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A Study on Enhancing Efficiency of a Filter for Arsenic Removal

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ABSTRACT

Arsenic (*As*), a naturally occurring element found in rocks and in the Earth's crust, is one of the most dangerous things that can get into drinking water. A unique system for filtering arsenic has been developed that makes use of naturally available laterite soil. To scale this technology and realize its full potential, a deep understanding of the relationship between filter life and operating regime (inlet concentration of arsenic, filter size, and laterite material) is required. We aim to study a mathematical model that presents relationship between the concentration (c) and adsorption (Γ) with dimensionless time. Based on Langmuir kinetics, the model integrates intra-particle mass transport of the contaminant within the filter medium. The model predicts the dependency of the filter adsorption rate and outlet concentration on several dimensionless parameters. Filter efficiency for both treated laterite and raw laterite are also studied here.

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1 Introduction

Decontamination is the process of eliminating pollutants such as chemicals, bacteria, and radioactive substances from an object or region. Since there is no permanent source of fresh water that is safe from arsenic, people are still drinking water that is contaminated by arsenic. The consumption of safe drinking water is a requirement for a sustainable way of life.

Arsenic (*As*) is designated as the most significant chemical contaminant in drinking water globally [1] as well as a class I probable carcinogen by the International World Cancer Research Fund [2]. Bangladesh is one of the most affected countries in the world. As a result of increased usage of phosphate-rich fertilizer and other agricultural activities in Bangladesh, arsenic-binding bonds in diverse rocks have been broken down. Consequently, *As* is released into groundwater, creating a serious public health threat. Arsenic causes acute to chronic carcinogenic (a substance that causes cancer) impacts on human health, and is primarily received via water supply. Long-term exposure to elevated

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concentrations of *As* in drinking water can result in skin sores, muscle weakness, and brain problems. Arsenicosis, multi-organ diseases, poor pregnancy outcomes, and a lower IQ among children are all caused by *As* [1]. Arsenic in the water supply has been linked to an increased risk of diseases including skin, lung, bladder, liver, and kidney malignancies [3], neurological disease, coronary heart disease, and other non-malignant ailments [4].

Arsenic (*As*) is found in variable amounts in the air, soil, groundwater, valuable minerals and rocks, and bio-accumulation [5]. It is found in 200 mineral species [6], with arsenates accounting for 60%, sulfides and sulfosalts for 20%, and arsenides, arsenites, oxides, silicates, and elemental arsenic (*As*) accounting for the remaining 20% [7]. Natural processes like volcanic emissions, the burning of coal, bioactivity, and the erosion of arsenic-bearing rocks and minerals cause elevated levels of *As* in groundwater [8]. Natural weathering contributes about 40,000 tons [7] of *As* to the worldwide environment each year, whereas human activities release double that amount.

Drinking water sources, both underground and on the surface, are extremely vulnerable to contamination from physical, chemical, biological, or radioactive sources [9]. To remove the contaminants, numerous water-purification technologies are used around the world. The acceptable limit of arsenic in drinking water, according to WHO guidelines, is 10 μg/L [10]. Even though Bangladesh considers a higher acceptable limit, arsenic-contaminated tube wells contain high levels of arsenic [11]. To efficiently address this issue, a water-purification technology that meets safe drinking-water criteria while using minimal energy and providing high throughput is required. Using readily available laterite soil, some can remove arsenic from water [12].

A significant amount of research around the world has been done on the removal of arsenic from drinking water. Recently, Mondal et al. [10] proposed a mathematical model that describes the elimination of arsenic from drinking water, where they discussed a way to predict how long a filter will last and how that depends on the parameters of the filter. Abhijit et al. [13] presented a work that uses abundantly available laterite to build a low-cost and efficient porous adsorbent. A hypothesis was proposed by Maji et al. [14] that describes several processes involved in arsenic removal from groundwater by adsorption. S. Mondal [15] conducted a study on laterite-based technology for reducing arsenic and evaluated its impact, Whereas Borsi [16] illustrated how hollow-fibre membrane filters work for filtering contaminated water. JY Hahn [17] did research to find a way to use visual analysis to figure out the amount of carbon in experimental filters, as well as the flowrate and the amount of contaminants that were removed. In a stirred tank adsorber, the adsorption kinetics of arsenite and arsenate species are predicted using a simple shrinking core model, which was proposed by Maiti [18]. Bohaienko et al. presented a model of the two-dimensional initial-boundary value problem of convective diffusion in the context of vertical steady-state groundwater filtration [19]. In order to remove *As*(*V*) from water, Usman et al. evaluated the possible performance of a particular adsorbent material in a filter and proposed a mathematical model to represent overall performance [20]. Bretzler et al. demonstrated that if specific parameters are well controlled, nail-based filters have the capacity to remove more than 90% of arsenic in a field setting [21]. In this study, we have developed a mathematical model for filtration that will address the decontamination of arsenic. Both raw and treated laterite will be considered for decontamination, and how different sizes affect the filter life will be investigated. To the best of our knowledge, it has not been studied before.

2 Methodology

Using porous water filters to remove arsenic ions is an example of a classic transportation problem. At the pore scale, this problem is modeled using the traditional convection-diffusion equation. It shows how very few species move through the water (tracers). The volume-averaging method is used to scale up the phenomenon of solute transport in homogeneous porous media, which includes both diffusion and advection [22]. Some advanced numerical schemes for reaction-diffusion equations are investigated by Lima et al. [23], Ali et al. [24]. In this study, arsenic adsorption in porous media is determined by the maximum amount of adsorption by laterite that can adsorb arsenic. Laterite is a layer of soil that is high in iron oxide and comes from a wide range of rocks that have broken down under conditions that are highly oxidizing and leaching. It grows in tropical and subtropical places with humid weather. Lateritic soils may have clay minerals, but they usually don't have much silica because silica is lost when water flows through the soil. Laterite is usually porous and like clay. It is made up of the iron oxide minerals goethite ($HFeO_2$), lepidocrocite (FeO(OH)), and hematite (Fe_2O_3). It also has titanium oxides and hydrated aluminum oxides, the most common of which is gibbsite, which has the formula $Al_2O_3.3H_2O$ [25]. Goethite, hematite, quartz, and other clay minerals make up the majority of the minerals that make up laterite, which is a solidified product of humid tropical weathering [26]. Arsenic from a fluid flow tainted with arsenic can accumulate in the many spaces and pores of laterite. We employ two types of laterite attributes in our model: treated laterite and raw laterite.

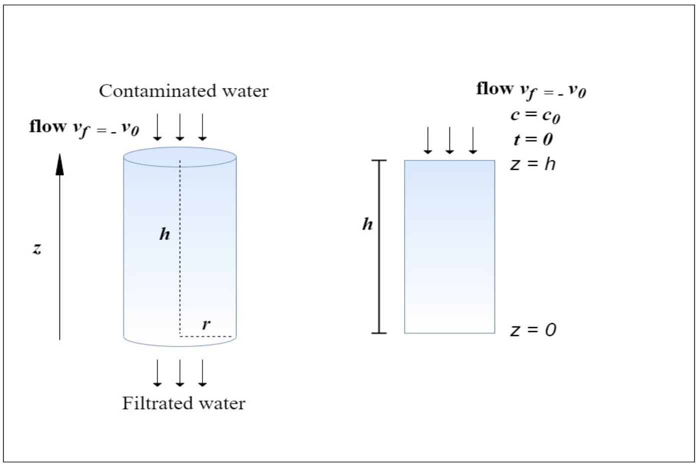


Figure 2.1: Schematic of a cylindrical-shaped filter. A cylinder of radius r and height h is filled with laterite material.

A typical filter consists of a cylindrical container filled with laterite soil. The fluid enters the filter from the top of the cylinder and is evenly distributed. As a way to model the column, we employ a (r, θ, z) cylindrical coordinate system. Using a simple cylinder model, we can visualize the filter's internal workings. The filter is modeled as a cylinder with a diameter of r and a height of h, as shown in Figure 2.1. The cylinder's axis is aligned with the z-axis. The filter's inlet is at $z = h$, and its outlet is at $z = 0$. At the inlet, we maintain a steady flow independent of the direction θ given by,

$$v_f(r;\theta;h) = -v_0$$

Here we consider two terms that are part of our model: the fluid that flows through the porous medium as well as the transportation of the contaminant. The fluid flow is stationary, but the transportation of contaminants through the porous medium changes over time. When a porous material is saturated, its porosity is defined as the percentage of its total volume occupied by pores or voids, and thus the volume of liquid that fills it. It is a nonstationary problem to deal with the transport of solutes, as they enter the flow at a certain time and location and are followed in their spatial dispersion through time [27].

2.1 Governing Equations

An incompressible fluid in a porous medium can be described using the Darcy model. For low-velocity flows, this is a generally accepted approximation of the filtration problem, where inertial effects can be ignored. We start with Darcy's Law for the fluid flow through a porous medium, given by

$$\phi_f(v_f - v_s) = -\frac{k}{\mu}\nabla p, \tag{2.1}$$

where ϕ_f denotes local porosity or fluid fraction, v_f is the fluid velocity, v_s is the solid velocity, k is the permeability of the solid, μ is the dynamic fluid viscosity and p is the fluid pressure. Here solid velocity $v_s = 0$.

Initially, we model the transportation of the contaminant via a simple advection-diffusion equation

$$\phi_f\left(\frac{\partial c}{\partial t}+v_f.\nabla c\right)=D\nabla^2 c-\rho_s\frac{\partial \Gamma}{\partial t}, \tag{2.2}$$

where c is the mass concentration of the contaminant, D is the diffusion coefficient in the porous medium, ρ_s denotes the density of the solid that adsorbs the contaminant, ϕ_f denotes the porosity of the laterite, v_f is the inlet fluid velocity, Γ is the amount of arsenic adsorbed to the solid. In general, adsorbent removal can be described by the Langmuir removal equation as

$$\frac{\partial \Gamma}{\partial t}=K^+c(\Gamma_{\max}-\Gamma)-K^-\Gamma, \tag{2.3}$$

where K^- is the backward rate constant. Because adsorption happens in a monolayer, Langmuir assumes that molecules competing for adsorption have no interaction and that all adsorption sites are equal. Sorption is likewise supposed to be a simple process. Here desorption is negligible, i.e., in an adsorption dominated regime, and

$$\frac{\partial \Gamma}{\partial t}=K^+c(\Gamma_{\max}-\Gamma), \tag{2.4}$$

controls the removal of contaminants from the fluid such that Γ is the amount of contaminant adsorbed to the solid, Γ_{max} is the maximum amount of contaminant that the solid can adsorb (i.e., the capacity of the solid for the given contaminant), and K^+ is the 'forward rate constant' such that $\rho_s K^+$ is the adsorption rate per second.

Initial and Boundary Conditions: We consider a steady flow at the entrance that does not depend on r or θ given by,

$$v_f(r;\theta;h)=-v_0$$

We enforce a non-penetration boundary condition at the cylinder boundary at $r = a$, such that $\frac{\partial v_f(a,\theta,z)}{\partial r} = 0$. Also, we take a no shear boundary condition on the cylinder edge, and as a result, we have $v_f = v_0\tilde{e_z}$ for all z, where $\tilde{e}$ is the unit axial vector. By using Darcy's law, we implicitly assume no shear boundary condition. Thus, from Darcy's law, we find that the flow induces a pressure difference ∇p across the cylinder given by

$$\nabla p=\frac{\phi_f \mu v_0 h}{k}$$

(i) In the beginning, we'll suppose that the filter is at full capacity- that is, $\Gamma = 0$ at $t = 0$ for all r, θ and z.

(ii) At the inlet, the concentration of the contaminant is constant, such that $c = c_0$ at $z = h$ for all r, θ and t.

(iii) At the outlet, we enforce a passive condition that says the concentration of the contaminant won't change at the bottom of the filter. This is given by,

$\frac{\partial c}{\partial z} = 0$ at $z = 0$ for all r, θ and t

2.2 Non-dimensionalisation

In an adsorption-dominated regime, we non-dimensionalize via

$$v_f = v_0 \tilde{v}_f, \quad z = h\tilde{z}, \quad \Gamma = \Gamma_{\max}\tilde{\Gamma}, \quad c = c_0\tilde{c}, \quad p = \left(\frac{h\mu v_0 \phi_f}{k}\right)\tilde{p} \quad \text{and,} \quad t = \frac{\tilde{t}}{c_0 K^+}$$

where *z* signifies spatial coordinates and variables denoted by a tilde are dimensionless. We are mostly interested in the filter's lifetime, which is determined by how close the material is to its maximum capacity (i.e., how close Γ is to Γ_{max}), as such, we have used Eq. (2.4) to determine the long-term timescale for *t*. Thus Eqs. (2.2) and (2.4) reduce to

$$\alpha\frac{\partial \tilde{c}}{\partial \tilde{t}} + \beta\tilde{v}_f . \tilde{\nabla}\tilde{c} = \gamma\tilde{\nabla}^2 \tilde{c} - \frac{\partial\tilde{\Gamma}}{\partial\tilde{t}}, \tag{2.5}$$

$$\frac{\partial\tilde{\Gamma}}{\partial\tilde{t}} = (1-\tilde{\Gamma})\tilde{c} - \delta\tilde{\Gamma}, \tag{2.6}$$

where the dimensionless parameters *α*, *β*, *γ* and *δ* are given by

$$\alpha = \frac{c_0\phi_f}{\rho_s\Gamma_{\max}}, \qquad \beta = \frac{v_0\phi_f}{h\rho_s\Gamma_{\max}K^+}, \qquad \gamma = \frac{D}{h^2\rho_s\Gamma_{\max}K^+}, \qquad \delta = \frac{K^-}{K^+c_0}$$

We work primarily in dimensionless quantities, so tildes are dropped for simplicity. It's worth noting that we use a constant fluid flow profile so that $v_f = 0\hat{\imath} + 0\hat{\jmath} - 1\hat{k}$. Hence, for the given boundary and initial conditions, we find Eqs. (2.5) and (2.6) reduce to a coupled partial differential equation (PDE) for *c* and Γ in terms of *z* and *t* given by,

$$\alpha\frac{\partial c}{\partial t} - \beta\frac{\partial c}{\partial z} = \gamma\frac{\partial^2 c}{\partial z^2} - \frac{\partial\Gamma}{\partial t} \tag{2.7}$$

$$\frac{\partial\Gamma}{\partial t} = (1-\Gamma)c - \delta\Gamma \tag{2.8}$$

with the initial and boundary conditions are,

$$\begin{aligned} &\Gamma = 0 \quad at \quad t = 0 \qquad \forall\, \mathrm{r}, \theta \text{ and z}, \\ &c = c_0 \quad at \quad z = h \qquad \forall\, r, \theta \ and\ t, \\ &\frac{\partial \mathrm{c}}{\partial \mathrm{z}} = 0 \ at\ z = 0 \qquad \forall\, r, \theta \ and\ t, \end{aligned} \tag{2.9}$$

To remove arsenic from contaminated water, we make a filter out of particles of treated laterite and raw laterite. The parameter values for treated laterite and raw laterite for the fluid with contaminants are given in Table-2.1 [15], [12], [10].

Table 2.1: The values of system parameters

Properties of the adsorbent	Treated laterite	Raw laterite
Maximum adsorption capacity, Γ_{max} (kg/kg)	0.0128	1.4×10^{-4}
Forward rate constant, K^{+} ($m^3/kg.s$)	1.44×10^{-5}	2.21×10^{-2}
Backward rate constant, K^{-} (1/s)	1.23×10^{-6}	3.12×10^{-6}
Average density of adsorbent, ρ_s (kg/m^3)	1050	1325
Inlet concentration of As, c_0 (g/L)	80×10^{-6}	80×10^{-6}
Linear inflow rate, v_0 (m/s)	3.7×10^{-5}	3.7×10^{-5}
Average bed porosity, φ_f	0.32	0.48
Diffusivity of As through the medium, D (m^2/s)	2.7×10^{-11}	3.7×10^{-10}

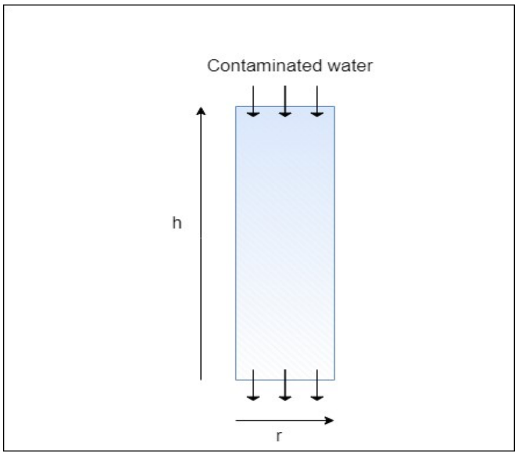
Using the above values, we estimate the dimensionless parameter values for both treated and raw laterite that are given in Tables- 3.1 and 3.2.

3 Results and Discussion

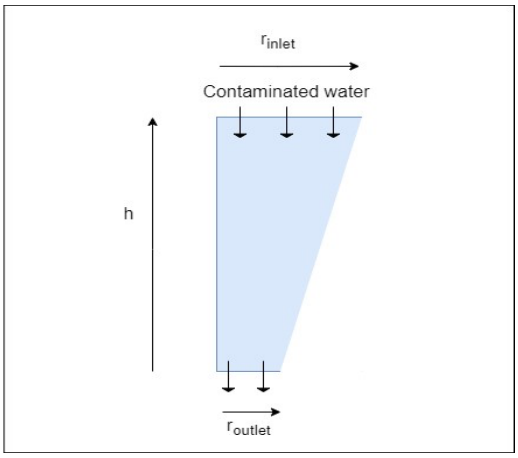
We solve the system of Eqs. (2.7) and (2.8) along with the initial and boundary conditions (2.9) using the finite element method based PDE software COMSOL Multiphysics 5.5.

To build our model, we combine two physics interfaces called coefficient form PDE to define the concentration of arsenic and the amount of arsenic adsorption. After choosing the physics, we set up the geometry of the domain. Here, we set up our model in terms of 2D geometry. We adjust the built-in equations and apply the boundary conditions according to our governing equations. Then, we pick a mesh that works for the domain. In our case, we have chosen free triangular meshing with some fine-tuning (to adjust precisely to get the effectiveness) around the edges and the ”extra fine” setting in COMSOL. We choose Lagrangian shape functions with quadratic elements as the basis functions or shape functions for both the advection-diffusion equation and the Langmuir equation. For Lagrange elements, the values of all the variables at the nodes are called degrees of freedom (dof). In this case, our specific discretization leads to a minimum of 11694 degrees of freedom (N_{dof}) (plus 896 internal DOFs) for regular rectangular shaped geometry and 13176 DOFs (plus 908 internal DOFs) for cone shaped geometry.

We present our results in graphs. To compute our results, we consider two laterite materials and two different geometric shapes with different heights.



(a) Regular rectangular shaped filter



(b) Cone shaped filter with $r_{inlet} > r_{outlet}$

Figure 3.1: Two different geometries

3.1 Treated laterite

At first, we considered treated laterite as a laterite material for the filter. We have plotted the concentration (c) and amount of arsenic adsorption profiles (Γ) for both geometry and different heights. The β values for various inlet velocities and heights for treated laterite are shown in Table 3.1. Table 3.2 presents the dimensionless parameter values for heights $h = 0.1\ m,\ 0.2\ m,\ 0.3\ m$ with $v_0 = 3.7014\times10^{-5}\ m/s$.

Table 3.1: β values for treated laterite for different inlet velocities

$v_0\ (m/s)$	$\beta\ (h = 0.1)$	$\beta\ (h = 0.2)$	$\beta\ (h = 0.3)$
4.3546×10^{-5}	0.72	0.36	0.24
3.7014×10^{-5}	0.612	0.306	0.204
3.1450×10^{-5}	0.52	0.26	0.173
2.9514×10^{-5}	0.488	0.244	0.163
1.4757×10^{-5}	0.244	0.122	0.0813
8.2253×10^{-6}	0.136	0.068	0.045

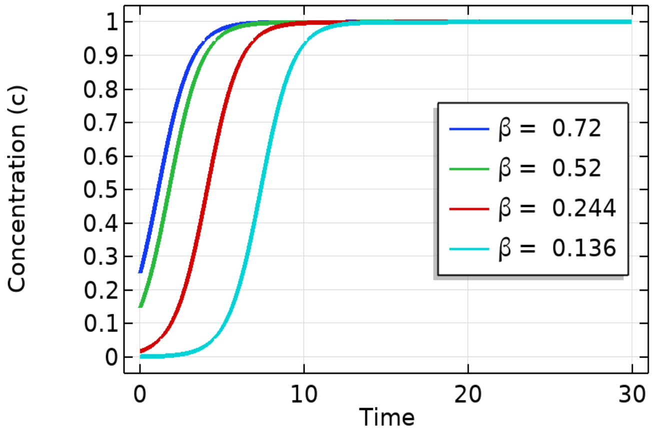
Table 3.2: Dimensionless parameter values for treated laterite

h (m)	α	β	γ	δ
0.1	1.9048×10^{-6}	0.612	1.395×10^{-5}	1.0677×10^{3}
0.2	1.9048×10^{-6}	0.306	3.4877×10^{-6}	1.0677×10^{3}
0.3	1.9048×10^{-6}	0.204	1.550×10^{-6}	1.0677×10^{3}

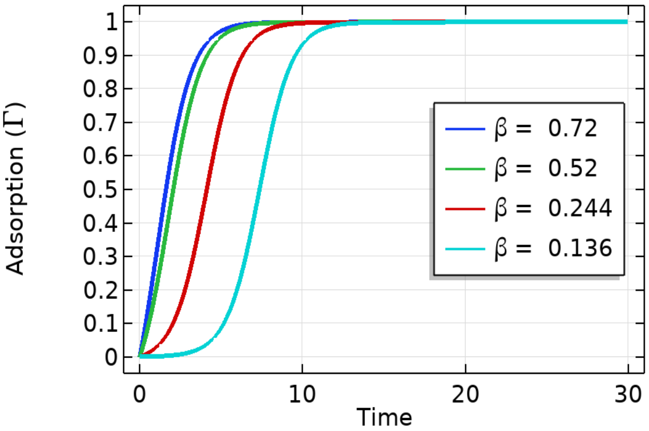
Figure 3.2 shows the effect of changing the heights of regular rectangular shaped filter made of treated laterite. When we examine the concentration profile 3.2(a) for height $h = 0.1$ m closely, we can see that for higher β values, the concentration profile begins above the point 0.2 in the y axis, whereas for decreasing β values, the starting point of the profile also decreases. For $\beta = 0.136$, the concentration profile starts nearly from zero.

However, in the adsorption profile 3.2(b), all graphs are starting from zero. Due to a higher β value, the profile initially rises and after a certain period of time it reaches its maximum value. When we decrease the β value, it takes longer time for both concentration and adsorption profile to reach the maximum value, $c = 1$ and $\Gamma = 1$.

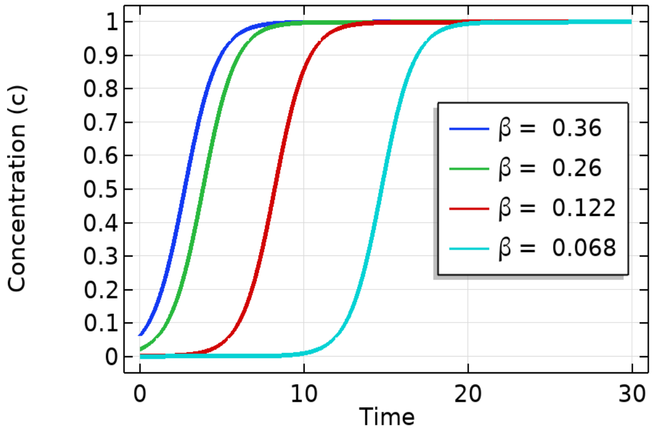
For height $h = 0.2$m, the concentration and adsorption profiles are depicted in Figure 3.2(c) and 3.2(d), respectively. We can also see that when β values decrease, it takes longer to reach the maximum point.



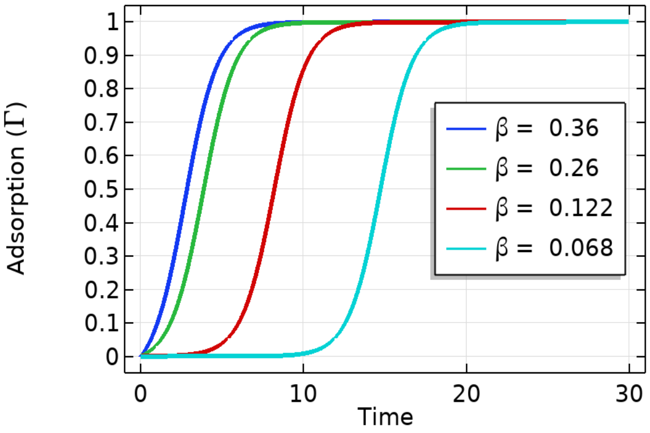
(a) Concentration (c) with $h =$ 0.1m



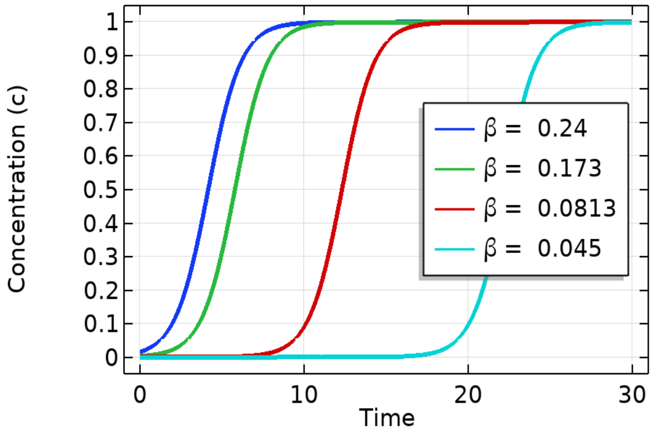
(b) Adsorption (Γ) with $h =$ 0.1m



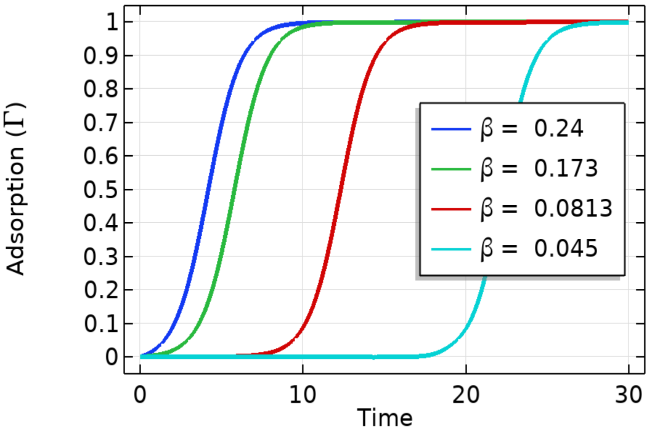
(c) Concentration (c) with $h =$ 0.2m



(d) Adsorption (Γ) with $h =$ 0.2m



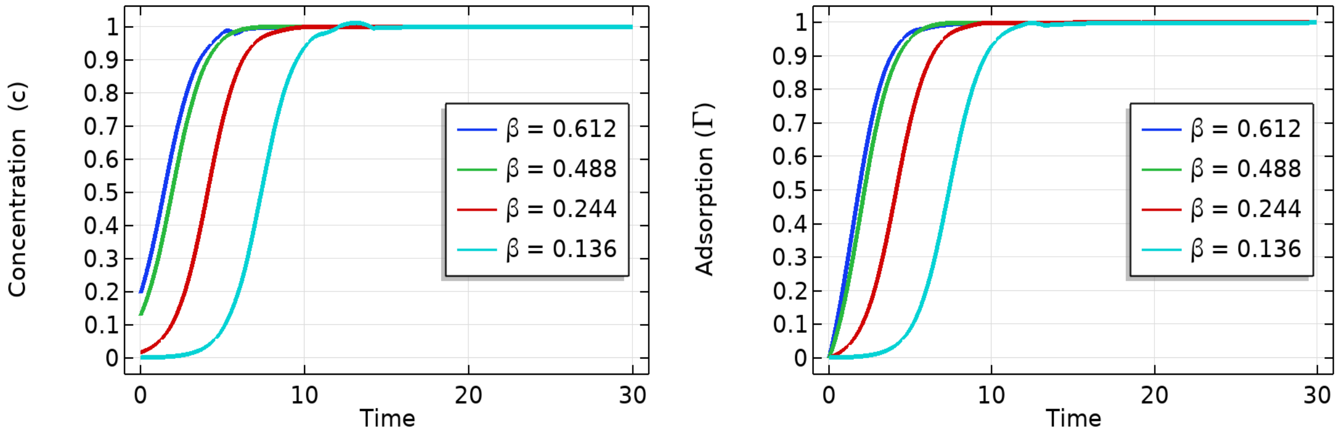
(e) Concentration (c) with $h =$ 0.3m



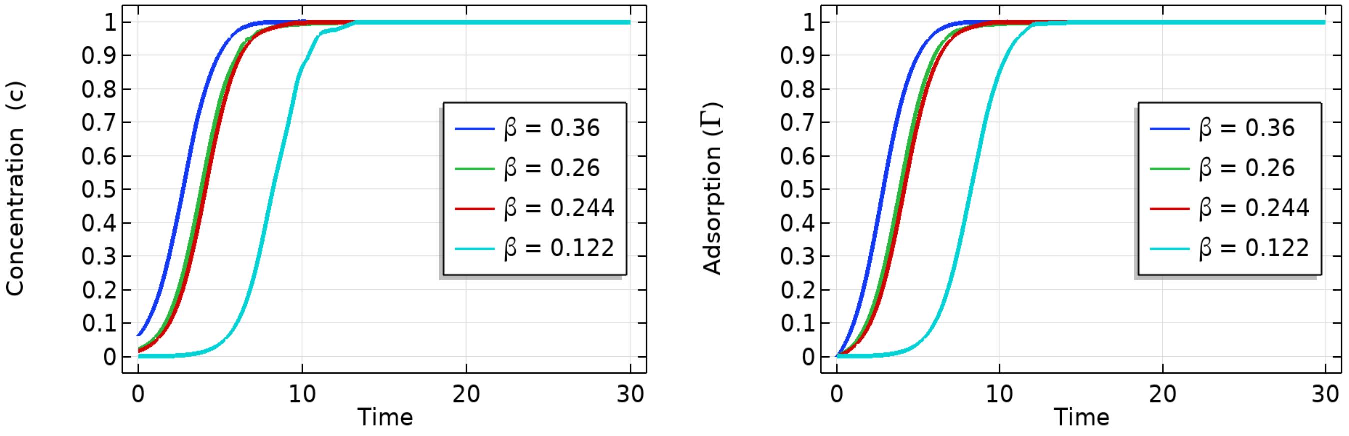
(f) Adsorption (Γ) with $h =$ 0.3m

Figure 3.2: Outlet concentration (c) and amount of arsenic adsorption (Γ) against dimensionless time for different β values with $\alpha = \delta = 0$ for different heights of the regular rectangular shaped filter with treated laterite

Figures 3.2(e) and 3.2(f) present the results for height $h = 0.3$m. As β value is inversely proportional with height, we get lower values for height $h = 0.3$m. When we look at the starting points of the graphs in 3.2(e), we can see that almost all profiles begin close to zero. All profiles for height $h = 0.3$m require more time to get their maximum value. As a result, if we compare the concentration and adsorption profiles for heights $h = 0.1$m, 0.2m and 0.3m, we can conclude that the time needed to reach the highest value ($c = 1$ and$\Gamma = 1$), increases as the height of a regular rectangular shaped geometry increases.



(a) Concentration (c) with $h = 0.1$m (b) Adsorption (Γ) with $h = 0.1$m



(c) Concentration (c) with $h = 0.2$m (d) Adsorption (Γ) with $h = 0.2$m

Figure 3.3: Outlet concentration (c) and amount of arsenic adsorption (Γ) against dimensionless time for different β values with $\alpha = \delta = 0$ for different heights of the cone shaped filter ($r_{inlet} > r_{outlet}$) with treated laterite.

Next we plot our results for cone shaped filter with $r_{inlet} > r_{outlet}$ in Figure 3.3. After examining the graphs 3.3(a) and 3.3(b) for height $h = 0.1$m, we conclude that concentration profiles begin far from zero for higher β values. When β value decreases, the profile starts nearly from zero and takes longer to reach the value $c = 1$ and $\Gamma = 1$.

The concentration profile for small β value starts close to zero and after a time period, it quickly increases to reach the maximum value, as shown in Figures 3.3(c) and 3.3(d). As β value decreases, the profile shifts from left to right gradually, and for lower β value, it takes longer time to hit the maximum value $c = 1$ and $\Gamma = 1$.

If we take a look at all profiles of both geometry (Figures 3.2 and 3.3), we can conclude that decreasing the β values, which means decreasing the filter's inlet velocity, increases the time required to obtain the maximum c and Γ values. The lifespan of a filter with a lower inlet velocity can be increased by increasing the filter's height.

3.2 Raw laterite

Next, we consider raw laterite as a decontaminant into the filter which is a layer of soil that is high in iron oxide and comes from a wide range of rocks that have broken down under conditions that are highly oxidizing and leaching. We have plotted concentration (c) and amount of arsenic adsorption (Γ) for height $h = 0.1m, 0.2m$. Table 3.3 presents the dimensionless parameter values for height $h = 0.1m, 0.2m$ for raw laterite which are evaluated using the parameters values for raw laterite from Table 2.1 for different h.

Table 3.3: Dimensionless parameter values for raw laterite

h (m)	α	B	γ	δ
0.1	2.0701×10^{-4}	0.043	9.03×10^{-6}	1.7647
0.2	2.0701×10^{-4}	0.02	2.2563×10^{-6}	1.7647

For raw laterite, we get different β with respect to different inlet velocity (v_0) and we get different δ values with respect to different inlet concentration (c_0). We have shown the values in Table 3.4.

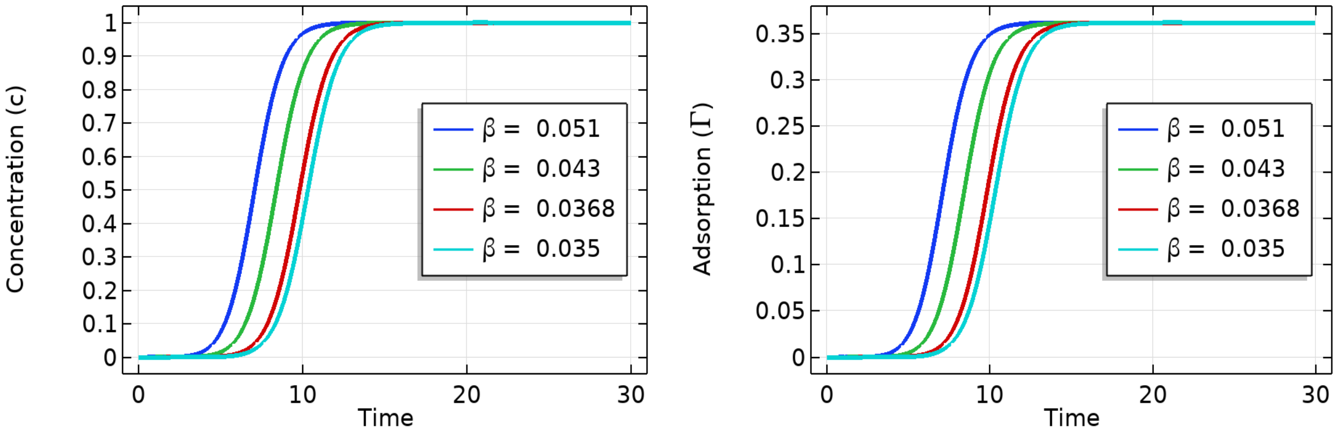
Table 3.4: Values of β and δ for various v_0 and c_0

v_0 (m/s)	c_0 ($\mu g/L$)	β ($h = 0.1$)	β ($h = 0.2$)	δ
4.3546×10^{-5}	80	0.051	0.026	1.7647
3.7014×10^{-5}	85	0.043	0.022	1.6551
3.1450×10^{-5}	93.1	0.0368	0.018	1.5164
2.9514×10^{-5}	99.7	0.035	0.017	1.4160

3.2.1 Concentration and adsorption profiles for different beta (β)

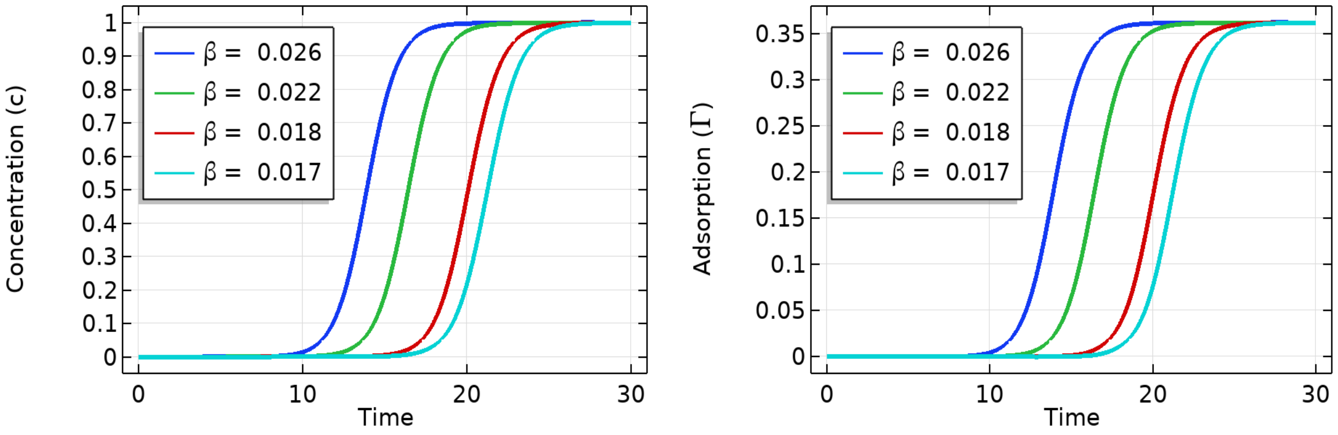
In Figure 3.4, the impact of changing the height of filter with raw laterite is shown for various β values with $\delta = 1.7647$. If we take a look at Figures 3.4 (a) and 3.4(b), we can see that the β values are comparatively much smaller than for treated laterite, and all concentration profiles start from zero. Initially, all profiles run very slowly, but after a certain period of time, the graphs are accelerated sharply and reach their highest value. For height $h = 0.1$m, decreasing β value increases the time required to reach the maximum value of $c = 1$ and $\Gamma \approx 0.36$. It is to be noted that, for raw laterite, the maximum amount of arsenic adsorption is $\Gamma \approx 0.36$ whereas for treated laterite, it was $\Gamma = 1$.

The concentration and adsorption profiles for four different β values for a filter of height $h = 0.2m$ are shown in Figures 3.4(c) and 3.4(d). It is quite evident that as the height of the filter is increased, the time needed to reach the maximum of c and Γ also increases.



(a) Concentration (c) with h= $0.1m$

(b) Adsorption (Γ) with h= $0.1m$



(c) Concentration (c) with h= $0.2m$

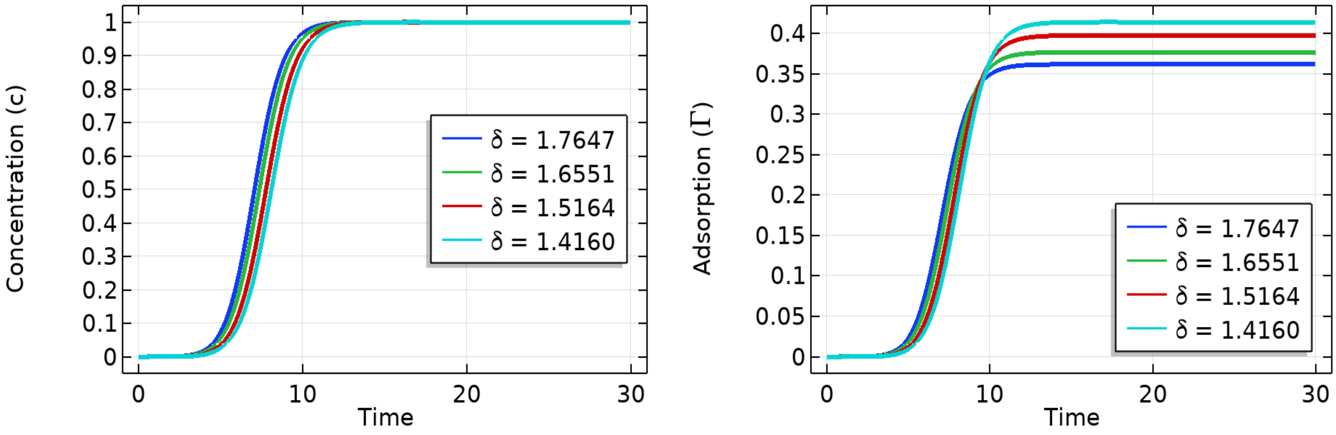
(d) Adsorption (Γ) with h= $0.2m$

Figure 3.4: Outlet concentration (c) and amount of arsenic adsorption (Γ) against dimensionless time for different β values with $\delta = 1.7647$ for a regular rectangular shaped filter with raw laterite

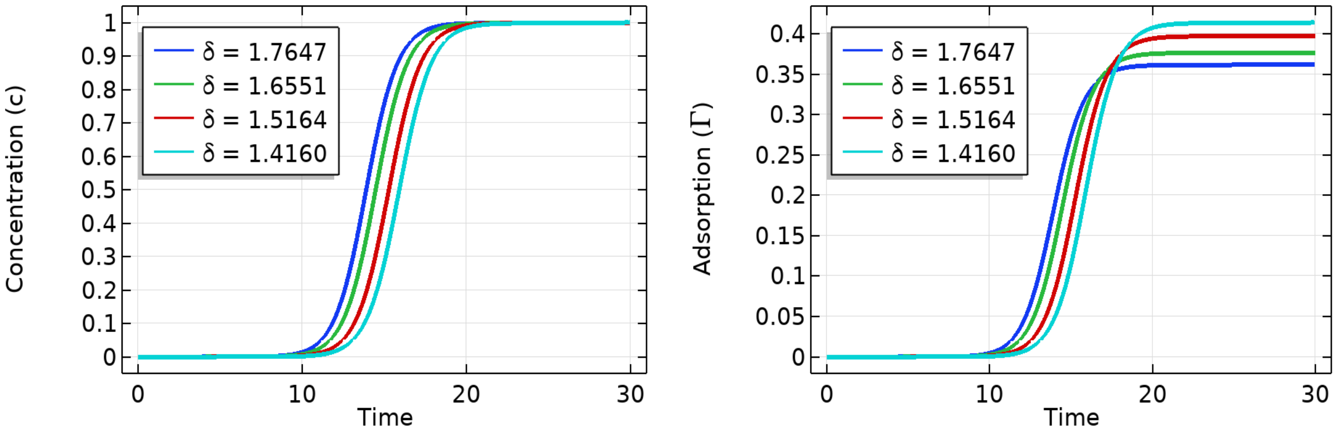
3.2.2 Concentration and adsorption profiles for different delta (δ)

In Figure 3.5 the concentration and adsorption profiles are depicted for various δ values. It should be mentioned that, for both $h = 0.1$m and $h = 0.2$m, we have considered the same set of δ variation. In our work, we use $\beta = 0.052$ for $h = 0.1$m and $\beta = 0.026$ or $h = 0.2$m. The variation of δ implies the variation of inlet concentration c_0. The values of δ increases as c_0 decreases, as shown in Table 3.4. From the Figures 3.5(a) and 3.5(b), we observe that the effects on the profiles due to changes in the δ values are comparatively less than due to β variation. All of these profiles start slowly from a starting point that is close to zero. Following that, the graphs quickly ascend until they reach their highest c and Γ values. The maximum amount of arsenic adsorption is reduced when the δ value is increased, as shown in Figure 3.5(b).

Figures 3.5(c) and 3.5(d) presents the outcome with same set of δ values for height $h = 0.2$m. If we compare these graphs with Figures 3.5(a) and 3.5(b), then we can say that takes longer to reach the maximum values.



(a) Concentration (c) with h= 0.1m (b) Adsorption (Γ) with h= 0.1m



(c) Concentration (c) with h= 0.2m (d) Adsorption (Γ) with h= 0.2m

Figure 3.5: Outlet concentration (c) and amount of arsenic adsorption (Γ) against dimensionless time for different δ values. Considering $\beta = 0.052$ for $h = 0.1$m and $\beta = 0.026$ for $h = 0.2$m

For both β variation and δ variation, we found that the adsorption profiles for raw laterite cannot exceed the value 0.5. Thus raw laterite has less than half the amount of arsenic adsorption capacity than the treated laterite.

4 Conclusion

Finding an efficient filter for arsenic decontamination is one of the biggest challenges of the developing countries. There is no complete decontamination approach until its long-term behavior and safe operating lifetime are determined. In this study, we have taken an attempt to construct a model based on Langmuir kinetics that can be used to predict the behavior of concentration and adsorption rate at the outlet of filter. The effectiveness of this model in predicting system performance under various conditions, including inlet fluid velocity which is used for calculating β values, contaminant concentration for different δ values, and filter height, has been demonstrated.

- For a specific height of the filter as β decreases, the outlet concentration, c and adsorption rate, Γ takes longer to reach $c = 1$ and $\Gamma = 1$ in case of treated laterite.

- In case of raw laterite, for a fixed height of the filter, as β decreases, the outlet concentration c and adsorption rate Γ takes longer to reach $c = 1$ and the maximum value Γ_{max} respectively.

- For raw laterite, as δ value decreases, outlet concentration and adsorption take longer to reach $c = 1$ and the maximum value of Γ which is around 0.36.

- For a specific geometry, as h increases, it takes longer for both c and Γ to reach the maximum value.

- The profiles for Γ indicate that the treated laterite is more efficient than raw laterite as treated laterite has strong adsorption capacity.

Therefore, we can conclude that this model aids in our understanding of the behavior of arsenic transportation and adsorption within filter media. We have shown how the model can predict the dependency of the filter lifetime on the core parameters. As our governing equations are only z dependent, the filter shape has no significant impact on the concentration and adsorption profiles. It will be useful to consider two-dimensional governing equation in which the transportation of contaminants is affected by both radius, r and height, z. A dual-layer bed of both treated laterite and raw laterite in a filter might also be an interesting idea for future study. We hope that the findings of this study will be used in the development and implementation of arsenic filters in the future.

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Data Availability Statement

The data that supports the findings of this study are generated by the authors and they are available within the article. The data set is also available on request from the corresponding author.

References

[1] WHO Report (2018). Arsenic, health effects. https://www.who.int/news-room/ fact-sheets/detail/arsenic.

[2] IARC (2004). *Some drinking-water disinfectants and contaminants, including arsenic*, volume 84. IARC.

[3] Hopenhayn-Rich, C., Biggs, M. L., and Smith, A. H. (1998). Lung and kidney cancer mortality associated with arsenic in drinking water in cordoba, argentina. *International Journal of Epidemiology*, 27(4):561–569.

[4] Argos, M., Ahsan, H., and Graziano, J. H. (2012). Arsenic and human health: epidemiologic progress and public health implications. *Reviews on environmental health*, 27(4):191–195.

[5] Balakumar, P. and Kaur, J. (2009). Arsenic exposure and cardiovascular disorders: an overview. *Cardiovascular toxicology*, 9(4):169–176.

[6] Beck, R., Styblo, M., and Sethupathy, P. (2017). Arsenic exposure and type 2 diabetes: micrornas as mechanistic links? *Current diabetes reports*, 17(3):1–10.

[7] Chowdhury, M. and Islam, R. (2015). *Removal of Arsenic from Contaminated Water by Granular Activated Carbon Embedded with Nano scale Zero-valent Iron.* PhD thesis, Concordia University.

[8] Society for Mining, Metallurgy and Exploration, Inc. (2015). The role of arsenic in the mining industry. https://me.smenet.org/docs/Publications/ME/Issue/ TheRoleofArsenicintheMiningIndustry.pdf.

[9] Tran, H. N., You, S.-J., Hosseini-Bandegharaei, A., and Chao, H.-P. (2017). Mistakes and inconsistencies regarding adsorption of contaminants from aqueous solutions: a critical review. *Water research*, 120:88–116.

[10] Mondal, R., Mondal, S., Kurada, K. V., Bhattacharjee, S., Sengupta, S., Mondal, M., Karmakar, S., De, S., and Griffiths, I. M. (2019). Modelling the transport and adsorption dynamics of arsenic in a soil bed filter. *Chemical Engineering Science*, 210:115205.

[11] Ahmad, S. A., Khan, M. H., and Haque, M. (2018). Arsenic contamination in groundwater in bangladesh: implications and challenges for healthcare policy. *Risk management and healthcare policy*, 11:251.

[12] Maiti, A., DasGupta, S., Basu, J. K., and De, S. (2008). Batch and column study: adsorption of arsenate using untreated laterite as adsorbent. *Industrial & engineering chemistry research*, 47(5):1620–1629.

[13] Maiti, A., Basu, J. K., and De, S. (2012). Experimental and kinetic modeling of as (v) and as (iii) adsorption on treated laterite using synthetic and contaminated groundwater: Effects of phosphate, silicate and carbonate ions. *Chemical Engineering Journal*, 191:1– 12.

[14] Maji, S. K., Pal, A., Pal, T., and Adak, A. (2007). Sorption kinetics of arsenic on laterite soil in aqueous medium. *Journal of Environmental Science and Health, Part A*, 42(7):989–996.

[15] Mondal, S., Roy, A., Mukherjee, R., Mondal, M., Karmakar, S., Chatterjee, S., Mukherjee, M., Bhattacharjee, S., and De, S. (2017). A socio-economic study along with impact assessment for laterite based technology demonstration for arsenic mitigation. *Science of the Total Environment*, 583:142–152.

[16] Borsi, I. and Fasano, A. (2009). Modelling waste water flow in hollow fibre filters. In *COMSOL Conference CD–2010 Edition*.

[17] Hahn, J. Y. (2015). *Characterizing the influence of carbon on the flowrate and disinfection efficacy of household ceramic water filters*. Missouri University of Science and Technology.

[18] Maiti, A., Sharma, H., Basu, J. K., and De, S. (2009). Modeling of arsenic adsorption kinetics of synthetic and contaminated groundwater on natural laterite. *Journal of hazardous materials*, 172(2-3):928–934.

[19] Bohaienko, V., & Bulavatsky, V. (2020). Simplified mathematical model for the description of anomalous migration of soluble substances in vertical filtration flow. *Fractal and Fractional*, *4*(2), 20.

[20] Usman, M., Belkasmi, A. I., Kastoyiannis, I. A., & Ernst, M. (2021). Pre-deposited dynamic membrane adsorber formed of microscale conventional iron oxide-based adsorbents to remove arsenic from water: application study and mathematical modeling. *Journal of Chemical Technology & Biotechnology*, *96*(6), 1504-1514.

[21] Bretzler, A., Nikiema, J., Lalanne, F., Hoffmann, L., Biswakarma, J., Siebenaller, L., ... & Hug, S. J. (2020). Arsenic removal with zero-valent iron filters in Burkina Faso: Field and laboratory insights. *Science of the Total Environment*, *737*, 139466.

[22] Whitaker, S. (1986). Flow in porous media i: A theoretical derivation of Darcy's law. *Transport in porous media*, 1(1):3–25.

[23] Lima, S.A., Kamrujjaman, M. and Islam, M.S., 2021. Numerical solution of convection–diffusion–reaction equations by a finite element method with error correlation. *AIP Advances*, *11*(8), p.085225.

[24] Ali, H., Kamrujjaman, M., & Islam, M. S. (2022). An Advanced Galerkin Approach to Solve the Nonlinear Reaction-Diffusion Equations With Different Boundary Conditions. *Journal of Mathematics Research*, *14*(1).

[25] Britannica, T. (2020). Editors of encyclopaedia. *Argon. Encyclopedia Britannica*.

[26] Haldar, S. et al. (2013). Mineral exploration. *Mineral Exploration*, pages 193–222.

[27] H. and Selim, H. (2008). Reaction and transport of arsenic in soils: equilibrium and kinetic modeling. *Advances in Agronomy*, 98:45–115.