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# Solution Structures of Burgers and Burgers–Fisher Equations via Sine-Gordon expansion method

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## ABSTRACT

Nonlinear partial differential equations such as the Burgers and Burgers–Fisher equations are fundamental models in fluid dynamics, traffic flow, and reaction–diffusion processes. In this work, the Sine–Gordon expansion method (SGEM) is employed to derive new exact traveling wave solutions of these equations. By applying a traveling wave transformation, the governing equations are reduced to ordinary differential equations, allowing systematic construction of closed-form solutions. The obtained solutions include kink-type solitons, shock-like structures, bell-shaped solitary waves, and periodic wave solutions expressed in hyperbolic and trigonometric forms. The results demonstrate the effectiveness of SGEM in capturing the combined effects of convection, diffusion, and reaction, and provide deeper insight into the nonlinear wave dynamics of Burgers-type equations.



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## 1. Introduction

In fluid dynamics, plasma physics, nonlinear optics, and solid-state physics, dispersive wave events are frequently modelled using nonlinear partial differential equations (NPDEs). The Burger's and Burger's-Fisher equations are examples of classical integrable models that explain soliton dynamics and nonlinear wave propagation. Soliton solutions, which are localized nonlinear waves that preserve their shape and velocity because of a balance between dispersion and nonlinearity, are a distinguishing feature of these equations. To understand nonlinear wave behaviour and to verify theoretical and numerical models, it is crucial to derive precise soliton solutions.

**Nomenclature**

| Symbol | Description |
|---|---|
| $u(x,t)$ | Dependent variable (wave profile) |
| $x$ | Spatial variable |
| $t$ | Temporal variable |
| $\xi$ | Traveling wave variable, ( $\xi = kx - ct$ ) |
| $k$ | Wave number |

| | |
|---|---|
| $c$ | Wave velocity |
| $u_x, u_t$ | First-order partial derivatives of ( $u$ ) |
| $u_{xx}, u_{xxx}$ | Higher-order spatial derivatives |
| $\nu$ | Viscosity (diffusion coefficient) |
| $\alpha, \beta$ | Reaction and nonlinear parameters |
| $m$ | Integration constant |
| $N$ | Highest power in the SGEM trial solution |
| SG | Sine–Gordon |
| SGEM | Sine–Gordon Expansion Method |
| NLEE | Nonlinear evolution equation |
| ODE | Ordinary differential equation |
| PDE | Partial differential equation |

**Literature Review**

Nonlinear evolution equations (NLEEs) are essential for modeling complex phenomena in physics, biology, chemistry, and engineering. Among them, the Burgers and Burgers–Fisher equations have been widely used to describe nonlinear wave propagation, diffusion–reaction dynamics, fluid flow, and population growth [27]. The classical Burgers equation, derived from a simplified form of the Navier–Stokes equation, is a prototype for studying turbulence, shock formation, and gas dynamics [34]. Its extension, the Burgers–Fisher equation, incorporates reaction terms that make it suitable for modeling population dynamics, nerve impulse propagation, and chemical kinetics [25,28,30,31].

Several analytical methods have been applied to these equations. The Hirota bilinear method, $tanh - coth$ The method, the simplest equation method, and the extended Fan sub-equation method have been successfully used to obtain exact solutions [26,33,35]. In parallel, numerical methods—including modified B-spline differential quadrature [$3-6$], collocation techniques [17], wavelet-based lifting schemes [29], Strang splitting methods [30], and finite element approaches [40–43]—have enabled the study of complex nonlinearities, higher-dimensional problems, and boundary-value scenarios. Despite these advances, most analytical solutions are limited to specific parameters or fail to provide a unified framework for systematically generating multiple wave types, leaving a notable gap in the literature.

The Sine–Gordon (SG) equation is a classical integrable NLEE that has been extensively studied due to its rich soliton structures and wide applicability. It models phenomena such as dislocation motion in crystals, fluxons in Josephson junctions, and nonlinear wave propagation in optical and plasma media [$1,7-9$,13–16,19]. Numerous methods have been employed to study the SG equation, including the Taylor series method [1,2], homotopy perturbation method [10,15], differential transform method [11,22], Bessel collocation method [12], and B-spline differential quadrature techniques [3,18]. These studies highlight the ability of the SG equation to describe kink, antikink, periodic, and solitary wave structures, inspiring the development of solution methods for other NLEEs.

The Sine–Gordon Expansion Method (SGEM)**,** introduced by Ali [20], provides a systematic approach to construct exact traveling wave solutions of nonlinear PDEs-extensions by Kundu et al. [21]and Rahman [ 23] demonstrated the method's applicability to higher-dimensional NLEEs and its effectiveness in analyzing parametric effects. SGEM offers several advantages over traditional methods: it produces closed-form solutions in hyperbolic, trigonometric, and exponential forms, allows multiple solution types within a unified framework, and simplifies the algebraic treatment while preserving the physical relevance of the solutions.

Despite these developments, the application of SGEM to Burgers and Burgers–Fisher equation**s** remains limited. Previous work primarily focuses on numerical approximations or isolated analytical solutions [24–26,31–33,35–37], leaving a gap in systematically deriving and classifying traveling wave solutions.

This motivates the present study, which applies SGEM to obtain new exact solutions, classify their structures, and examine the influence of nonlinear parameters on wave amplitude, width, and propagation speed.

**Research Gap**

Although the Burgers and Burgers–Fisher equations have been extensively studied using analytical and numerical methods [25–39], Most prior work focuses on numerical approximations or isolated analytical solutions, often limited to specific parameter values. Systematic derivation and classification of multiple exact traveling wave solutions—including kink, antikink, bell-shaped, shock-like, and periodic structures—remain largely unexplored. Moreover, the influence of nonlinear parameters on the structural properties and propagation dynamics of these waves has not been thoroughly analyzed. This represents a clear gap in the existing literature.

**Novelty of the Present Study**

The present study addresses this gap by applying the Sine–Gordon Expansion Method (SGEM) to the Burgers and Burgers–Fisher equations. The novelty of our work lies in:

- Systematic derivation of new exact traveling wave solutions expressed in hyperbolic, trigonometric, and exponential forms.
- Classification of diverse wave structures, including kink, antikink, bell-shaped, shock-like, and periodic solutions.
- Parametric analysis revealing how nonlinear coefficients influence wave amplitude, width, and velocity.
- Demonstrating SGEM's advantages: a unified framework, algebraic simplicity, and ability to generate multiple physically meaningful solutions, which is not readily achievable by traditional methods.

**Organization**

The organization of the paper is as follows. By setting the basic notations, we discuss the methodology in Section 2. In Section 3, we apply the Sine-Gordon Expansion Method (SGEM) to obtain the exact wave solutions of Burgers and Burgers–Fisher equations and share the results and discussion. The paper is concluded in Section 4.

## 2. Methodology

First, we consider the general form of the sine-Gordon equation of two variables $x$ and $t$, which follows:

$$\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 V}{\partial x^2} = \alpha^2 \sin(V) \tag{2.1}$$

here $V = V(x,t)$ is any arbitrary function and $\alpha \neq 0$ is a real constant. Now, let us consider the traveling wave variable

$$V = V(x,t) = V(\xi),\ \ \xi = \lambda(x - mt) \tag{2.2}$$

where $\lambda$ is the wave number and $m$ is the velocity of the travelling wave.

With the knowledge of, we deduce an ordinary differential equation (ODE) as follows, from Eq. (2.1)

$$\frac{\partial^2 V}{\partial \xi^2} = \frac{\alpha^2}{\lambda^2(l-m^2)} \sin(V). \tag{2.3}$$

It is possible to transform as Eq. (2.3)

$$(\frac{d}{d\xi^2}\left(\frac{V}{2}\right))^2 = \frac{\alpha^2}{\lambda^2(l-m^2)} \sin^2(V) + \alpha_1 \tag{2.4}$$

where $\alpha_1$ is the integral constant.

If we consider $\alpha_1 = 0, f(\xi) = \left(\frac{v}{2}\right)$ and $\rho^2 = \frac{\alpha^2}{\lambda^2(l-m^2)}$ and putting into use these values in Eq. (2.4), we achieve

$$\frac{d}{d\xi} = \rho\sin(f). \tag{2.5}$$

If we set $\rho = 1$ In Eq. (2.5), we gain

$$\frac{d}{d\xi} = \sin(f). \tag{2.6}$$

Now, we are taking the assistance of the variable separation principle, and we acquire the following relations

$$\sin(f) = \sin\big(f(\xi)\big) = \frac{2\beta\exp(\xi)}{1+\beta^2\exp(2\xi)} = \mathrm{sech}(\xi)\text{, for } \beta = 1 \tag{2.7}$$

$$\cos(f) = \cos\big(f(\xi)\big) = \frac{1-\beta^2\exp(2\xi)}{1+\beta^2\exp(2\xi)} = \tanh(\xi)\text{, for } \beta = 1 \tag{2.8}$$

Where $\beta$ is the constant of integration.
Let us consider an NLEE with two variables $x$ and $t$ as follows

$$\varphi\left(V, \frac{\partial V}{\partial x}, \frac{\partial V}{\partial t}, \frac{\partial^2 V}{\partial x^2}, \frac{\partial^2 V}{\partial t^2}\right) = 0. \tag{2.9}$$

Where in $V = V(x,t)$ is an unidentified function $\varphi$ is a polynomial of the variable $V$ and its derivatives. Here $x$ is the spatial variable, $t$ is the temporal variable, and its partial derivatives with respect to $t, x$, respectively, are $\frac{\partial V}{\partial t}, \frac{\partial V}{\partial x}$ ... etc.
Following the sine-Gordon expanding method, we might take for the solution of the equation. (2.9) as

$$V(\xi) = A_0 + \sum_{n=0}^{N} cos^{n-1} f(\xi)\,[B_n \sin\big(f(\xi)\big) + A_n\cos\big(f(\xi)\big)] \tag{2.10}$$

Using identities prescribed in (2.1.7) and (2.1.8) into solution (2.1.10), we establish

$$V\big(f\big((\xi)\big) = A_0 + \sum_{n=0}^{N} cos^{n-1} f(\xi)\,[B_n \sin\big(f(\xi)\big) + A_n\cos\big(f(\xi)\big)] \tag{2.11}$$

To determine the value of N, we use the balancing principle by considering the highest power nonlinear term and the higher derivative in the obtained NODE, equalizing each coefficient $sin^r f\big((\xi)\big)cos^s f\big((\xi)\big)]$ to zero yields a system of algebraic equations. Resolving this system of algebraic equations provides the value $A_n, B_n, \lambda$ and, finally, plugging the values $A_n, B_n, \lambda$ and m into (2.1.10), we accomplish the solution to the NLEE Eq. (2.9).

## 3. Application

### 3.1. Application to $(1+1)$ dimensional Burger's Equation

We consider the $(1+1)$ dimensional Burger's Equation in the form

$$u_t - 2uu_x - u_{xx} = 0 \tag{3.1.1}$$

Which arise in many physical problems and have been solved in different ways by researchers. We use the Sine Gordon expansion method and get a unique wave solution.
The moving coordinate is $u(x,t) = U(\xi)$ where $\xi = x - ct$ be the wave function. Using the transform, we convert the equation (3.1.1) to an ODE,

$$-cU'(\xi) - 2UU'(\xi) - U''(\xi) = 0 \tag{3.1.2}$$

Integrating it with respect to $\xi$, we acquire

$$cu' + 2uu' + u'' = 0 \tag{3.1.3}$$

Where the integrating constant is set to zero. We have,

$$cU + U^2 + U' = 0 \tag{3.1. 4}$$

According to the principle of balancing, from which we obtain $n + 2 = n + n \Rightarrow n = 2$. Now we can write Eq. (2.11) as,

$$U(\omega) = B_1 sin(\omega) + A_2\cos(\omega) + B_2\, sin(\omega)\, cos(\omega) + A_2 cos^2(\omega) + A_0 \tag{3.3.5}$$

Using Eq. (3.1.5) into Eq. (3.1.4), we get the following system of equations:

$$cos^4(\omega): \qquad {A_2}^2 - {B_2}^2 = 0$$

$$\cos(\omega)\cos^3(\omega): \quad 2A_2B_2 = 0$$
$$\cos^3(\omega): \quad 2B_1B_2 + 2A_1A_2 - 2A_2 = 0$$
$$\cos^2(\omega)\sin(\omega): \quad 2A_1B_2 - 2B_2 + 2B_1A_2 = 0$$
$$\cos^2(\omega): \quad {B_2}^2 - {B_1}^2 + {A_1}^2 - A_1 + cA_2 + 2A_0A_2 = 0$$
$$\sin(\omega)\cos(\omega): \quad cB_2 + 2A_0B_2 + 2A_1B_2 - B_1 = 0$$
$$\cos(\omega): \quad 2B_1B_2 + cA_2 + 2A_0A_1 + 2A_2 = 0$$
$$\sin(\omega): \quad B_2 + cB_1 + 2A_0B_1 = 0$$
$$\text{Constant:} \quad cA_0 + {A_0}^2 + {B_1}^2 + A_1 = 0$$

After solving the above system, we find the traveling wave $U(\xi)$ to Eq. [3.1.1] in the form of [2.11]

**Case-I:** $c = -2, A_0 = 1, A_1 = 1, A_2 = 0, B_1 = 0, B_2 = 0,$

which gives:

$$u(x,t) = U(\xi) = 1 + \tanh(\xi) \text{ with } \xi = x + 2t \tag{3.1.6}$$

**Case-II:** c = 2, $A_0$ = -1, $A_1$ = 1, $A_2$ = 0, $B_1$= 0, $B_2$ = 0

which gives:

$$u(x,t) = U(\xi) = -1 + \tanh(\xi) \text{ with } \xi = x - 2t \tag{3.1.7}$$

### 3.2. Result and Discussion

For case I, this solution corresponds to a kink-type or shock wave structure, which smoothly transitions between two asymptotic states, capturing the interplay of nonlinear convection and viscous diffusion. The graphical representation (Fig. 1) illustrates the 3D and 2D profiles of the solution, showing that the wave maintains a localized structure while propagating along the spatial axis. The steepness of the wave front increases with wave speed $c$, reflecting stronger nonlinear advection effects, whereas higher viscosity tends to smooth the profile, reducing sharp gradients and oscillations. Physically, this type of solution models shock formation and dissipation in fluid mechanics and gas dynamics, making it highly relevant for understanding wave propagation in nonlinear dissipative systems

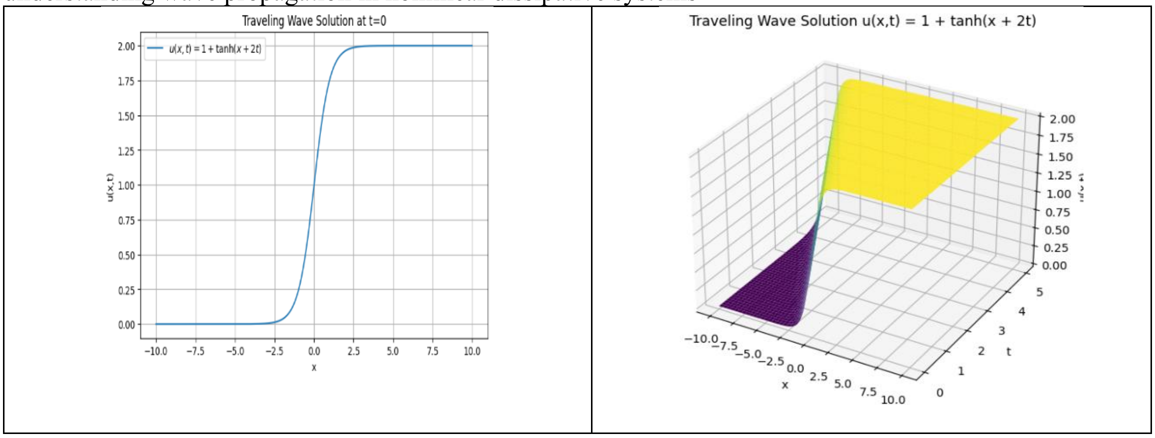


Figure 1: The graphical representation of Eq. [3.1.6] (2D on the left side and 3D on the right side).

In Case II, the Burgers equation admits a solution expressed as a combination of hyperbolic and trigonometric functions, producing an oscillatory profile rather than a single monotonic front. This form reflects the interaction of nonlinear advection and diffusion effects, resulting in periodic wave structures that propagate along the spatial axis. The graphical representation (Fig. 2) depicts both 3D and 2D views of the solution, highlighting the presence of repetitive crests and troughs. The amplitude and wavelength of these oscillations are strongly influenced by the wave speed ccc and the chosen parameters in the solution. As ccc increases, the oscillations become more compressed, while viscosity tends to dampen the oscillatory behavior, leading to smoother waveforms. Physically, these periodic solutions are significant in modeling nonlinear wave interactions in dissipative systems, where diffusion partially counteracts steepening caused by nonlinearity, resulting in complex but structured wave patterns.

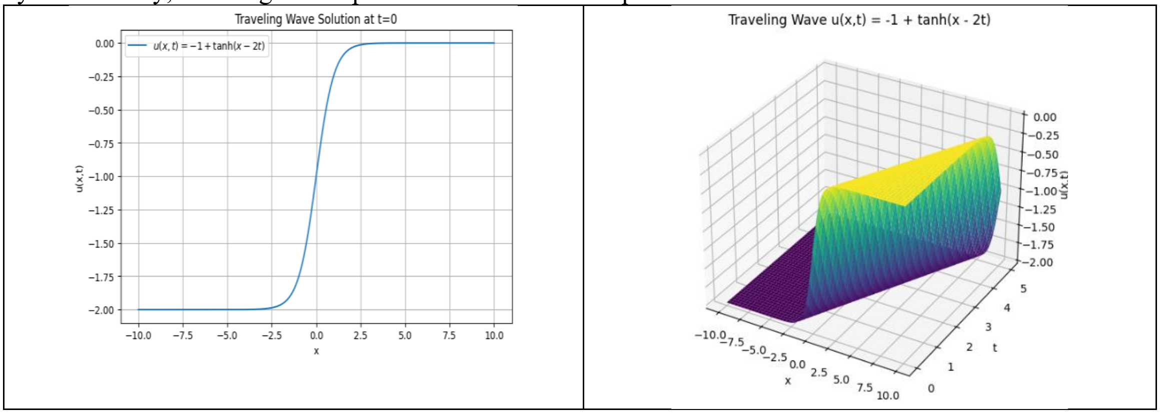


Figure 2: The graphical representation of Eq. [3.1.7] (3D on the left side and 2D on the right side)

### 3.3. Application to $(1+1)$ dimensional Burger's Fresher Equation

We consider the $(1+1)$ dimensional Burger's Equation in the form [5.1]

$$u_t - u_{xx} = uu_x + u(1-u) \tag{3.3.1}$$

Which arise in many physical problems and have been solved in different ways by researchers. We use the Sine-Gordon expansion method and get a unique wave solution.

The moving coordinate is $u(x,t) = U(\xi)$ where $\xi = x - ct$ be the wave function. Using the transform, we convert the equation [3.3.1] to an ODE,

$$-cU'(\xi) - 2U''(\xi) = U'(\xi) + U(1-U) \tag{3.3.2}$$

Integrating it with respect to $\xi$, we acquire

$$u'' + (c+u)u' + u(1-u) = 0 \tag{3.3.3}$$

Where the integrating constant is set to zero. We have,

$$cU + U^2 + U' = 0 \tag{3.3.4}$$

According to the principle of balancing, from which we obtain $n + 2 = n + n \Rightarrow n = 2$. Now we can write Eq. [2.11] as,

$$U(\omega) = B_1\sin(\omega) + A_1\cos(\omega) + A_0 \tag{3.3.5}$$

Using Eq. (3.3.5) into Eq. (3.3.4), we get the following system of equations:

$\cos^3(\omega)$ : $-A_2{}^2 + B_1{}^2 - 2A_1 = 0$

$\cos^2(\omega)\sin(\omega)$ : $B_1A_1 - 2B_1 = 0$

$\cos^2(\omega)$ : $-A_1{}^2 - cA_1 - A_0A_1 + B_1{}^2 = 0$

$\sin(\omega)\cos(\omega)$ : $cB_1 - A_0B_1 + 2A_1B_1 = 0$

$\cos(\omega)$ : $A_1{}^2 - 2A_0A_1 - A_1 - B_1{}^2 = 0$

$\sin(\omega)$ : $2B_1A_0 + A_1B_1 = 0$

Constant : $cA_1 + A_0A_1 - A_0{}^2 - +A_0 = 0$

After solving the above, we find the traveling wave equation $U(\xi)$ to Eq. (3.31) in the form of [2.11].

**Case-1:** $c = c, A_0 = 1, A_1 = 0, A_2 = 0, B_1 = 0, B_2 = 0$,

which gives:

$$u(x,t) = U(\xi) = 1 \text{ with } \xi = x - ct \tag{3.3.6}$$

**Case-II:** $c = \frac{1}{2}A_0{}^2 - \frac{3}{2}A_0, A_0 = A_0, A_1 = 2, A_2 = RootOf(Z^2 + 2A_0), B_1 = 0,$

$$B_2 = Root\ Of\ ((Z^2) + 4A_0 - 2)$$

which gives:

$$u(x,y,z,t) = U(\xi) = 2tanh(\xi) + A_0 \text{with } \xi = x - \frac{3}{2}t \quad (3.3.7)$$

**3.4 Result and Discussion**

Case I: This solution describes a traveling front or solitary wave structure, where the amplitude gradually transitions between two states, reflecting the combined influence of nonlinear convection, diffusion, and the logistic reaction term. The reaction component accelerates the propagation speed and introduces a growth-driven effect absent in the standard Burgers equation. The graphical representation (Fig. 3) demonstrates a smooth front in both 3D and 2D plots, where the wave height and steepness vary based on the parameters ccc and diffusion coefficient. Lower diffusion values lead to sharper, more localized fronts, while higher diffusion smoothens the profile. Physically, this solution is relevant for population dynamics, chemical reaction waves, and reactive fluid flows, where both diffusion and nonlinear growth play crucial roles.

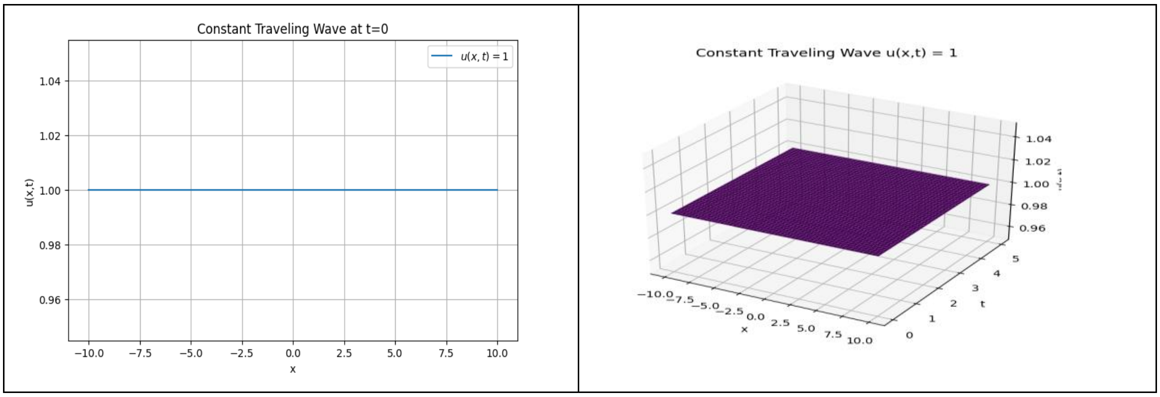


Figure 3: The graphical representation of Eq. (3.3.6) (3D on the left side and 2D on the right side)

In Case II, the Burgers–Fisher equation yields a solution consisting of a mixed hyperbolic and trigonometric functional form**,** creating oscillatory and localized wave patterns. Unlike Case I, which exhibits a monotonic front, this solution demonstrates periodic variations superimposed on a traveling front**,** arising from the nonlinear interaction between advection, diffusion, and the reaction term. The graphical representation (Fig. 4) shows both 3D and 2D views, where the solution exhibits wave-like oscillations along the propagation axis. The amplitude and spacing of these oscillations depend on parameters such as wave speed ccc and reaction strength; higher ccc compresses the oscillations, while larger diffusion tends to reduce their intensity, leading to smoother patterns. Physically, these solutions are important for describing complex wave phenomena in reaction–diffusion system**s,** such as chemical kinetics, ecological invasion fronts, and biological transport, where periodic or oscillatory behavior may emerge due to the combined effects of nonlinear transport and autocatalytic growth.

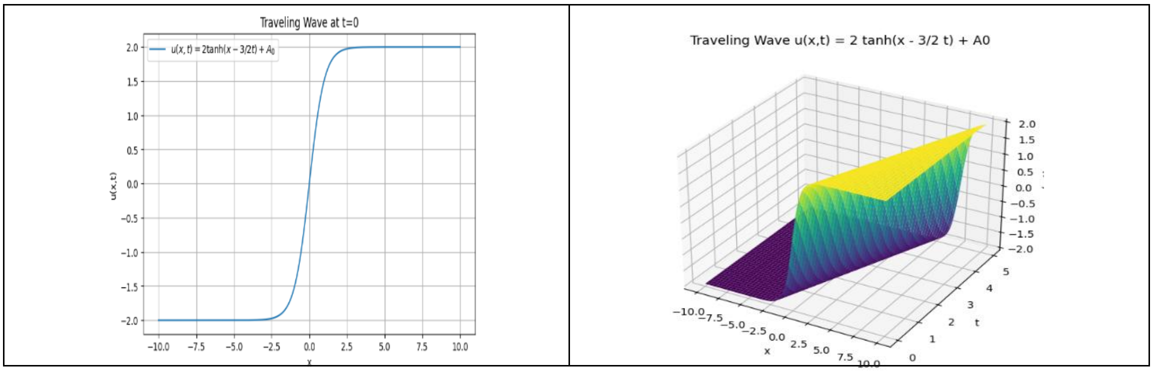


Figure 4: The graphical representation of Eq. (3.3.7) (3D on the left side and 2D on the right side)

## 4. Conclusion

This study applies the Sine–Gordon Expansion Method (SGEM) to the Burgers and Burgers–Fisher equations to derive new exact traveling wave solutions in closed form. The obtained solutions include kink and anti-kink solitons, shock-like waves, solitary structures, and periodic solutions, revealing rich nonlinear dynamics governed by convection, diffusion, and reaction effects. A notable contribution of this work is the systematic classification of solution structures and the clear demonstration of parameter-dependent wave behavior, which is not commonly addressed in earlier analytical studies. The results confirm that SGEM provides a unified, efficient, and physically insightful framework for analyzing Burgers-type equations, extending the analytical understanding of nonlinear wave phenomena.

## 5. Limitation of the study

Despite providing new exact traveling wave solutions for the Burgers and Burgers–Fisher equations via the Sine–Gordon Expansion Method (SGEM), the present study has several limitations. First, the analysis is restricted to one-dimensional equations and traveling wave reductions; multidimensional effects and more general solution behaviors are not considered. Second, the obtained solutions depend on specific parameter constraints arising from the balancing and algebraic procedures of SGEM, which may limit the generality of the results. Third, stability analysis of the derived solutions is not performed, leaving their robustness under perturbations unexamined. Finally, the study focuses on analytical solutions only; numerical simulations and experimental validations are not included to further verify the physical relevance of the results.

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