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Polynomial Deceleration Parameter Explaining Bianchi Type I Model

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ABSTRACT

The focus of the current research investigates the dynamical behavior of the universe within the framework of a new Bianchi type-I anisotropic, homogeneous astrophysical models with a modified theory of $f(R,T)$ gravity. The significant effect of the model is to find solutions to the field equations using Polynomial Deceleration Parameter (PDP) which is a powerful tool to analyze the cosmic expansion history. Through a combination of theoretical analysis and graphical representation, we explore the consequences of $f(R,T)$ gravity on the universe's evolution for $r = 1$ to 5 polynomial value of DP. Our study unveils intriguing features in the context of this modified gravity theory, shedding light on its impact on cosmic expansion, acceleration, and potential deviations from standard cosmological models. The graphical representations provide intuitive insights, enhancing our comprehension of the complex interplay between geometry and matter content in the universe. This research contributes to a deeper understanding of the universe's behavior in alternative gravitational theories.

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1 Introduction

In the realm of cosmology, researchers tried to develop various implications, especially polynomial expressions, within the context of Bianchi type-I cosmological model. The utilization of polynomials is profound for understanding various fundamental topics. This paper explores the contemporary problems of cosmological exploration, this study aspires to illuminate the previously uncharted terrain of the Polynomial Deceleration Parameter [1, 2].

Cosmological model Bianchi type I describes a spatially homogeneous yet anisotropic world [3]. This model assumes a three-dimensional cosmos that is flat and uniform in one direction but can change in the other two. Luigi Bianchi introduced this model in 1897 [4].

The normal deceleration value does not adequately represent the anisotropic effects on the pace of the universe's expansion, whereas the polynomial deceleration parameter does [5, 6]. The degree and direction of anisotropy in the cosmos are determined by the anisotropy parameters of the Bianchi type-I metric, which are used to build the polynomial function [7]. The $f(R,T)$ gravity framework, that incorporates a function of the Ricci scalar and the trace of the energy-momentum tensor into the gravitational action, generalizes the Bianchi type-I model with a polynomial deceleration component [8, 9]. The $f(R,T)$ function and its derivatives are incorporated into the updated field equation [10, 11].

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The Bianchi type-I model with polynomial deceleration parameter has the benefit of allowing for a more adaptable universe description, which may assist to explain some of the gaps between observations and theoretical predictions. A greater variety of cosmological scenarios, including inflation, dark energy, and gravitational waves, can be accommodated by this model [12, 13]. The complexity gains come with an expense, though, as more information is needed to properly limit the model's parameters and generate reliable predictions [14, 15].

This paper explores an analysis of a spatially uniform and directionally varying Bianchi type-I line element using $f(R,T)$ gravity with a polynomial deceleration parameter. To delineate the structure of the present manuscript, the subsequent sections of the paper are arranged in the following manner: Section 2, focusing on the functional form $f(R,T) = R+2f(T)$, and considering $f(t) = \gamma T$ as $f(R,T) = R+2\gamma T$, where γ is a constant that may not necessarily be positive, we establish the fundamental field equations for this model. Finally, the Bianchi type-I field equations' solutions along with expression for various parameter of cosmological importance are shown here. In Section 3, the polynomial deceleration parameter(PDP) is described, covering the model for $r = 1$ to 5. The section presents an overview of all cosmological and physical parameters. Subsequently, graphical representations and explanations of these cosmological parameters are provided in subsections 3.1 to 3.5. Finally, in Section 4, wrap up our discussions and conclusions of these models.

Objectives:

- In this paper, the primary aim is to discover the field equations for Bianchi type-I Universe model under $f(R,T)$ gravity theory.
- Using field equations and mathematical constraint, the goal is to find expressions for physical parameters such as pressure P, energy density ρ and EOS parameter ω.
- In the context of this manuscript, we have tried to find the expression of cosmological parameters such as deceleration parameter q, Hubble parameter H, Scale factor s, and Jerk parameter J by using polynomial deceleration parameter(PDP).
- Employing $MATLAB$ to clarify the variations in physical and cosmological parameters across different time intervals and considering universel accepted parameter values
- Ultimately, the aim is to elicit the behavior of the universe across different universe models within the polynomial range of $r = 1$ to 5 and categorize them based on their origins and whether they are predominantly matter-driven or dark energy-driven.

2 Metric and Field Equations for Bianchi type-I Universe Model

We take into account the homogeneous anisotropic Bianchi type-I metric, which is denoted by

$$ds^2 = dt^2 - C^2dx^2 - D^2\left(dy^2 + dz^2\right) \tag{2.1}$$

Here, C and D are merely scale variables and cosmic time functions in this context. This metric has an XY-plane-associated symmetric plane and a Z-axis located symmetry axis. Anisotropic dark energy's energy-momentum tensor is given as

$$T^i_j = diag\left[1, -\omega, -(\omega+\delta), -(\omega+\delta)\right]\rho \tag{2.2}$$

where ρ is the energy density of the fluid. $\omega = \frac{p}{\rho}$, equation of state parameter and δ is the skewness parameter deviation from ω which is not necessarily functions of cosmic time(t) or be constant.

Using the Hilbert-Einstein action principle, we may derive the field equations for the $f(R,T)$ gravity theory as

$$f_R(R)R_{ij} - \frac{1}{2}f(R)R_{ij} - \left(\nabla_i\nabla_j - g_{ij}\Pi\right)f_R(R) = (8\pi + f_T(T))T_{ij} + \left(f_T(T)p + \frac{1}{2}f(T)\right)g_{ij} \tag{2.3}$$

where $f(R,T)$ is an arbitrary function of Ricci scalar (R) and trace (T) of the energy-momentum tensor T_{ij} and $f_R = \frac{\partial f(R)}{\partial R}$, $f_T = \frac{\partial f(T)}{\partial T}$ and ∇_i refers the derivative that covariates [8, 16].

Consider the function $f(T) = \gamma T$, where γ is a constant, field equations of the $f(R,T)$ gravity of anisotropic universe is as follows:

$$2\frac{C_1D_1}{CD} + \frac{D_1^2}{D^2} = (2\delta + 3\omega - 1 - 8\pi - 2\gamma)\rho - 2\gamma P \tag{2.4}$$

$$2\frac{D_{11}}{D} + \frac{D_1^2}{D^2} = \left[(8\pi + 2\gamma)\,\omega - (1 - 3\omega - 2\delta)\,\gamma\right]\rho - 2\gamma P \tag{2.5}$$

$$\frac{C_{11}}{C} + \frac{D_{11}}{D} + \frac{C_1 D_1}{CD} = \left[(8\pi + 2\gamma)\,(\omega + \delta) - (1 - 3\omega - 2\delta)\,\gamma\right]\rho - 2\gamma P \tag{2.6}$$

By applying a mathematical constraint where the equation of state (EoS) parameter(ω), is directly proportional to the skewness parameter(δ), we obtain the relationship $\omega + \delta = 0$ [17].

To find mathematical expression for physical parameters ρ, ω, and p, we use equations (2.4) – (2.6). Now subtracting equation (2.5) from (2.6) we get

$$\omega = \frac{1}{\rho\,(8\pi + 2\gamma)}\left[\frac{D_{11}}{D} + \frac{D_1^2}{D^2} - \frac{C_1 D_1}{CD} - \frac{C_{11}}{C}\right] \tag{2.7}$$

and subtracting equation (2.4) from (2.5) we get

$$\omega = -1 + \frac{1}{\rho\,(8\pi + 2\gamma)}\left[2\frac{D_{11}}{D} - 2\frac{C_1 D_1}{CD}\right] \tag{2.8}$$

From equation (2.7) and (2.8) we get

$$\rho = -\frac{1}{8\pi + 2\gamma}\left[\frac{D_1^2}{D^2} - \frac{D_{11}}{D} - \frac{C_{11}}{C} + \frac{C_1 D_1}{CD}\right] \tag{2.9}$$

Using equation (2.7) we get

$$\omega = -\frac{\left(\frac{D_1^2}{D^2} + \frac{D_{11}}{D} - \frac{C_{11}}{C} - \frac{C_1 D_1}{CD}\right)}{\left(\frac{D_1^2}{D^2} - \frac{D_{11}}{D} - \frac{C_{11}}{C} + \frac{C_1 D_1}{CD}\right)} \tag{2.10}$$

By utilizing the equation $\omega = \frac{p}{\rho}$, we can derive the formula for pressure P as

$$P = \omega\rho = \frac{1}{8\pi + 2\gamma}\left[\frac{D_1^2}{D^2} + \frac{D_{11}}{D} - \frac{C_{11}}{C} - \frac{C_1 D_1}{CD}\right] \tag{2.11}$$

The spatial volume V is given by

$$V = s^3 = CD^2, \qquad \Rightarrow s = \left(CD^2\right)^{\frac{1}{3}} \tag{2.12}$$

where $s = s(t)$ is the scale factor of the universe and in this model considering scale factors C and D are related as follows

$$C = D^M \tag{2.13}$$

where M is a constant.

From (2.12) we have

$$s = D^{\frac{M+2}{3}} \tag{2.14}$$

The transitional phase such as from the decelerating to accelerating phase of the universe can be determined by using a parameter is called Jerk parameter which is denoted by

$$J(t) = \frac{1}{s\,H^3}\,\frac{d^3 s}{dt^3} \tag{2.15}$$

This scale factor s is defined in terms of redshift z by the expression Capozziello et al.

$$1 + z = \frac{1}{s(t)} \tag{2.16}$$

We will find all these parameters (C, D, H, ρ, P, J and ω) in terms of the PDP (q) in the following section.

3 Varying Polynomial Deceleration Parameter

For the evolution of the universe, the scalar expansion is observed, and for the Einstein field equations as well as cosmological parameters we may consider

$$\Theta(t) = -B/\sum_{n=1}^{r}(-1)^n b_n t^n, \qquad r \geq n \tag{3.1}$$

where $B,\ n,\ r,$ and b_n are real numbers [10, 11].

The form of the DP in terms of scalar expansion is given by

$$q(\Theta) = -\left(1 + \frac{3}{\Theta^2}\frac{d\Theta}{dt}\right) \tag{3.2}$$

$$q(t) = -\left(1 + 3\sum_{n=1}^{r}(-1)^n n b_n t^{n-1}/B\right) \tag{3.3}$$

All of these equations represent the cosmological models with varying deceleration parameters.

3.1 Bianchi type-I Model with $r = 1$

In this section, we use Eqs. (2.4 − 3.3) to analyze some hypothetical cosmological models. In the case of ($r = 1$), the DP given by Eq. (3.3) can be rewritten as

$$q = -\left(1 - \frac{3b_1}{B}\right) = \frac{3b_1}{B} - 1 = l - 1 \tag{3.4}$$

where $l = \frac{3b_1}{B}$. For observed kinematics the value of $l = 1.6$ and $b = 0.097$ which align with the observations of Khature and Shaikh [18] and Bakry, Moatimid, and Shafeek [19].

By using Eq. (3.4), we have the form of the Hubble parameter, Scale factor, and Jerk parameter as follows

$$H(t) = \frac{1}{lt} \tag{3.5}$$

$$s(t) = t^{\frac{1}{l}} \tag{3.6}$$

$$J(t) = (l-1)(2l-1) \tag{3.7}$$

Using equation (2.14) we get

$$D = t^{\frac{3}{l(M+2)}} \qquad \text{and} \qquad C = t^{\frac{3M}{l(M+2)}} \tag{3.8}$$

Using scale factors C and D with their derivatives we find the physical parameters from (2.9), (2.10) and (2.11) we get

$$\rho(t) = -\frac{3\left(-3(M-1)M + l\left(M^2+3M+2\right)\right)}{2l^2(M+2)^2 t^2(4\pi+\gamma)} \tag{3.9}$$

$$P(t) = \frac{3(l-3)(M-1)}{2l^2(M+2)t^2(4\pi+\gamma)} \tag{3.10}$$

$$\omega(t) = -\frac{3(l-3)(M-1)(M+2)}{-3(M-1)M + l(M^2+3M+2)} \tag{3.11}$$

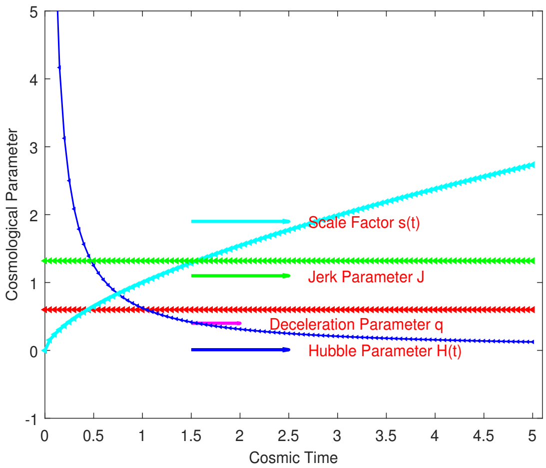
From (3.10) we see that pressure $P \geq 0$ if $l \geq 3$ and $M \geq 1$ or $l \leq 3$ and $M \leq 1$.

We see that if $l > 0$, then

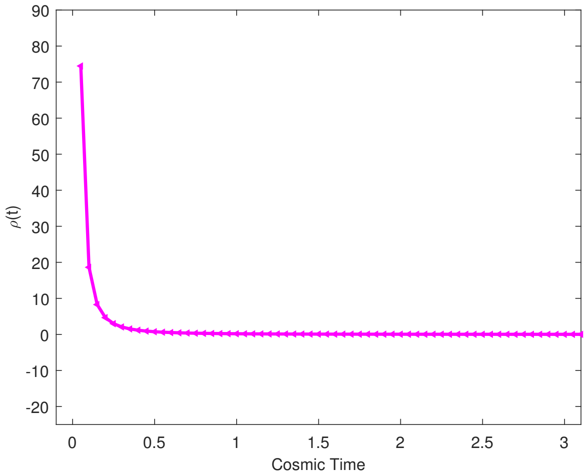
$$\lim_{t\to 0} H(t) = \infty \qquad \lim_{t\to\infty} H(t) = 0$$

Table 3.1: Constant Deceleration Parameter Model ($b_1 \neq 0$).

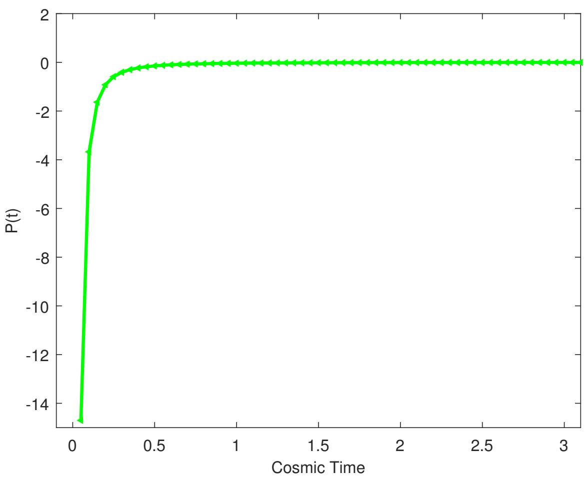
Applied Conditions	**Value of** $l = \frac{3b_1}{B}$	**Scale Factor**	**Cosmological Model**
$B = 3b_1$	$l = 1$	$s(t) = t$	Milne Model
$B = \frac{3b_1}{2}$	$l = 2$	$s(t) = t^{\frac{1}{2}}$	Radiation Model
$B = 2b_1$	$l = \frac{3}{2}$	$s(t) = t^{\frac{2}{3}}$	Einstein de Sitter Model
$B = 0$	$l = \infty$	$s(t)$ =constant	Einstein Model



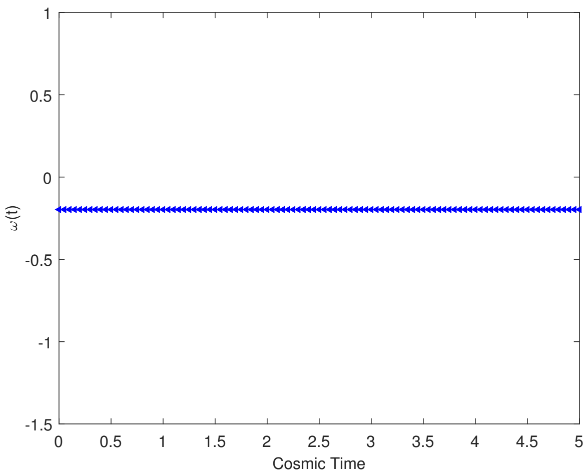
(a) Showing cosmological parameter with time at $l = 1.6$ and $b = 0.097$.



(b) Showing Energy density $\rho(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(c) Showing Pressure $P(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(d) Showing EOS parameter $\omega(t)$ with time at $M = 0.6$ and $\gamma = -16$.

Figure 3.1: $r = 1$ model.

$$\lim_{t\to 0} s(t) = 0 \qquad \lim_{t\to\infty} s(t) = \infty$$
$$\lim_{t\to 0} \rho(t) = \infty \qquad \lim_{t\to\infty} \rho(t) = 0$$
$$\lim_{t\to 0} P(t) = \infty \qquad \lim_{t\to\infty} P(t) = 0$$

Since the in Fig 3.1(a), DP (q) , Jerk parameter J and in Fig 3.1(d), EOS parameter (ω) are not time dependent so they depend on M and l and they are constant. The cosmological model ($r = 1$) represents the constant DP model. In Fig 3.1(b) and in Fig 3.1(c) show positive pressure and positive energy density respectively except at the begining of the universe. At the creation of the Universe, they diverge. Overall, the cosmological model ($r = 1$) represents the constant deceleration parameter model.

3.2 Bianchi type-I Model with $r = 2$

In the case of ($r = 2$), the DP given by Eq. (3.3) can be rewritten as follows

$$q = -1 + \frac{3b_1}{B} - \frac{6b_2}{B}t = l - 1 - bt \tag{3.12}$$

where $l = \frac{3b_1}{B}$ and $b = \frac{6b_2}{B}$.

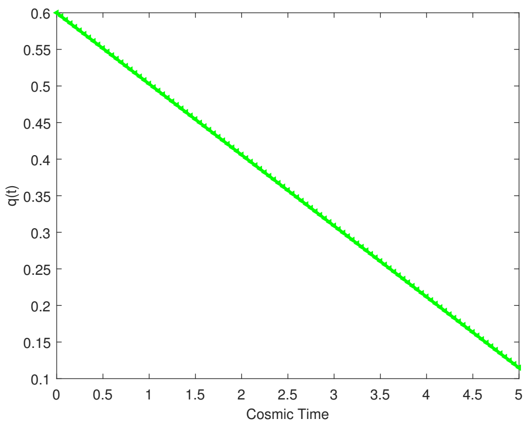


Figure 3.2: Showing Deceleration parameter $q(t)$ with time at $l = 1.6$ and $b = 0.097$.

From (3.12),

$$H(t) = \frac{2}{t\,(2l - bt)}$$

$$s(t) = \left(\frac{t}{2l - bt}\right)^{\frac{1}{l}}$$

$$J(t) = 1 + 2l^2 + 3bt + \frac{3b^2t^2}{2} - 3l(1 + bt)$$

and scale factors

$$D = \left(\frac{t}{2l - bt}\right)^{\frac{3}{l(M+2)}} \quad \text{and}$$

$$C = \left(\frac{t}{2l - bt}\right)^{\frac{3M}{l(M+2)}}$$

Using scale factors C and D with their derivatives we find the physical parameters from (2.9), (2.11) and (2.10) we get

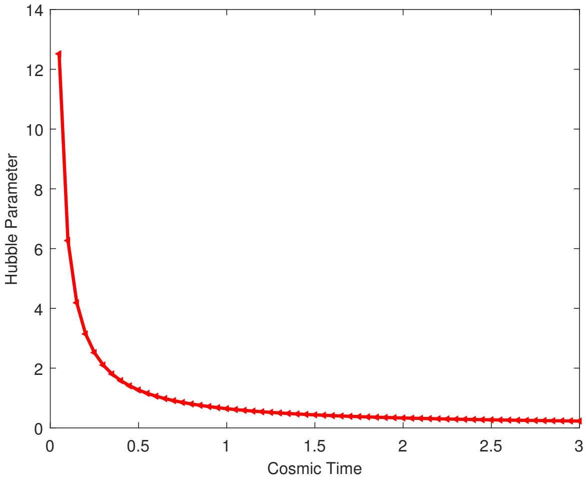
$$\rho(t) = \frac{6\left(-l\left(2 + 3M + M^2\right) + 2bt + 3M\,(bt - 1) + M^2\,(bt + 3)\right)}{(bt - 2l)^2\,(M + 2)^2\,t^2\,(4\pi + \gamma)} \tag{3.13}$$

$$P(t) = \frac{6\,(l - bt - 3)\,(M - 1)}{(bt - 2l)^2\,(M + 2)\,t^2\,(4\pi + \gamma)} \tag{3.14}$$

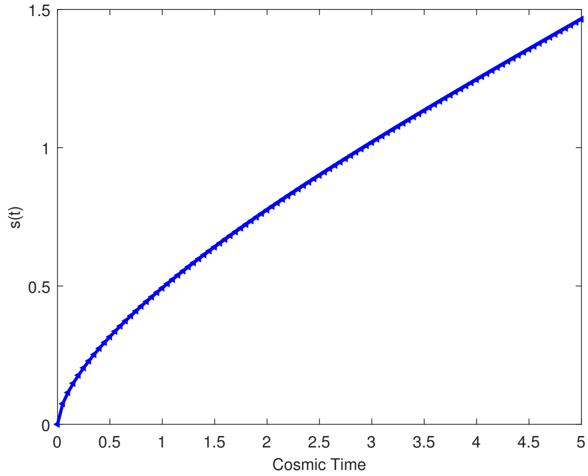
$$\omega(t) = \frac{(l-bt-3)(M-1)(M+2)}{-l(M^2+3M+2)+2bt+3M(bt-1)+M^2(bt+3)} \tag{3.15}$$

In Fig 3.2, the behavior of DP shows the universe udergoing with a linearly varying deceleration parameter q.

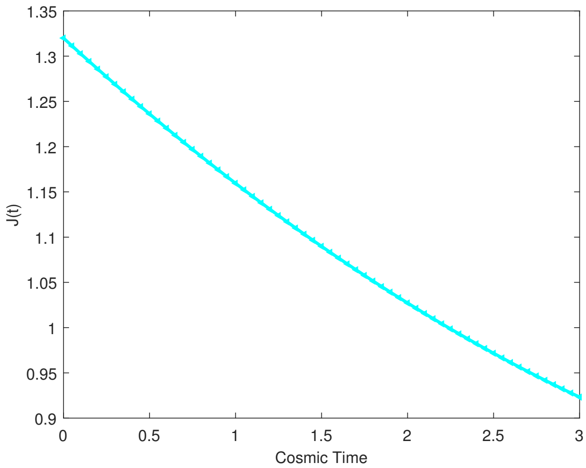
In Fig 3.3(a), at initial phase of the universe, Hubble parameter starts ($t = 0$) with a Big Bang, then undergoes a decelerating expansion through a steady-state universe as $H \to 0$.



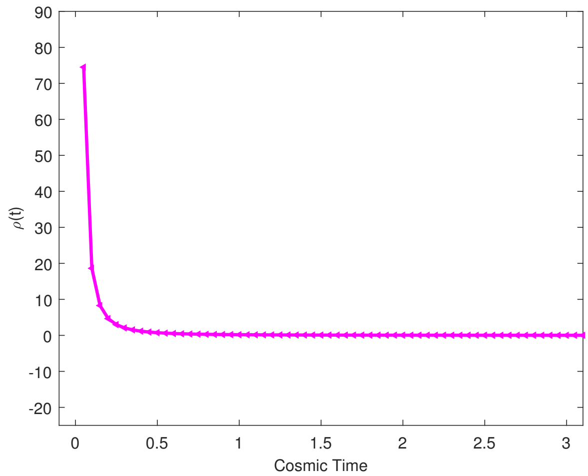
(a) Showing Hubble parameter $H(t)$ with time at $l = 1.6$ and $b = 0.097$.



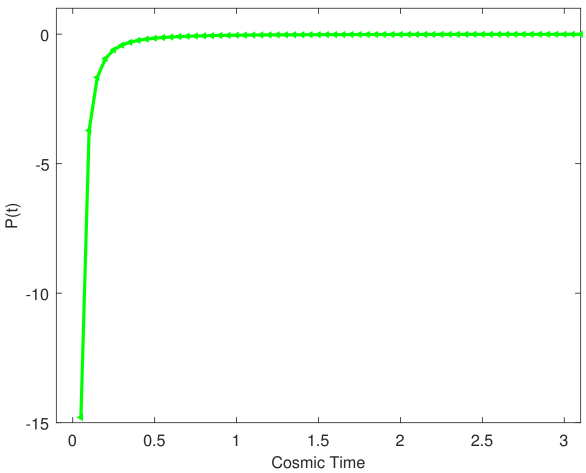
(b) Showing Scale factor $s(t)$ with time at $l = 1.6$ and $b = 0.097$.



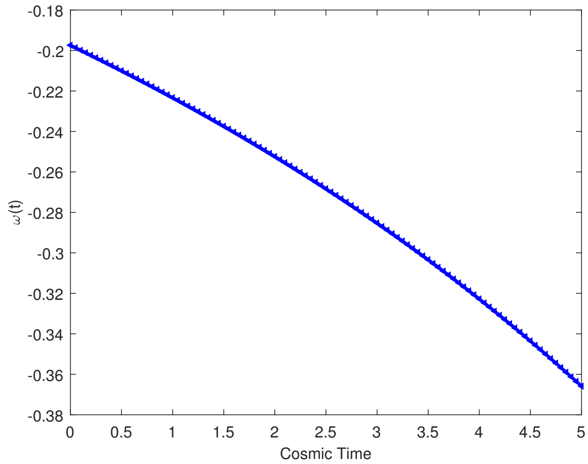
(c) Showing Jerk parameter $J(t)$ with time at $l = 1.6$ and $b = 0.097$.



(d) Showing Energy density $\rho(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(e) Showing Pressure $P(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(f) Showing EOS parameter $\omega(t)$ with time at $M = 0.6$ and $\gamma = -16$.

Figure 3.3: $r = 2$ model.

Scale factor with respect to time in Fig 3.3(b) represents when $s(t) = 0$, at $t = 0$, the universe starts with an initial singularity of the Big Bang and then undergoes an expanding universe to a big rip in a later epoch. At $t = 2.92$, the universe enters the present stage.

Jerk parameter $J(t)$ shows in Fig 3.3(c) the universe's expansion is accelerating at a decreasing rate within $t \in [0, 5]$.

In this model, energy density $\rho(t)$ is positive but pressure and EOS parameter both are negative. Which predicts a dark energy dominated universe shown in Fig 3.3(d), Fig 3.3(e) and Fig 3.3(f).

This model predicts that the lifespan of the cosmos is limited. At $t = 0$, there is a Big Bang, and it lasts until $t = \frac{2l}{b}$. We provide a specific law with a linear negative slope in time for the deceleration parameter. In the presence of isotropic and anisotropic fluid, the law we present can also be applied to Kantowski-Sachs spacetime and spatially homogeneous but anisotropic Bianchi-type spacetimes [20, 21]. $r = 2$ model represents LVDP model with negative pressure, which indicates the Universe is dominated by dark energy.

3.3 Bianchi type-I Model with $r = 3$

In the case of ($r = 3$), the DP can be rewritten as

$$q = -1 + \frac{3b_1}{B} - \frac{6b_2}{B}t + \frac{9b_3}{B}t^2 \tag{3.16}$$

To make this model consistent with the observed cosmological kinematics, we set $B = 6$ and select additional coefficient values according to the instructions in Katore and Shaikh [11, 18] as follows:

$$b_1 = 4, \qquad b_2 = 6 \qquad \text{and} \qquad b_3 = 2$$

Now for the Hubble parameter, consider the relation

$$\begin{aligned} q &= \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 1 - 6t + 3t^2 \\ \Rightarrow H &= \frac{1}{t\,(t-1)\,(t-2)} \end{aligned} \tag{3.17}$$

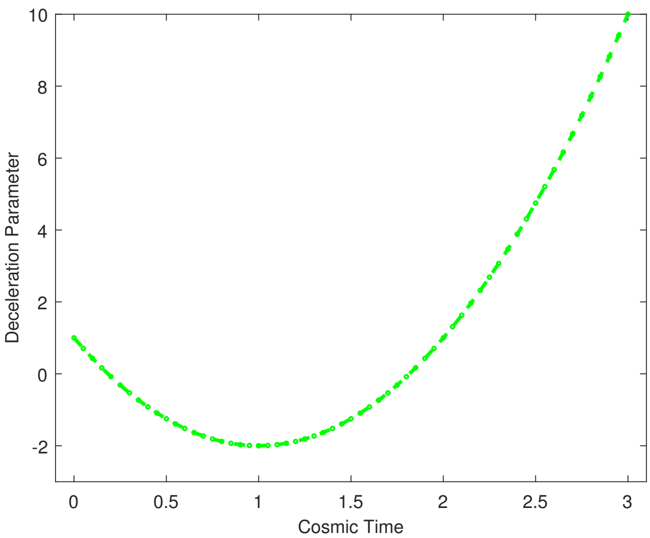
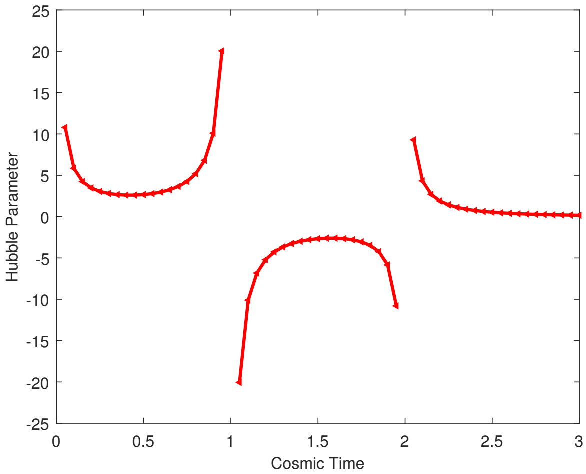


Figure 3.4: Showing Deceleration parameter $q(t)$ with time at $l = 1.6$ and $b = 0.097$.

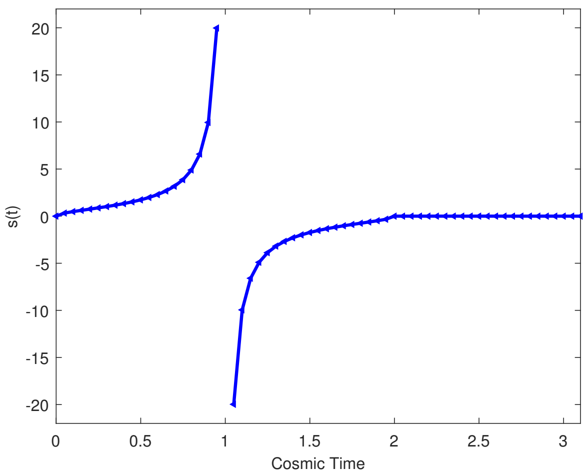
t	DP(q)	Phase
0	1	Big Bang
0.183503	0	Accelerating
0.42265	-1	De Sitter
1.57735	-1	De Sitter
1	-2	Big Rip
2	1	Decelerated expansion

From the relation between $H(t)$ and $s(t)$ we have

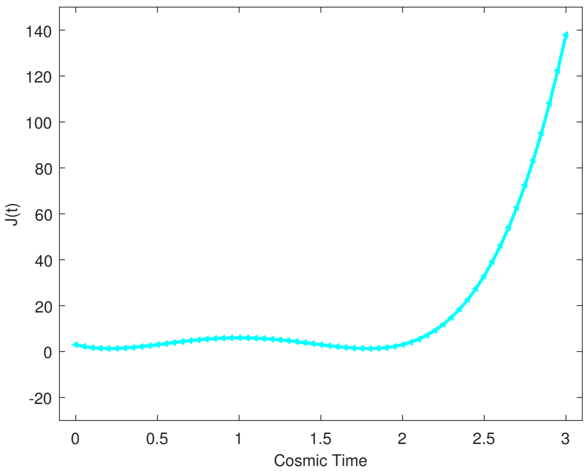
$$s(t) = \frac{\sqrt{-(-2+t)t}}{1-t} \tag{3.18}$$



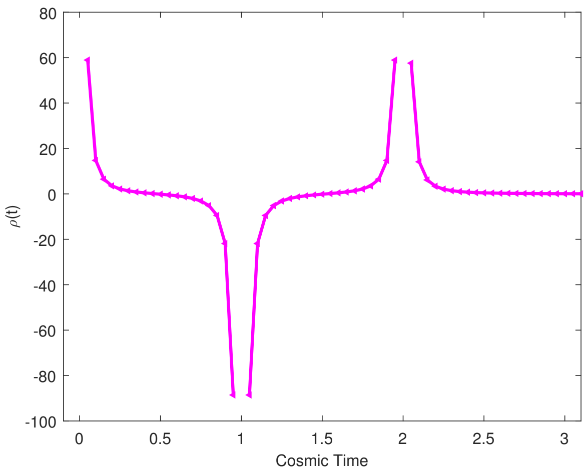
(a) Showing Hubble parameter $H(t)$ with time at $l = 1.6$ and $b = 0.097$.



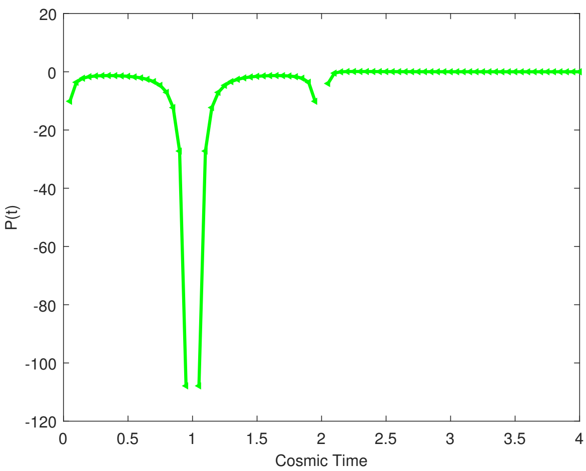
(b) Showing Scale factor $s(t)$ with time at $l = 1.6$ and $b = 0.097$.



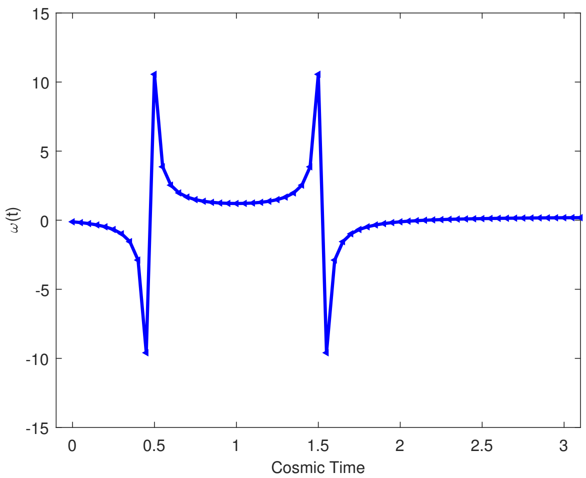
(c) Showing Jerk parameter $J(t)$ with time at $l = 1.6$ and $b = 0.097$.



(d) Showing Energy density $\rho(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(e) Showing Pressure $P(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(f) Showing EOS parameter $\omega(t)$ with time at $M = 0.6$ and $\gamma = -16$.

Figure 3.5: $r = 3$ model.

and the scale factors are

$$D = \left(\frac{\sqrt{-(-2+t)t}}{1-t}\right)^{\frac{3}{2+M}} \quad \text{and}$$
$$C = \left(\frac{\sqrt{-(-2+t)t}}{1-t}\right)^{\frac{3M}{2+M}} \tag{3.19}$$

Here Jerk parameter

$$J(t) = 3\left(1 - 6t + 19t^2 - 16t^3 + 4t^4\right)$$

Using scale factors C and D with their derivatives we find the physical parameters from (2.9), (2.10) and (2.11) we get

$$\rho(t) = -\frac{3\left(4 + 9M(-1+t)^2 - 12t + 6t^2 + M^2\left(-1 - 6t + 3t^2\right)\right)}{2(2+M)^2(\gamma + 4\pi)(-2+t)^2(-1+t)^2t^2} \tag{3.20}$$

$$P(t) = \frac{3(-1+M)\left(-1 - 6t + 3t^2\right)}{2(2+M)(\gamma + 4\pi)(-2+t)^2(-1+t)^2t^2} \tag{3.21}$$

$$\omega(t) = -\frac{\left(-2 + M + M^2\right)\left(-1 - 6t + 3t^2\right)}{4 + 9M(-1+t)^2 - 12t + 6t^2 + M^2\left(-1 - 6t + 3t^2\right)} \tag{3.22}$$

In Fig 3.4, DP with time shows a transition Universes expansion from deceleration to acceleration at $t \in [0, 1]$ and after that the process reverse. Universe behavior towards DP for different times also present along with Fig 3.4 in a table.

In Fig 3.5(a), Hubble parameter starts with a big bang at the initial singularity $t = 0$. Diverges at the Big Rip time $t = 1$ and $t = 2$. It shows the cyclic behavior of the universe.

The cosmological model at $r = 3$ is seen to have a singularity at $t = 0$ as $s(t) = 0$, followed by a diverging phase at $t = 1$. Big Rip occurs at time $t = 1$, as planned. As a result, the model exhibits cyclic behavior shown in Fig 3.5(b).

In Fig 3.5(c), Jerk parameter shows universe's expansion is accelerating at an increasing rate. This model shows energy density positive, and pressure negative. Energy density ρ and pressure p decrease monotonically with cosmic time (t) shown in Fig 3.5(d) and in Fig 3.5(e). EoS parameter ω evolves with time t representing in Fig 3.5(f). Dark energy dominates the Universe when ω approaches 1 late, accelerating its expansion.

This model indicates accelerated expansion of the Universe containing Big Bang and Big Rip stages with negative pressure and negative ρ. It concludes that the nature of the Universe is not matter-dominated; eventually, it is a dark energy-dominated model.

3.4 Bianchi type-I Model with $r = 4$

In the case of ($r = 4$), the DP given by Eq. (3.3) can be rewritten as

$$q = -1 + \frac{3b_1}{B} - \frac{6b_2}{B}t + \frac{9b_3}{B}t^2 - \frac{12b_4}{B}t^3 \tag{3.23}$$

To make this model consistent with the observed cosmological kinematics, we set $B = 6$ and select additional coefficient values according to the instructions in Katore and Shaikh [18] as follows:

$$b_1 = 4, \qquad b_2 = 10 \qquad b_3 = 6 \qquad \text{and} \qquad b_4 = 1$$

Now, we can rewrite (3.23) as

$$q(t) = 1 - 10t + 9t^2 - 2t^3 \tag{3.24}$$

Now, the form of the Hubble parameter and scale factor is given by

$$H = -\frac{2}{t\,(t-2)\,(t^2-4t+2)} \tag{3.25}$$

$$s(t) = \frac{\left(\frac{t(t-2)}{t^2-4t+2}\right)^{\frac{1}{2}}}{\left(\frac{\frac{1}{\sqrt{2}}(t-2)-1}{\frac{1}{\sqrt{2}}(t-2)+1}\right)^{\frac{1}{2\sqrt{2}}}} \tag{3.26}$$

$$J(t) = 3 - 30t + 159t^2 - 238t^3 + 153t^4 - 45t^5 + 5t^6 \tag{3.27}$$

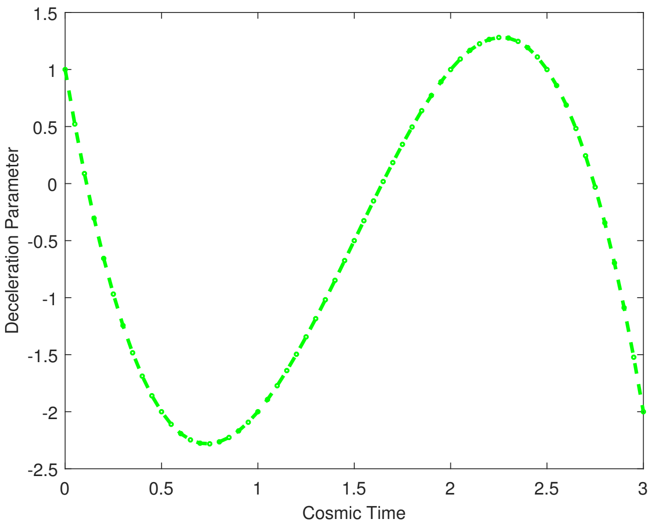


Figure 3.6: Showing Deceleration parameter $q(t)$ with time at $l = 1.6$ and $b = 0.097$.

Using equation (2.14) we get

$$D = \frac{\left(\frac{t(t-2)}{t^2-4t+2}\right)^{\frac{3}{2(M+2)}}}{\left(\frac{\frac{1}{\sqrt{2}}(t-2)-1}{\frac{1}{\sqrt{2}}(t-2)+1}\right)^{\frac{3}{2\sqrt{2}(M+2)}}} \quad \text{and}$$

$$C = \frac{\left(\frac{t(t-2)}{t^2-4t+2}\right)^{\frac{3M}{2(M+2)}}}{\left(\frac{\frac{1}{\sqrt{2}}(t-2)-1}{\frac{1}{\sqrt{2}}(t-2)+1}\right)^{\frac{3M}{2\sqrt{2}(M+2)}}} \tag{3.28}$$

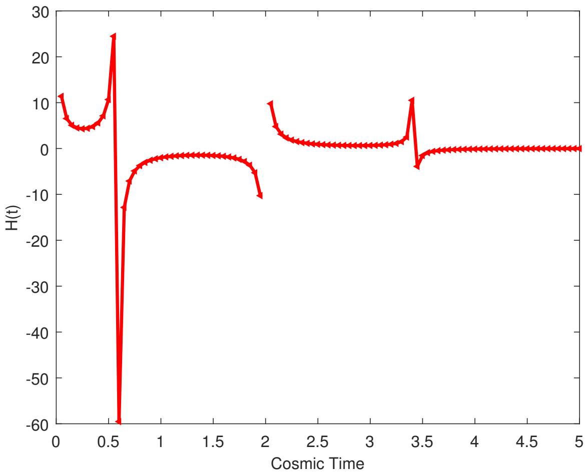
The specifics of the deceleration parameter (q) in **Figure 3.6** are presented in **Table 3.2** through the utilization of $MATLAB$, thereby clarifying the notion of universal behavior.

Table 3.2: Expliaining Universes behavior for $r = 4$ model.

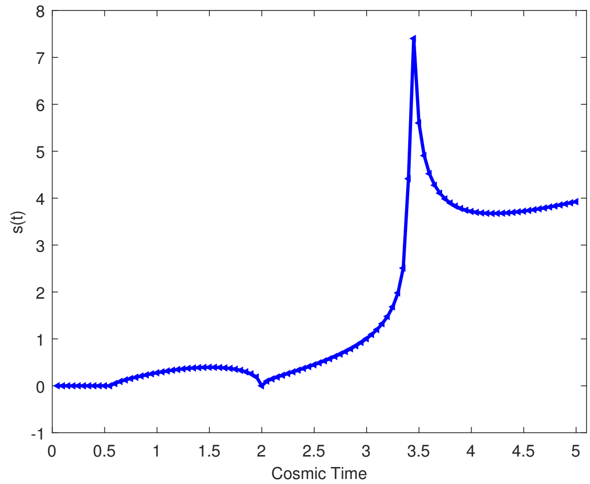
t	**q**	**Universe Behavior**
0	1	Big Bang
2.27	1.282	Big Rip
0.73	-2.282	Big Rip
0.11, 1.65 and 2.74	0	Translation
0.26, 1.4 and 2.9	-1	De Sitter

Using scale factors C and D with their derivatives, we find the physical parameters from (2.9), (2.10) and (2.11) we get

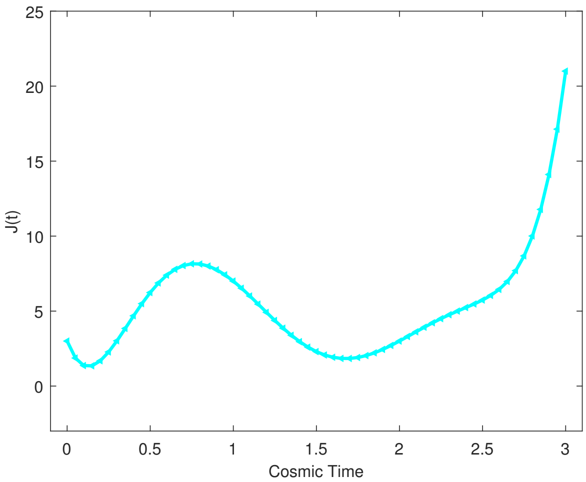
$$\rho(t) = \frac{6\left(-4+20t-18t^2+4t^3+3M\left(-3+10t-9t^2+2t^3\right)+M^2\left(1+10t-9t^2+2t^3\right)\right)}{(2+M)^2(\gamma+4\pi)\left(2+\sqrt{2}-t\right)^2(-2+t)^2t^2\left(-2+\sqrt{2}+t\right)^2} \tag{3.29}$$



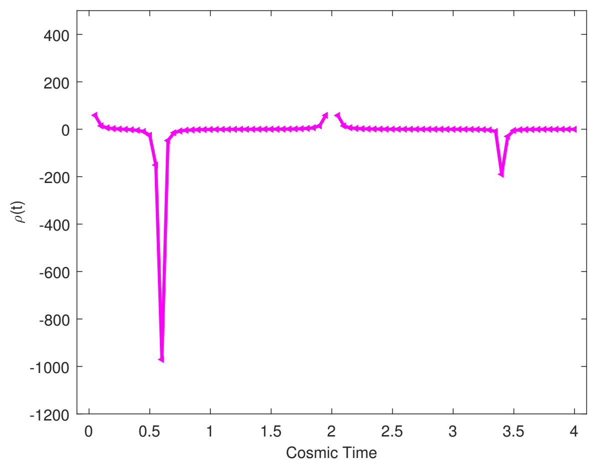
(a) Showing Hubble parameter $H(t)$ with time at $l = 1.6$ and $b = 0.097$.



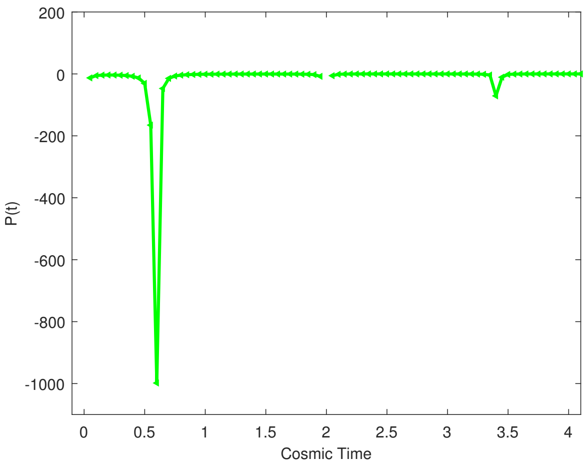
(b) Showing Scale factor $s(t)$ with time at $l = 1.6$ and $b = 0.097$.



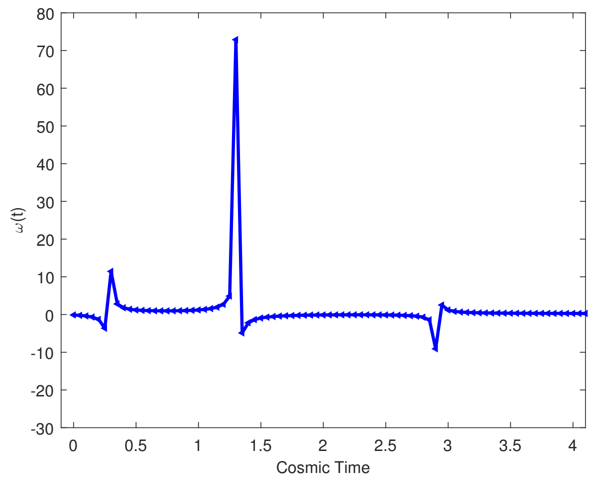
(c) Showing Jerk parameter $J(t)$ with time at $l = 1.6$ and $b = 0.097$.



(d) Showing Energy density $\rho(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(e) Showing Pressure $P(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(f) Showing EOS parameter $\omega(t)$ with time at $M = 0.6$ and $\gamma = -16$.

Figure 3.7: $r = 4$ model

$$\omega(t) = -\frac{(-1+M)(2+M)\left(1+10t-9t^2+2t^3\right)}{-4+20t-18t^2+4t^3+3M\left(-3+10t-9t^2+2t^3\right)+M^2\left(1+10t-9t^2+2t^3\right)} \tag{3.30}$$

$$P(t) = -\frac{6(-1+M)\left(1+10t-9t^2+2t^3\right)}{(2+M)(\gamma+4\pi)t^2\left(-4+10t-6t^2+t^3\right)^2} \tag{3.31}$$

The specifics of the parameters in **Figure 3.7** are presented in **Table 3.3** through the utilization of $MATLAB$, thereby clarifying the notion of universal behavior.

Table 3.3: Expliaining Universes behavior for $r = 4$ model.

Parameter	**Universe Behavior**
$H(t)$	Starting the universe with a big bang at the initial singularity $t = 0$ and undergoing a decelerating expansion to a big rip to an accelerated expansion. Diverges at the big rip time $t = 0.6$, $t = 2$ and $t = 3.4$. It also shows the cyclic behavior of the universe, but the cycle requires less time than $r = 3$ model. (Fig 3.7(a))
$S(t)$	The model starts with a singularity at $t = 0$ and the Big Bang, after which $s(t)$ vanishes. The scale factor diverges at the Big Rip time $t = 2 - \sqrt{2}$ and returns to $s(t) = 0$ at $t = 2$. At $t = 3$, the universe enters the present age and at $t = 3.45$ it reaches translation phase. (Fig 3.7(b))
$J(t)$	Jerk parameter is positive with time indicating that the Universe's expansion is accelerating. (Fig 3.7(c))
$\rho(t)$	This model satisfies positivity energy conditions, starting with a big bang at $t = 0$. It enters the big rip at $t = 0.6$, $t = 2$ and $t = 3.4$ where ρ diverges with time and re-enters the Big Bang stage. (Fig 3.7(d))
$P(t)$	The big bang occurs at $t = 0$ with negative pressure, then enters big rip at $t = 0.6$, $t = 2$ and $t = 3.4$ where P diverges with time and re-enters the Big Bang stage. Throughout the time interval pressure violates universe positive conditions. (Fig 3.7(e))
$\omega(t)$	The model starts with a singularity at $t = 0$ and the Big Bang, after which it enters the first big rip at $t = 0.3$, second at $t = 1.3$ and third big rip at $t = 2.9$. The EOS parameter diverges at each Big Rip stage. (Fig 3.7(f))

This model's simulation of a positive DP $q = 1$ at an initial epoch is straightforwardly observed to progress into a negative value. At the beginning and conclusion of each phase of the universe's history, the Hubble parameter diverges. And scale factor, pressure, energy density, and EOS are functions of cosmic time t. The $r = 4$ model encompasses all preceding models and subsequently includes the periodic deceleration parameter PVDP model. This is a new model and also indicates dark energy dominated universe with accelerated expansion [7, 11].

3.5 Bianchi type-I Model with $r = 5$

In the case of ($r = 5$), the DP is given by Eq. (3.3) can be rewritten as follows

$$q = -1 + \frac{3b_1}{B} - \frac{6b_2}{B}t + \frac{9b_3}{B}t^2 - \frac{12b_4}{B}t^3 + \frac{15b_5}{B}t^4 \tag{3.32}$$

To make this model consistent with the observed cosmological kinematics, we set $B = 6$ and select additional coefficient values according to the instructions in Katore and Shaikh [18] as follows:

$$b_1 = 4, \quad b_2 = 16, \quad b_3 = \frac{40}{3}, \quad b_4 = 4 \text{ and } b_5 = \frac{2}{5}$$

Then (3.32) can be rewritten as

$$q(t) = 1 - 16t + 20t^2 - 8t^3 + t^4 \tag{3.33}$$

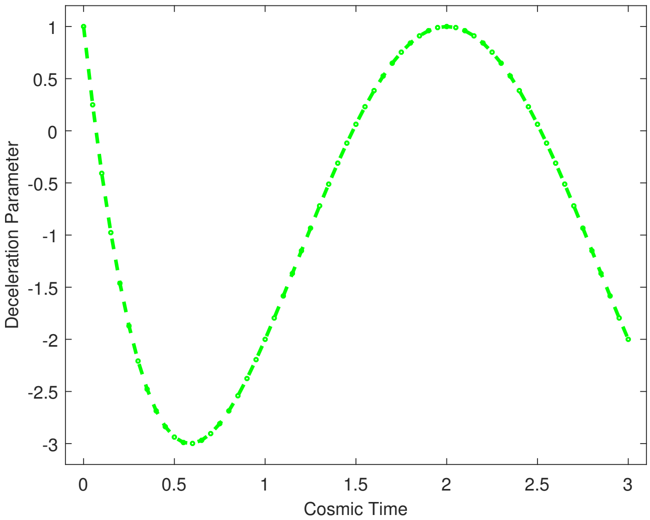


Figure 3.8: Showing Deceleration parameter $q(t)$ with time at $l = 1.6$ and $b = 0.097$.

Using equation (51) we get

$$D = \left[\frac{t^{1.5}(t-4.15603)^{0.598284}(t-2.13497)^{1.55645}}{(t-0.333923)^{2.14575}(t-3.37508)^{1.509}}\right]^{\frac{3}{M+2}}$$

$$C = \left[\frac{t^{1.5}(t-4.15603)^{0.598284}(t-2.13497)^{1.55645}}{(t-0.333923)^{2.14575}(t-3.37508)^{1.509}}\right]^{\frac{3M}{M+2}}$$

Now the Hubble parameter,

$$H(t) = \frac{15}{t(t-0.333923)(t-3.37508)(t-4.15603)(t-2.13497)} \tag{3.34}$$

The relation between the scale factor and the Hubble parameter is given by

$$s(t) = \frac{t^{1.5}(t-4.15603)^{0.598284}(t-2.13497)^{1.55645}}{(t-0.333923)^{2.14575}(t-3.37508)^{1.509}} \tag{3.35}$$

$$J(t) = 3 - 48t + 404t^2 - \frac{2536t^3}{3} + \frac{2455t^4}{3} - \frac{2144t^5}{5} + \frac{376t^6}{3} - \frac{96t^7}{5} + \frac{6t^8}{5} \tag{3.36}$$

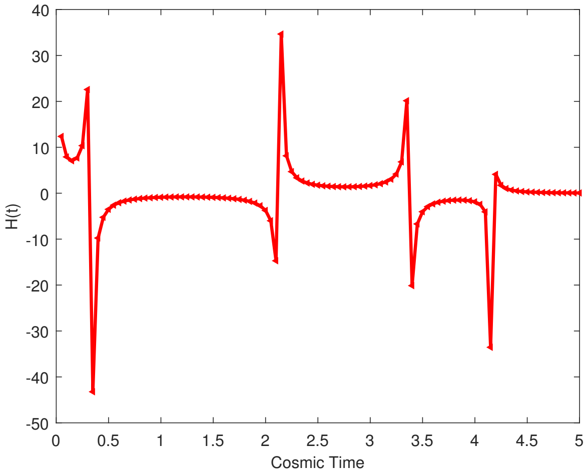
The specifics of the deceleration parameter (q) in **Figure 3.8** are presented in **Table 3.4** through the utilization of $MATLAB$, thereby clarifying the notion of universal behavior.

Table 3.4: Explaining Universes behavior for $r = 5$ model

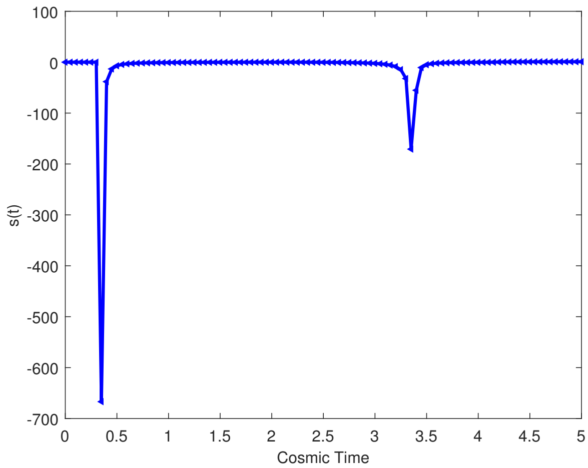
t	DP (q)	Universe Stage
0 and 2	1	Big Bang
0.59	-3	Big Rip
0.07, 1.48 and 2.52	0	Translation
0.15, 1.23 and 2.77	-1	De Sitter

Physical parameters energy density, pressure, and equation of state parameter can be written as

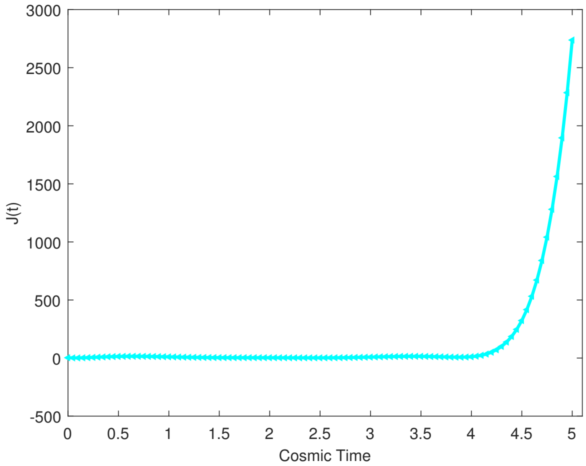
$$\rho(t) = -\frac{675\left(M^2\left(-1-16t+20t^2-8t^3+t^4\right)+2\left(2-16t+20t^2-8t^3+t^4\right)+3M\left(3-16t+20t^2-8t^3+t^4\right)\right)}{2(2+M)^2(\gamma+4\pi)t^2(30-120t+100t^2-30t^3+3t^4)^2} \tag{3.37}$$



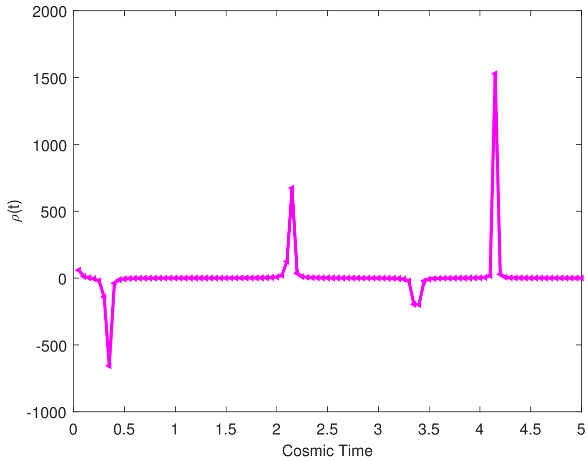
(a) Showing Hubble parameter $H(t)$ with time at $l = 1.6$ and $b = 0.097$.



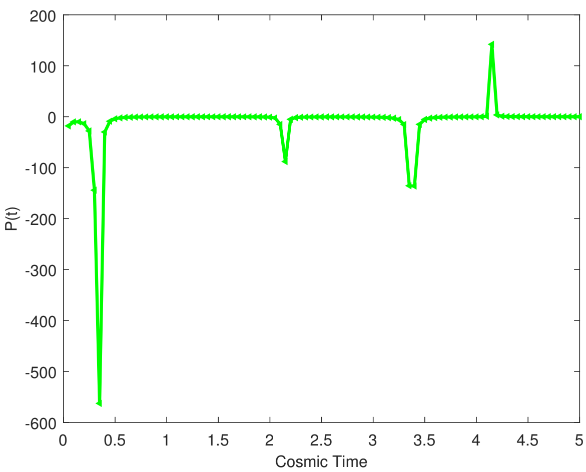
(b) Showing Scale factor $s(t)$ with time at $l = 1.6$ and $b = 0.097$.



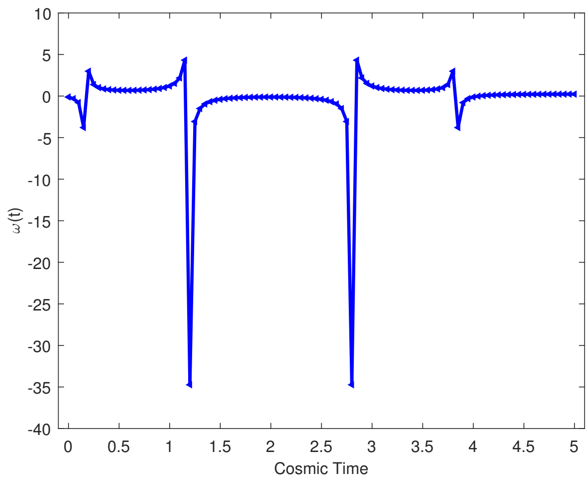
(c) Showing Jerk parameter $J(t)$ with time at $l = 1.6$ and $b = 0.097$.



(d) Showing Energy density $\rho(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(e) Showing Pressure $P(t)$ with time at $M = 0.6$ and $\gamma = -16$.



(f) Showing EOS parameter $\omega(t)$ with time at $M = 0.6$ and $\gamma = -16$.

Figure 3.9: $r = 5$ model.

$$P(t) = \frac{675(-1+M)\left(-1-16t+20t^2-8t^3+t^4\right)}{2(2+M)(\gamma+4\pi)t^2\left(30-120t+100t^2-30t^3+3t^4\right)^2} \tag{3.38}$$

$$\omega(t) = -\frac{(-1+M)(2+M)\left(-1-16t+20t^2-8t^3+t^4\right)}{M^2(-1-16t+20t^2-8t^3+t^4)+2(2-16t+20t^2-8t^3+t^4)+3M(3-16t+20t^2-8t^3+t^4)} \tag{3.39}$$

The specifics of the parameters (q) in **Figure 3.9** are presented in **Table 3.5** through the utilization of $MATLAB$, thereby clarifying the notion of universal behavior.

Table 3.5: Explaining Universes behavior for $r = 5$ model

Parameter	**Universe Behavior**
$H(t)$	Starting the universe with a big bang at initial singularity $t = 0$ and undergoing an accelerating expansion to a big rip to decelerated expansion. Diverges at the big rip time $t = 0.334$, $t = 2.1$, $t = 3.4$ and $t = 4.1$. It also shows the cyclic behavior of the universe, but the cycle requires less time than the lower-degree polynomial model. (Fig 3.9(a))
$S(t)$	The universe's first Big Bang occurred at $t = 0$, and the first Big Rip occurred at $t = 0.333923$. At $t = 2.13497$, the Universe goes back to the first stage (the second Big Bang), at $t = 3.37508$, it goes to the second stage, and at $t = 4.15603$, it returns to the first stage. The scale factor diverges at both the first Big Rip $t = 0.333823$ and the second Big Rip $t = 3.37508$. (Fig 3.9(b))
$J(t)$	Jerk parameter is positive with time indicating that the Universe's expansion is accelerating. (Fig 3.9(c))
$\rho(t)$	This model does not satisfy the positive energy conditions throughout the time interval from 0 to 5. It begins with a Big Bang at $t = 0$, characterized by a positive energy density (ρ). Subsequently, ρ decreases, taking on negative values before reaching the first Big Rip stage at $t = 0.35$, where $\rho = -655.8$. Afterward, ρ starts to increase and reaches a second Big Rip at $t = 2.15$, with $\rho = 673.1$. There are two additional Big Rip events: one at $t = 3.4$, where $\rho = -198.8$, and another at $t = 4.15$, with $\rho = 1528$. (Fig 3.9(d))
$P(t)$	The pressure starts with a big bang at $t = 0$ with a negative value. The plot represents negative pressure at times 0 to 4 and becomes positive at $t > 4$. Big Rip stages occur at $t = 0.35$, $t = 2.15$, $t = 3.4$ with negative pressure, and at $t = 4.15$ with positive pressure. (Fig 3.9(e))
$\omega(t)$	The universe enters a big bang stage initially and enters a Big Cranch stage at $t = 0.15$. On the phase of $t = 1.2$ to 2.8 and the universe is dominated by Phantom energy since $0 < \omega < -1$ which drives an ever-accelerating expansion leading to the catastrophic "Big Rip" scenario. Otherwise it signifies the presence of exotic matter or energy in cosmology since $\omega > 1$. (Fig 3.9(f))

At $t = 0$ (the initial Big Bang) in the $r = 5$ scenario, the Universe enters a long period of deceleration with $q = 1$. At both the start and the finish of the two phases, Hubble's parameter exhibits a divergence. The Big Rip phase is included in the $r = 5$ model, which is a generalization of the PVDP model.

The described $r = 5$ universe model is characterized by an increasing jerk parameter, signifying an accelerated expansion. Negative pressure, typically associated with dark energy, influences this expansion. The nearly constant scale factor during a diverging stage suggests a significant cosmic transition. The positive energy density points to the presence of matter or energy. This model captures the late-time universe, with dark energy dominating expansion, possibly driving cosmic acceleration. The scale factor's behavior hints at a unique epoch, potentially marking a transition between deceleration and acceleration. Together, these characteristics paint a picture of a universe undergoing dynamic changes, with dark energy playing a dominant role in driving its accelerated expansion at a specific stage of cosmic history.

4 Conclusion

In our paper, Bianchi type-I space-time is investigated by assuming PDP with $f(R, T)$ gravity theory. The findings of this study indicate that a Universe may remain static, neither expanding nor contracting, when the DP follows a variable polynomial function and eventually settles at a constant value for $r = 1$ model. This phenomenon aligns with the predictions made by established cosmological models such as the Milne model, the radiation model, the Einstein de Sitter model, and the Einstein model. From $r = 2$ model, DP is linear and all other parameters are time dependent, which indicates the Universes accelerated expansion with negative pressure. The DP is appeared in second degreee when we consider VPDP for $r = 3$ model. Where we see the universe transition from deceleration to acceleration with singularity at $t = 0$ and $t = 1$ and also this model predicts universe's accelerated expansion dominated by dark energy.Cubic DP arises from a model with $r = 4$, indicating an accelerated universe that undergoes periodic changes. It begins with a Big Bang, featuring accelerated expansion, then transitions to a Big Rip phase with decelerated expansion as time progresses. Eventually, it returns to the Big Bang stage and repeats the cycle. During this process, the pressure violates the positive energy conditions. Finally, $r = 5$ model shows multiple Big Rip stage with accelerated expansion. The universe's energy density oscillates, transitioning from positive to negative and then back to positive, and vice versa. This cyclic pattern suggests a dynamic, oscillatory universe, where expansion and contraction phases continually repeat for $t = 0$ to 4 where pressure is negative. Big Crunch appears in $r = 5$ model.

For Bianchi type-I model $r = 1$ to 4, the universe is dominated by dark energy with accelerated expansion, since pressure $P < 0$ and energy density $\rho > 0$. But the $r = 5$ model shows an unusual form of the universe.

It's important to note that the exact nature and properties of dark energy have been partially understood. Its presence and behavior in the universe are based on current cosmological models and observational data. Further research and observations are needed to refine our understanding of dark energy and its implications for the future evolution of the universe.

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