_1 = 0$ or $d_2 = 0$ which is the assertion of the theorem.

In case $D_1 = D_2$ Theorem 2.5 can be proved for semiprime Γ-rings.

Theorem 2.6. Let M be a 2-torsion free semiprime Γ-ring. Suppose there exists such a symmetric bi-derivation D: $M \times M \to M$ that $D(d(x), x) = 0$ holds for all $x \in M$, where d denotes the trace of D. Then $D = 0$.

Proof. In this case (14) reduces to $d(x)\alpha y\beta d(x) = 0$ for x, $y \in M$, α, $\beta \in \Gamma$, which implies that $d(x) = 0$ for all $x \in M$, by semiprimeness of Posner [10] has proved a result which states that in case M is a 2-torsion free prime Γ-ring and D_1, D_2 are nonzero derivations on M, then the mapping $x \to D_1(D_2(x))$ cannot be a derivation.

The result below was motivated by Posner's result mentioned above.

Theorem 2.7. Let M be a 2 and 3-torsion free prime Γ-ring satisfying the condition (*). Let D_l: $M \times M \to M$ and D_2: $M \times M \to M$ be symmetric bi-derivations. Suppose further that there exists a symmetric bi-additive mapping B: $M \times M \to M$ such that $d_l(d_2(x)) = f(x)$ holds for all $x \in M$, where d_l and d_2 are the traces of D_1 and D_2, respectively, and f is the trace of B. Then $D_1 = 0$ or $D_2 = 0$.

Proof. The linearization of the relation

$$d_1(d_2(x)) = f(x) \text{ for all } x \in M. \tag{15}$$

gives us

$$d_l(d_2(x) + d_2(y) + 2D_1(x, y)) = f(x) + f(y) + 2B(x, y)$$

and

$$d_l(d_2(x)) + d_l(d_2(y)) + 4d_l(D_2(x, y)) + 2D_1(d_2(x), d_2(y)) + 4D_1(d_2(x), D_2(x, y)) + 4D_1(d_2(y), D_2(x, y)) = f(x) + f(y) + 2B(x, y).$$

Using (15) we arrive at

$$2d_l(D_2(x, y)) + D_1(d_1(x), d_1(y)) + 2D_1(d_2(x), D_2(x, y)) + 2D_1(d_2(y), D_2(x, y)) = B(x, y).$$

Substituting in the equation above x by $-x$ we obtain by comparing this new equation with the equation above that

$$2D_l(d_2(x), D_2(x, y)) + 2D_1(d_2(y), D_2(x, y)) = B(x, y) \text{ for all } x, y \in M. \tag{16}$$

Let us replace in (16) x by $2x$. We have

$$8D_1(d_2(x), D_2(x, y)) + 2D_1(d_2(y), D_2(x, y)) = B(x, y) \text{ for all } x, y \in M. \tag{17}$$

By comparing (16) and (17) we obtain

$$6D_1(d_2(x), D_2(x, y)) = 0$$

$$\Rightarrow D_1(d_2(x), D_2(x, y)) = 0 \text{ for all } x, y \in M. \quad (18)$$

since M is 2 and 3-torsion free. From (18) it follows that both terms on the left side of the relation (16) are zero, which means that $B = 0$. Hence (15) reduces to

$$d_1(d_2(x)) = 0 \text{ for all } x \in M. \quad (19)$$

Let in (18) y be $y\alpha x$. We have

$$0 = D_1(d_2(x), D_2(x, y\alpha x))$$
$$= D_1(d_2(x), D_2(x, y)\alpha x + y\alpha d_2(x))$$
$$= D_1(d_2(x), D_2(x, y)\alpha x) + D_1(d_2(x), y\alpha d_2(x))$$
$$= D_1(d_2(x), D_2(x, y))\alpha x + D_2(x, y)\alpha D_1(d_2(x), x) + D_1(d_2(x), y)\alpha d_2(x) + y\alpha d_1(d_2(x))$$

for all $x, y \in M$, $\alpha \in \Gamma$.

This implies

$$D_1(d_2(x), y)\alpha d_2(x) + D_2(x, y)\alpha D_1(d_2(x), x) = 0 \text{ for all } x, y \in M, \alpha \in \Gamma. \quad (20)$$

according to (18) and (19). Let us replace in (20) y by $x\beta y$. We have

$$0 = D_1(d_2(x), x\beta y)\alpha d_2(x) + D_2(x, x\beta y)D_1(d_2(x), x)$$
$$= D_1(d_2(x), x)\alpha y\beta d_2(x) + x\beta D_1(d_2(x), y)\alpha d_2(x) + d_2(x)\alpha y\beta D_1(d_2(x), x)$$
$$+ x\alpha D_2(x, y)\beta D_1(d_2(x), x)$$
$$= D_1(d_2(x), x)\alpha y\beta d_2(x) + d_2(x)\alpha y\beta D_1(d_2(x), x) + x\beta(D_1(d_2(x), y)\alpha d_2(x)$$
$$+ D_2(x, y)\alpha D_1(d_2(x), x)) \text{ for all } x, y \in M, \alpha, \beta \in \Gamma.$$

Now, by (20), we arrive finally at

$$D_1(d_2(x), x)\alpha y\beta d_2(x) + d_2(x)\alpha y\beta D_1(d_2(x), x) = 0 \text{ for all } x, y \in M, \alpha, \beta \in \Gamma. \quad (21)$$

From the relation above one can conclude that $D_1(d_2(x), x) = 0$ is fulfilled for all $x\in M$. Namely, if $D_1(d_2(x), x) \neq 0$ for some $x\in M$, then $d_2(x) = 0$ according to (21) and Lemma 2.2, contrary to the assumption $D_1(d_2(x), x) \neq 0$. Therefore, since $D_1(d_2(x), x) = 0$ for all $x\in M$, the proof of the theorem is complete since all the requirements of Theorem 2.5 are fulfilled.

In case $D_1 = D_2$ Theorem 2.7 can be proved for semi-prime Γ-rings.

Theorem 2.8. Let M be a 2, 3-torsion free semiprime Γ-ring satisfying the condition (*). Let $D: M \times M \to M$ and $B: M \times M \to M$ be a symmetric bi-derivation and a symmetric bi-additive mapping, respectively. Suppose that $d(d(x)) = f(x)$ holds for all $x \in M$, where d is the trace of D and f is the trace of B. Then $D = 0$.

Proof. Obviously, we can use the beginning of the proof of Theorem 2.5. In this case relations (18) and (19) can be written in the form

$$D(d(x), D(x, y)) = 0 \text{ for all } x, y \in M. \tag{22}$$

and

$$d(d(x)) = 0 \text{ for all } x \in M. \tag{23}$$

Let us write in (22) $y\alpha z$ instead of y. We have

$0 = D(d(x), D(x, y\alpha z))$

$= D(d(x), D(x, y)\alpha z + y\alpha D(x, z))$

$= D(d(x), D(x, y)\alpha z) + D(d(x), y\alpha D(x, z))$

$= D(d(x), D(x, y))\alpha z + D(x, y)\alpha D(d(x), z) + D(d(x), y)\alpha D(x, z) + y\alpha D(d(x), D(x, z))$ for all $x, y, z \in M$, $\alpha \in \Gamma$.

Hence by (22) we have

$$D(x, y)\alpha D(d(x), z) + D(d(x), y)\alpha D(x, z) = 0$$

and, in particular, for $z = d(x)$ we obtain

$$D(d(x), y)\alpha D(x, d(x)) = 0 \text{ for all } x, y \in M, \alpha \in \Gamma. \tag{24}$$

according to (23). Replace in (24) y by $x\beta y$. We have $0 = D(d(x), x\beta y)\alpha D(x, d(x)) = D(d(x), x)\beta y\alpha D(x, d(x)) + x\beta D(d(x), y)\alpha D(x, d(x))$ which leads to

$D(d(x), x)\alpha y\beta D(d(x), x) = 0$; $x, y \in M$, $\alpha, \beta \in \Gamma$; and we obtain $D(d(x), x) = 0$ for all $x \in M$ by the semiprimeness of M. Thus by Theorem 2.6 the proof is complete.

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