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Approximate Shortest Distance and Direction between two Places on the Spherical Earth and the Oblate Spherical Earth

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Abstract— Differential geometry is an important aspect of current physics and the study of any planar surface. Quantum mechanics and modern science are developing with the help of differential Geometry in daily life. In this paper, we discuss two mathematical formulas named Haversine formula for spherical earth and Lambert's formula for oblate spherical earth. These formulas are used to determine the shortest distance between two points on Earth, as well as the sine formula for determining the direction of various locations towards the North Pole. Some examples of different locations are illustrated in this literature which gives a better overview.

Index Terms - Differential Geometry, Oblate sphere, Latitude, Longitude, Haversine formula, Lambert's formula.

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I. INTRODUCTION

EARTH is a real-life approximation of a sphere. But the earth is not a perfect sphere, it has an extensive surface [1]. Since the earth is slightly flatted at the ends of its axis of rotation it is called 'oblate spheroid'. Humans need to calculate the distance between two locations on earth for many reasons and the calculation is not very easy. An algorithm named the Haversine algorithm can be used to calculate the shortest distance between two coordinates that are on the surface of the earth [2], [3], [4]. Latitude and longitude are common things in every place on the earth. These comprehend a coordinate scheme that can detect or recognize geographic positions on the surfaces of planets like earth. Latitude can be defined concerning the equatorial reference plane which passes through the center of the earth, and also contains the great circle representing the equator [5]. Longitude can be defined in terms of meridians which are considered as half-circles running from pole to pole. Longitudes are defined by a reference meridian named prime meridian [6]. Haversine algorithm is used to calculate the shortest distance by applying those and with the help of spherical geometry which also estimates the value for travel the shortest distant traveling [7], [8].

With the progress of technology, data and information in cyberspace are becoming more and more accessible through regular handheld gadgets like a smart phone without complication. Knowing those two points on the earth is sufficient to measure the distance and hence find ways to destination with the help of specialized calculating tools [9].

There are different formulas for determining the distance between two locations on the flat and round planes [10], [11]. But, spherically curved planes like earth require the involvement of trigonometry which makes these types of problems sophisticated [12]. This formula is one such tool to approach solving these types of problems.

Determining the best route to the destination requires an intelligent route search. The suitable selection of branches is also a problem in route searching. Using this single algorithm can help determine the shortest distance to the destination.

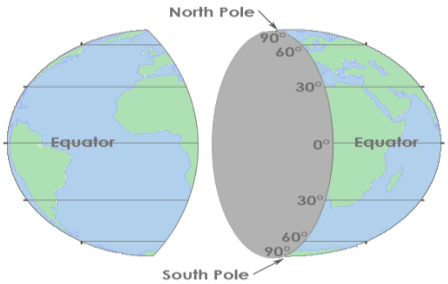

***Fig 4**:* Longitude (Vertical Circles)

F. *Geodesic*

The Geodetic Reference System 1967 will be superseded by the Geodetic Reference System 1980, which will be based on the theory of the geocentric equipotential ellipsoid and described by the following conventional constants:

- Equatorial radius of the Earth:
  $a = 6378137\ m$
- Polar radius of the Earth:
  $b = 6356752.314\ m$
- Flattering for the Earth:
  $f = \frac{a-b}{a}$
  $= 0.003352810703188$
- Mean radius of the Earth:
  $r = 6\ 371\ 008.7714\ m$

G. *Oblate Sphere*

The equatorial distance is higher than the distance between the poles in this planetary configuration. The Earth is a planet with an oblate shape. An oblate spheroid is a well-known shape. It is the form of the Earth and a few other planets. It looks like a sphere that has been squished from the top such that the diameter around the poles is less than the circumference around the equator [7]. This sort of shape is known as an ellipsoid. The flatter the oblate spheroid, the faster the spin.

It has the form of an oblate spheroid. This simply implies that it is flatter towards the poles and wider at the equator. Because of centrifugal force during rotation, the Earth bulges near the equator. The bulk pushes outwards and flattens out along the axis of rotation, much to spinning a pizza on, the 5th of January, 2021.

A well-known shape is an oblate spheroid. The Earth and some other planets have this form. It's like a sphere that's been crushed from the top so that the circumference around the poles is less than the equator's circumference. This type of shape is known as an ellipsoid.

III. Formulation

Theorem 1.0

If the arc $s = AB$ of the circle r subtends an angle θ at the center, then $s = r\theta$, where θ is measured in radians.

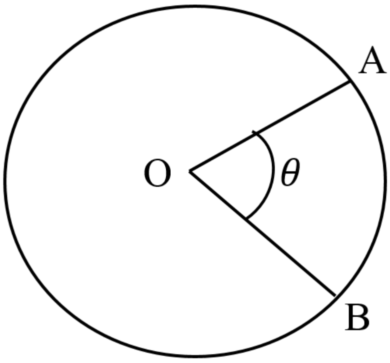

***Fig 5**:* Circle subtends an angle θ at the centre

A. *Haversine's Formula for Spherical Earth Distances*

Calculate the central angle θ in radians between two points (φ_1, λ_1) and (φ_2, λ_2)on a sphere using the Great-circle distance method (law of Haversine formula)

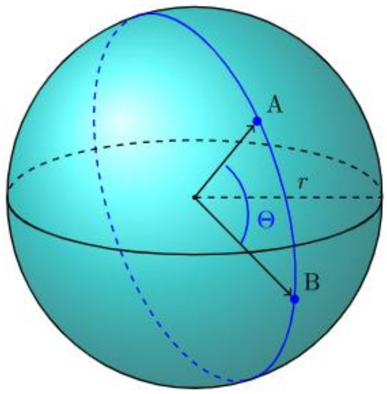

***Fig 6**:* Two points A(φ_1, λ_1) and B(φ_2, λ_2) on Spherical Earth

Central angle θ:
The Haversine formula follows the Haversine of θ (that is, hav(θ)) to be computed directly from the latitude (represented by φ) and longitude (represented by λ) of the two points:

$$\text{hav}(\theta) = \text{hav}(\varphi_2 - \varphi_1) + \cos(\varphi_1)\cos(\varphi_2)\,\text{hav}(\lambda_2 - \lambda_1)$$

One can prove the formula by transforming the points given by their latitude and longitude into Cartesian coordinates, then taking their dot product.