

## AN IMPECCABLE SOLUTION PROCEDURE OF STOCHASTIC PROGRAMMING PROBLEMS BY INSERTING SCENARIOS IN DETERMINISTIC CASE

Sajal Chakroborty<sup>1,\*</sup> and M. Babul Hasan<sup>2</sup>

Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh.

\*Corresponding author: sajal.math@yahoo.com

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### ABSTRACT

In this paper, a new technique has been developed for solving stochastic programming problems by using the idea of decomposition based pricing method. A computer code has also been developed by using a mathematical programming language AMPL and a real life oriented model has been developed. The technique has been demonstrated by analyzing the model for different data sets which has been collected for different scenarios. To our knowledge, this is the first work for solution procedure of stochastic programming problems by using decomposition based pricing method.

**Keywords:** SP, DBP, AMPL, Scenarios

### 1. Introduction

Stochastic programming (SP) deals with uncertainty and probability of occurring that uncertainty. It is a special case of linear programming (LP) problems and contains some characteristics of LP [6]. Basic difference between LP and SP is that ideas of LP can be applied only for deterministic cases whether SP works with uncertain cases. The beginning of SP dates back to the 50's and 60's of the last century. G. B. Dantzig first formulated the general problem of LP with uncertain data [14] and he is considered as a pioneer to establish SP as a branch of mathematics. SP models extend the scope of linear and nonlinear programming to include probabilistic or statistical information about one or more uncertain problem parameters. It has lots of implementations in real life [17].

Due to uncertainty, volume of SP increases dramatically. That's why decomposition techniques are more effective for solving SP problems because it helps to solve problem by splitting into many sub-problems. Slyke and Wets developed L-shaped method [16] for solving SP problems. They used Benders decomposition technique [1]. But when the number of scenarios increase, it becomes quite difficult to solve the problem by their technique. Hagle and Sen [8] developed a technique by integrating the elements of decomposition techniques and statistical approximation techniques which is renowned as stochastic decomposition. Most of the solution techniques for SP or multi stage SP problems were developed by using Benders decomposition technique. But the row

generation technique makes the solution procedure more complicated because at each stage new constraints are added.

The objective of this paper is to find an easier way to solve SP problems. In this paper, a new technique has been developed for solving SP problems by using decomposition based pricing (DBP) method. It has been applied on a real life oriented model. A profit analysis is presented for that problem. The analysis is also extended for multi period case. From this analysis one can make a forecast about the future scenario and make proper plane to prevent any obstacles. The rest of the paper is organized as follows. In Section 2, decomposition techniques have discussed. Then have focused on some existing solution procedures of SP in Section 3. After that the deterministic case of our developed model, its formulation and solution has presented in Section 4 and 4.1 respectively. Then in Section 4.2 and 4.3, this idea has extended into stochastic formulation. The developed algorithm and AMPL code has presented in Section 5 and 5.1 respectively. In Section 6, optimal solution of the stochastic model and in Section 6.1 a profit comparison has presented graphically. After that in Section 6.2, a profit analysis for several years has presented. Finally, a conclusion has drawn about our work.

## 2. Preliminaries

In this section, some necessary definitions and important techniques relevant to the work have been discussed.

### 2.1. SP and its relation with LP

In this section, a relationship between LP and SP has presented. Consider the following LP problem [15].

$$\text{Maximize (or Minimize)} z = c^T x \quad (2.1)$$

subject to

$$Ax = b \quad (2.2)$$

$$x \geq 0 \quad (2.3)$$

The  $m \times n$  matrix  $A = (a_{ij})$  is the coefficient matrix of the equality constraints (2.2),  $b = (b_1, b_2, \dots, b_m)^T$  is the right hand side vector of the constraints (2.2), the components of  $c$  are the profit factors and,  $x = (x_1, x_2, \dots, x_n)^T \in R^n$  is the vector of variables, called the decision variables and  $x \geq 0$  are called non negative restrictions.

When some or all of the parameters of LP problem contains uncertainty then it is called SP problem. A mathematical formulation of SP problems has given below [14].

$$\text{Maximize (or Minimize)} z = c_{\omega}^T x_{\omega}$$

subject to

$$Ax_{\omega} = b$$

$$x_{\omega} \geq 0$$

Here the profit factor  $C_{\omega}$  contains uncertainty for different scenarios  $\omega$  have considered and  $x_{\omega}$  represents corresponding stochastic decision variables. There are different types of SP. In this paper, mainly the focus has given on scenario based SP. In the next section, a discussion has presented on scenario based SP.

### 2.2. Scenario Based SP

A scenario for a stochastic model is a collection of outcomes for all the stochastic events taking place in the model, along with the associated probability of the scenario to occur. Khan and Weiner (1967) [18], probably first originated scenario analysis, defined a scenario as a hypothetical sequence of events constructed to focus on casual processes and decision points. Scenarios can be of several types, the most common being “favorability to sponsor,” e.g. optimistic or pessimistic and “Probability of occurrence,” e.g. most likely or least likely, is also very popular although very subjective [5]. Sources of data for scenarios are historical data, expert opinions, simulation based on a mathematical or statistical model.

SP associated with scenarios for respectable uncertainties inserted in any or all parameters is called scenario based SP. A mathematical formulation of this type SP has given below.

$$\begin{aligned} & \text{Maximize } z = P_{\sigma} C_{\sigma} X_{\sigma} \\ & \text{subject to} \\ & AX_{\sigma} \leq \bar{b} \\ & X_{\sigma} \geq 0 \end{aligned}$$

In this problem,  $P_{\sigma}$  is representing the probability for scenario  $\sigma$ ,  $C_{\sigma}$  is for profit factor obtained for different scenarios  $\sigma$  and  $X_{\sigma}$  are the corresponding stochastic decision variables for scenarios  $\sigma$ . In the next section, the basic idea of decomposition technique has discussed.

### 2.3. Decomposition technique

It is a solution procedure for LP problems and more effective for solving large scale LP problems. Consider the following LP problem [19].

$$\begin{aligned} & \text{Maximize } z = C^1 X_1 + C^2 X_2 + \dots + C^n X_n \\ & \text{subject to} \\ & A_1 X_1 + A_2 X_2 + \dots + A_n X_n \leq b \\ & B_1 X_1 \leq b_1 \\ & B_2 X_2 \leq b_2 \\ & \dots \dots \dots \\ & B_n X_n \leq b_n \end{aligned}$$



$$\begin{aligned} \sum_{i=1}^k \theta_1^i &= 1 \\ \sum_{i=1}^k \theta_2^i &= 1 \\ &\dots\dots\dots \\ \sum_{i=1}^k \theta_n^i &= 1 \\ \theta_i^i &\geq 0, i = 1 \dots n \end{aligned}$$

Here  $\theta_i^i$  are representing new variables achieved for master problems.

**Step 3:** Finally, optimal solution obtained when sum of sub-problem values become equal the master problem value *i.e.*

$$V(S_1) + V(S_2) + \dots + V(S_n) = V(M)$$

Here  $V(S_1), V(S_2), \dots, V(S_n)$  are representing values of sub-problems and  $V(M)$  represents value of the master problem. There are several types of decomposition techniques in use. Among them Dantzig-Wolfe decomposition, Benders decomposition [3] and Triangular decomposition are noteworthy. Ideas of DBP have used to develop our own technique for solving SP, which is the faster and latest decomposition technique. In the next section, DBP algorithm has discussed.

2.4. DBP algorithm

This procedure iteratively solves a relaxed sub-problem to identify potential entering basic columns [7]. The sub-problem is chosen to exploit special structure, rendering it is easy to solve. Mamer and McBride [12] developed DBP for multi-commodity flow problems. To demonstrate this algorithm the LP presented in Section 2.3 has considered.

**Step 1:** Set iteration  $k = 1$ . We use three alternative methods to pick an initial set of prices  $\lambda^k$ .

- (i) Start with  $\lambda^1 = 0$ . Or,
- (ii) Start with  $\lambda^1 > 0$  as the dual prices from the relaxed constraints of the LP relaxation. Or,
- (iii) Start with  $\lambda^1 > 0$  such that  $\text{Max}(C_i - \lambda^k A_i) > 0$ .

**Step 2:** Solve the sub-problem,

$$\begin{aligned} S(x, \lambda^k): \quad &\text{Maximize } C_i X_i - \lambda^k (A_i X_i - b) \\ &\text{subject to} \\ &B_i X_i \leq b_i \end{aligned}$$

$$X_i \geq 0$$

for  $X_i > 0, i = 1, \dots, n$  and put  $i$  in  $I^k$ .

**Step 3:** Solve the restricted master problem,

$$\mathbf{M}(X, k): \text{ Maximize } z = \sum_{i=1}^n C_i X_i$$

subject to

$$\sum_{i=1}^n A_i X_i \leq b$$

$$B_i X_i \leq b_i$$

$$X_i \geq 0$$

**Step 4:** For stopping criterion, we use two alternate methods.

- (i) Stop when the objective value of the sub-problem and the restricted master problem are equal i.e.  $v(S^k) = v(M^{k+1})$ . Else set  $k = k + 1$ , go to step-1.Or,
- (ii) Stop when no new variables come into the restricted master. Else go to step-2.

### 3. Chronicles of Solving SP

In the current section, some existing techniques for solving SP problems has discussed briefly.

#### 3.1. L-shaped Method

This method was developed by Slyke and Wets [16]. It was also developed by using a decomposition technique named Bender's decomposition [1]. It contains the following steps.

- It decomposes the whole problem into two stages first. The first stage problem leads to a master problem and the second stage problem leads to a sub-problem.
- The second stage objective function is approximated by using a piecewise linear convex function.
- Then approximation is developed iteratively, and convex function is typically represented in a master program using a cutting plane approximation.

One of the drawbacks of this method is that, when the number of scenarios is much too high it is quite difficult to solve the problem by using this method [10].

#### 3.2. Nested Decomposition Technique

It is a solution procedure of multi-stage SP problems. It just likes a recursive version of L-shaped method. In this procedure, a problem mainly solved sequentially and it can be represented by a scenario tree representation with the change of associated probability with time. It has discussed briefly in the following steps [13].

- Parent nodes send proposals for solutions to their children nodes.
- Child nodes send cuts to their parent nodes.
- There are different “sequence procedures” that tell in which order the problems corresponding to different nodes in the scenario tree are solved.

### 3.3. Stochastic Decomposition

This technique was developed by Hagle and Sen [8]. It specifically integrates elements of decomposition techniques and statistical approximation techniques. It consists of the following steps.

- It operates with an adaptive sample size.
- Unlike the L-shaped method, which solves a sub-problem for each scenario for each cutting plane constructed, it uses recursive approximation methods based on previously solved problems to bypass the solution of the vast majority of the sub-problems that would otherwise be solved [9].

In the next section, our developed real life oriented model by collecting sample data from a rural area of Bangladesh has been presented.

## 4. A Deterministic Model

A Bangladeshi farmer has 500 acres land and he wants to grow corn, wheat and beets. He grows and sells his crops by himself in the market. He wants to make a proper allocation of his land and maximizes his profit. He requires a minimum amount of wheat and corn to feed his livestock.

- ❖ The farmer requires 200 tons of wheat and 240 tons of corn to feed his livestock.
  - This can be grown on his land or bought from wholesaler.
  - Any production in excess of these amounts can be sold for TK.170/ton (wheat) and TK.150/ton (corn).
  - Any shortfall must be bought from wholesaler at a cost of TK.238/ton (wheat) and TK.210/ton (corn).
- ❖ Farmer can also grow beets as follows :
  - Beets sell at TK.36/ton for the first 6000 tons.
  - Due to economic quotas on beet production, beets in excess of 6000 tons can only be sold at TK.10/ton.

Required data of the problem has shown in following table.

**Table 1.** Data for real life oriented model

	Wheat	Corn	Beets
Yield(T/acre)	2.5	3	20
Planting cost (TK/acre)	150	230	260
Selling price	170	150	360 under 6000 T 10 above 6000T
Purchasing price	238	210	N/A
Minimum requirements	200	240	N/A

Data for the corresponding model has collected from “Pachuria” village of “Tangail” district of Bangladesh by our direct communication of year 2011.

#### 4.1. Optimal Solution of Deterministic model

In the current section, optimal solution of the deterministic model presented in the previous section has discussed. This model has solved by using DBP method [11]. Optimal solution with details formulation of this model has presented below.

##### Decision variables

$x_w$  = acres of land devoted to wheat

$x_c$  = acres of land devoted to corn

$x_b$  = acres of land devoted to beet

$y_w$  = Tons of wheat sold

$y_c$  = Tons of corn sold

$y_b$  = Tons of beets sold at favourable price

$y_a$  = Tons of beet sold at lower price

$z_w$  = Tons of wheat purchased

$z_c$  = Tons of corn purchased

##### LP formulation

**Maximize Profit  $z$  = Selling price – Planting cost – Purchasing cost**

$$= 170y_w + 150y_c + 360y_b + 10y_a - 150x_w - 230x_c - 260x_b - 238z_w - 210z_c$$

subject to

$$x_w + x_c + x_b \leq 500$$

$$2.5x_w + z_w - y_w \geq 200$$

$$3x_c + z_c - y_c \geq 240$$

$$20x_b - y_b - y_a \geq 0$$

$$y_b \leq 6000$$



$$x_w, x_c, x_b, y_w, y_c, y_b, z_w, z_c \geq 0$$

In the next table, we have presented optimal solution of the model.

**Table 2.** Optimal solution for deterministic problem

	Wheat	Com	Beet
Plant (acres)	120	80	300
Production(Tons)	300	240	6000
Sales (Tons)	100	0	6000
Purchase (Tons)	0	0	0
• Profit : Tk. 118600			

From Table 2, it can be observed that the farmer can make profit of TK.118,600 if he produced 300 tons of Wheat, 240 tons of Corn and 6000 tons of Beet and if he sales and devotes land for each crop according to the rate and distributions presented in this table.

In the next section, idea of deterministic model has extended by inserting uncertainty.

*4.2. Stochastic Extension of Deterministic Model*

Cultivation rate actually depends on weather. Due to the change of weather cultivation rate changes automatically and then profit changes dramatically. We have considered this uncertainty into our existing model. We considered three scenarios “good weather”, “normal weather”, and “bad weather”. Here “bad weather” means “lack of sunshine and more drought”, “normal weather” means “sunshine and lack of drought”, and “good weather” means “sunshine and no drought”. Corresponding data for these three scenarios of the stochastic formulation of deterministic model has presented in the following tables.

**Table 3.** Data for Stochastic formulation

	Wheat	Corn	Beets
Planting cost (TK/acre)	150	230	260
Selling price (TK/Tons)	170	150	36 under 6000 Tons 10 above 6000 Tons
Purchase price (TK/Tons)	238	210	N/A
Minimum requirement (Tons)	200	240	N/A

**Table 4.** Scenario based data

	Scenario 1 “Bad weather”	Scenario 2 “ Normal weather”	Scenario 3 “Good weather”
Probability	0.3	0.2	0.5
Wheat yield (Tons)	2.0	3.5	4.0
Corn yield (Tons)	3.5	3.7	3.8

Beets yield (Tons)	16	27	29
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Sources of data are same as the deterministic model stated in Section 4.

#### 4.3. Formulation of Stochastic Extension

In this section, the stochastic extension of the deterministic model has formulated. Different indices have introduced first and then different stochastic decision variables which have used in our work has discussed.

##### Indices

- $W$  , amount of wheat.
- $C$  , amount of corns.
- $b$  , amount of beat have to sell in favorable price.
- $e$  , amount of beat have to sell in lower price.
- $i$  , number of scenarios.

##### Stochastic decision variables

$x_{w,i}$  = acers of land devoted to wheat for scenario  $i = 1, 2, 3$ .

$x_{c,i}$  = acers of land devoted to corn for scenario  $i = 1, 2, 3$ .

$x_{b,i}$  = acers of land devoted to beet for scenario  $i = 1, 2, 3$ .

$y_{w,i}$  = Tons of wheat sold for scenario  $i = 1, 2, 3$ .

$y_{c,i}$  = Tons of corn sold for scenario  $i = 1, 2, 3$ .

$y_{b,i}$  = Tons of beet sold at favourable price for scenario  $i = 1, 2, 3$ .

$y_{e,i}$  = Tons of beet sold at lower price for scenario  $i = 1, 2, 3$ .

$z_{w,i}$  = Tons of weat purchased for scenario  $i = 1, 2, 3$ .

$z_{c,i}$  = Tons of corn purchased for scenario  $i = 1, 2, 3$ .

##### Stochastic formulation

Maximize

Profit

$$Z = 0.3(170y_1(w,1) + 150y_1(c,1) + 36y_1(b,1) + 10y_1(e,1) - 150x_1(w,1) - 230x_1(c,1) -$$

$$260x_1(b,1) - 238z_1(w,1) - 210z_1(c,1)) + 0.2(170y_1(w,2) + 150y_1(c,2) + 36y_1(b,2) + 10y_1(e,2)$$

$$- 150x_1(w,2) - 230x_1(c,2) - 260x_1(b,2) - 238z_1(w,2) - 210z_1(c,2)) + 0.5(170y_1(w,3) + 150y_1(c,3)$$

$$+36y_1(b,3) + 10y_1(o,3) - 150x_1(w,3) - 230x_1(c,3) - 260x_1(b,3) - 238z_1(w,3) - 210z_1(c,3))$$

subject to

$$\left. \begin{aligned} x_{w,1} + x_{c,1} + x_{b,1} &\leq 500 \\ 2.0x_{w,1} + z_{w,1} - y_{w,1} &\geq 200 \\ 3.5x_{c,1} + z_{c,1} - y_{c,1} &\geq 240 \\ 16x_{b,1} - y_{b,1} - y_{e,1} &\geq 0 \\ y_{b,1} &\leq 6000 \end{aligned} \right\} \leftarrow \text{Scenario 1}$$

$$\left. \begin{aligned} x_{w,2} + x_{c,2} + x_{b,2} &\leq 500 \\ 3.5x_{w,2} + z_{w,2} - y_{w,2} &\geq 200 \\ 3.7x_{c,2} + z_{c,2} - y_{c,2} &\geq 240 \\ 27x_{b,2} - y_{b,2} - y_{e,2} &\geq 0 \\ y_{b,2} &\leq 6000 \end{aligned} \right\} \leftarrow \text{Scenario 2}$$

$$\left. \begin{aligned} x_{w,3} + x_{c,3} + x_{b,3} &\leq 500 \\ 4.0x_{w,3} + z_{w,3} - y_{w,3} &\geq 200 \\ 3.8x_{c,3} + z_{c,3} - y_{c,3} &\geq 240 \\ 29x_{b,3} - y_{b,3} - y_{e,3} &\geq 0 \\ y_{b,3} &\leq 6000 \end{aligned} \right\} \leftarrow \text{Scenario 3}$$

$$x_{w,i} + x_{c,i} + x_{b,i} + y_{w,i} + y_{c,i} + y_{b,i} + y_{e,i} + z_{w,i} + z_{c,i} \geq 0, \text{ where } i = 1,2,3.$$

### 5. Algorithm of our Developed Method

In this section, algorithm of our developed method has presented. The algorithm has discussed by the following steps.

**Step 1:**  $N \leftarrow$  Set number of unknown decision variables.

**Step 2:**  $\sigma \leftarrow$  Set number of scenarios.

**Step 3:**  $P \leftarrow$  Set number of probabilities for corresponding scenario  $\sigma$ .

**Step 4:** Decompose the whole problem into sub-problem and a master problem.

**Step 5:** Apply idea of DBP to solve the decomposed problem (step 1 to 4 of Section 2.4).

### 5.1. Computer Code in AMPL

In this section, an AMPL [20] code has developed according to the algorithm developed in Section 5. This code consists of an (a) AMPL model file, (b) AMPL data file and (c) AMPL run file. Due to the limitation of pages code is not present here. But if the referees are interested then they can contact with the corresponding authors.

In the next section, optimal solution of the stochastic extension of deterministic model obtained by our developed code has presented.

## 6. Optimal Solution of Stochastic Extension of Deterministic Model

In the current section, optimal solution of the stochastic extension of deterministic problem has presented. The optimal solution has shown in Table 5.

**Table 5.** Optimal solutions for different scenarios

Scenarios		Wheat	Corn	Beets	Profit
Bad Weather	Plant (acres)	100	68.5714	331.429	TK. 22188
	Production(Tons)	200	239.9999	5302.864	
	Sales (Tons)	0	0	5302.86	
	Purchase (Tons)	0	0	0	
Normal weather	Plant (acres)	212.913	64.8649	222.222	TK. 40809.9
	Production (Tons)	745.1955	240.00012	5999.994	
	Sales (Tons)	545.195	0	6000	
	Purchase (Tons)	0	0	0	
Good weather	Plant (acres)	229.946	63.1579	206.897	TK.117776
	Production (Tons)	919.784	240.00002	6000.013	
	Sales (Tons)	719.782	0	6000	
	Purchase (Tons)	0	0	0	

From the above table, it can be observed that for bad weather at 100 acres land 200 tons wheat is produced and it requires approximately 69 acres and 332 acres land to produce 240 tons corns and 5302 tons beet respectively. Profit 22188 TK has been made for this scenario. But from the table, it can be observed that the production rate of corn and beat is batter when weather remains normal and good compared to bad weather. Production rate of wheat is also increased with the change of scenario. By making overall comparisons it is seen that highest profit has made when weather was good.

### 6.1. Profit Comparison

In this section, a comparison between the profits obtained for each scenario has presented. From the above discussions it can be observed that profit changes depending on an uncertain parameter

weather. From Table 5, it can be observed that maximum profit is TK. 117776 which is obtained for good weather and minimum profit is TK.22188 which is obtained for bad weather. Now the comparison between profits for each scenario has presented graphically below.

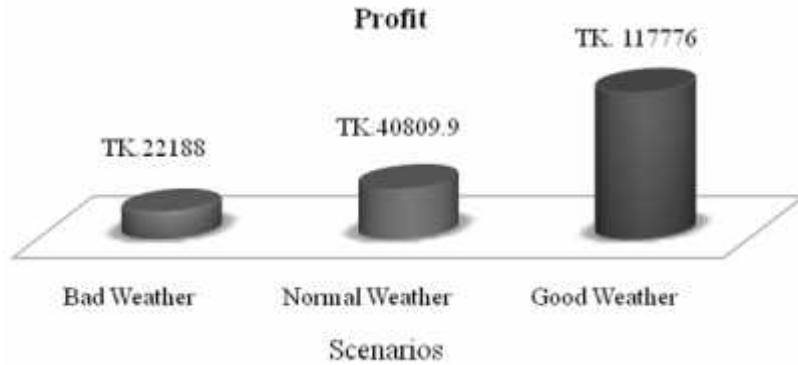


Fig. 1 Scenario based comparison between profits

6.2. Extension of Profit Comparison

In this section, comparison between profits for year 2012, 2013 and 2014 has presented. Due to the volume whole data sets are not presented here. If referees are interested please contact with authors via editor. Only optimal solution has presented in Table 6.

Table 6. Optimal solutions for different years

Scenarios	Year					
	2012		2013		2014	
	Probability	Profit	Probability	Profit	Probability	Profit
Bad Weather	0.4	TK. 8272	0.25	TK. 41054.3	0.25	TK.32200
Normal Weather	0.25	TK. 46116.6	0.3	TK.55339.9	0.4	TK.73786.5
Good Weather	0.35	TK.70525	0.45	TK.157770	0.35	TK.122710

From above table, it can be observed that highest profit is made at year 2013 when probability of occurring good weather was 0.45, which is the highest probability of occurring good weather comparing to other years. Sum of all probability in each year is always 1. Graphically this comparison has presented below.

In Fig.2, highest profit TK.157770 has indicated by red bar chart obtained for good weather. From the comparisons presented in Fig.2, we observe that smallest profit has obtained due to the bad weather.

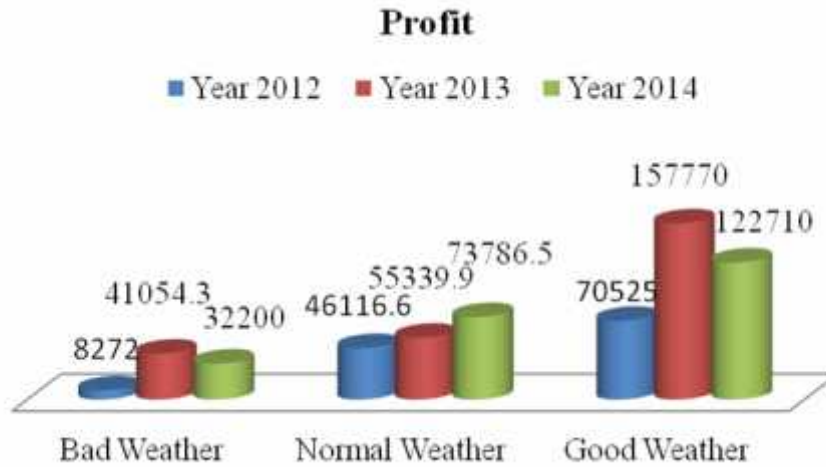


Fig.

2 Profit Comparisons for different years

Although it's depending on nature but it can make a serious problem for a farmer. They cannot stop it but they can make some plan to prevent it and can get remedy from making a loss. They can take help of some technologies like water pump to remove droughts, make some small channels in field to remove excess water caused by rain etc. In the next section, we a conclusion has drawn.

## 7. Conclusion

There are many existing techniques for solving SP problems. In most of the techniques Bender's decomposition method was used. In this paper, we developed a new technique for solving SP problems by using DBP method. Data were collected from a rural area of Bangladesh for several years and then a real life oriented model had developed. First the model was presented for deterministic case only and then the idea was extended by considering three different scenarios involving an uncertainty. The model then analyzed by our developed technique and then several graphical comparisons had presented. The stochastic model was developed by inserting scenarios in parameter related to cultivation only. This idea can be further extended to analyze the profit by considering uncertainty in the cost parameter or in the minimum requirement parameter. This idea can also be extended to apply in different areas of operations research. We hope that it will be a new attractable system for solving SP problems.

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