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LINEAR NUCLEAR ACOUSTIC WAVES IN DEGENERATE QUANTUM PLASMA

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***Abstract*—A rigorous theoretical investigation has been made on the linear propagation of electrostatic perturbation modes of degenerate pressure driven modified nucleus-acoustic (DPDMNA) 'waves in a degenerate quantum plasma (DQP) system. It contains cold inertia-less degenerate electron species (DES), cold inertial non-degenerate light nucleus species (LNS) and stationary heavy nucleus species (HNS) which maintains the quasi-neutrality condition at equilibrium only. The mass density of the cold LNS provides the inertia and the cold inertia-less cold LNS provides the inertia and the cold inertia-less DES gives rise to the restoring force. The reductive perturbation method has been used for the study of nonlinear propagation of the DPDMNA waves. The basic features of linear waves are supervised in a theoretical manner. It has been observed that the phase speed of the DPDMNA waves changes with the change of charge density of the stationary HNS for both non-relativistic and ultra-relativistic DES; The NA waves with their dispersion properties which are consequential in various astrophysical and laboratory plasmas, have been broadly considered.**

***Index Terms*—Degenerate electron, Nucleus-acoustic waves, Quantum plasma.**

I. Introduction

In recent years, modern plasma physics is very much concentrated on the degenerate quantum plasma (DQP) system [1]. Through plasma physics is sometime considered as classical field, it may be quantum because of its particle's quantum nature [2]. The DQP systems are significantly different from other plasma systems because of their extra-ordinarily high density particularly, for astrophysical plasma systems [3, 4, 5, 6, 7, 8] and low temperature particularly, for laboratory plasma systems [9, 10, 11, 12, 13]. They usually contain inertialess non-relativistically (NR) or ultra-relativistically (UR) degenerate electron species (DES), cold inertial non-degenerate light nucleus species (LNS) like ${}^{4}_{2}\mathrm{He}$ [5] or ${}^{12}_{6}\mathrm{C}$ or ${}^{16}_{8}\mathrm{O}$ [4, 6] and heavy nucleus species (HNS) like ${}^{56}_{26}\mathrm{Fe}$, ${}^{85}_{37}\mathrm{Rb}$ or ${}^{96}_{42}\mathrm{Mo}$ [14, 15].

The degeneracy of the cold electron species (in such DQP systems) arises due to Heisenberg's uncertainty principle, and the uncertainty in momenta of extremely DES are infinitely large. This introduces a pressure called 'degenerate pressure' which depends only on the cold electron number density, but not on the electron thermal temperature. The degenerate pressure $\mathcal{P}$ exerted by the DES is given by [16, 17, 18, 19, 20, 21, 22]

$$\mathcal{P} = K n_e^{\gamma}, \tag{1}$$

where n_e is the number density of the DES, and γ and K are

$$\gamma = \frac{5}{3}, \quad K = \frac{3}{5}(\frac{\pi}{3})^{\frac{1}{3}}\frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c\hbar c, \tag{2}$$

for the non-relativistic limit [8, 17] , and

$$\gamma = \frac{4}{3}, \quad K = \frac{3}{4}(\frac{\pi^2}{9})^{\frac{1}{3}}\hbar c \simeq \frac{3}{4}\hbar c, \tag{3}$$

for the ultra-relativistic limit [8, 17],

where $\Lambda_c = \pi\hbar/m_e c$, $\hbar$ is the reduced Planck's constant, and m_e is the rest mass of the cold DES.

The DQP systems are significantly different from other plasma systems not only because they have unique properties (extra-ordinarily low density and temperature), but also because they introduce new kinds of waves. One of these kinds of waves is predicted here. The present work is attempted to find the possibility for the existence of such a new kind of waves, called here degenerate pressure-driven modified nucleus-acoustic (DPDMNA) waves, in which the inertia is provided by the mass density of the cold inertial LNS, and the restoring force is provided by the degenerate pressure of the cold inertia-less NR or UR DES, and to identify the dispersion properties of the linear DPDMNA waves propagating in such a DQP system. A linear system is a system in which the output is linearly proportional to input. For an example, an amplifier (or any other system) to an input signal

$$y = y_o \cos(\omega t) \tag{4}$$

can be considered, where ω is the signal frequency resulting in an output which can be expressed as output α input in case of small amplitude i.e the response of the system is linear to its input. In linear theory, the wave amplitude is assumed to be sufficiently small to ignore contributions of terms of second order or higher (i.e., nonlinear terms) in wave

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amplitude. However, when the wave amplitude becomes larger, the linear approximation breaks down and nonlinear effects must be taken into account.

A DQP system is considered containing non-inertial DES [5] represented by the subscript e, and inertial LNS (which can be ${}^{4}_{2}\mathrm{He}$ or [4, 8] or ${}^{12}_{6}\mathrm{C}$ or ${}^{16}_{8}\mathrm{O}$ [4, 6]) represented by the subscript l, and the HNS (which can be ${}^{56}_{26}\mathrm{Fe}$, ${}^{85}_{37}\mathrm{Rb}$ or ${}^{96}_{42}\mathrm{Mo}$ [14, 15]) represented by the subscript h. Thus, at equilibrium we have $n_{e0} = Z_l n_{l0} + Z_h n_{h0}$, where n_{e0} (n_{l0}) n_{h0} is the number density of the DES (LNS) HNS e (l) h, and Z_l (Z_h) is the charge state of the NLNS (SHNS). The dynamics of the DES is describe by the balance between the electrostatic force ($n_e e \partial\phi/\partial x$) and the degenerate pressure force ($\partial\mathcal{P}/\partial x$). Thus, substituting (1) into $n_e e\partial\phi/\partial x = \partial\mathcal{P}/\partial x$, the number density n_e of the DES can be expressed as

$$n_e = n_{e0}\left[1 + \left(\frac{\gamma-1}{\gamma}\right)\frac{e\phi}{\mathcal{E}_{e0}}\right]^{\frac{1}{\gamma-1}}, \tag{5}$$

where $\mathcal{E}_{e0} = Kn_{e0}^{\gamma-1}$ is the energy associated with the electron degenerate pressure; ϕ is the electrostatic wave potential; x (t) is the space (time) variable; e is the charge of a proton. We note that (5) is valid for the DPDMNA waves, whose phase speed is much smaller than C_e [where $C_e = (\mathcal{E}_{e0}/m_e)^{1/2}$]

II. Mathematical Model

The dynamics of these DPDMNA waves is described by

$$\frac{\partial n_l}{\partial t} + \frac{\partial}{\partial x}(n_l u_l) = 0, \tag{6}$$

$$\frac{\partial u_l}{\partial t} + u_l\frac{\partial u_l}{\partial x} = -\frac{Z_l e}{m_l}\frac{\partial\phi}{\partial x} + \eta_l\frac{\partial^2 u_l}{\partial x^2}, \tag{7}$$

$$\frac{\partial^2\phi}{\partial x^2} = 4\pi e\left(n_e - Z_l n_l - Z_h n_{h0}\right), \tag{8}$$

where n_l (u_l) is the number density (fluid sped) of the cold inertial LNS l and η_l is the co-efficient of the viscous fluid of light nuclei. It may be noted here that (i) the effect of the self-gravitational field is neglected since it is inherently small in comparison with the electric field in the DQP systems under consideration; (ii) the effect of the degeneracy of the cold LNS is neglected since the DPDMNA wave phase speed is much larger than C_l [where $C_l = (\mathcal{E}_{l0}/m_l)^{1/2}$ and $\mathcal{E}_{l0} = K_l n_{l0}^{\gamma-1}$], $\mathcal{E}_{l0}$ is the degenerate energy of the cold LNS, $K_l = 3\Lambda_{cl}\hbar c/5m_l$, and $\Lambda_{cl} = \pi\hbar/m_l c$; (iii) the consideration of stationary HNS is valid as long as $\omega_{ph} \ll \omega$ [where $\omega_{ph} = (4\pi n_{h0}Z_h^2 e^2/m_h)^{1/2}$ is the heavy nucleus plasma frequency, ω is the frequency of the DPDMNA waves, and m_h is the mass of the HNS].

To find the linear dispersion relation for the DPDMNA waves, (6)−(8) are linearized to reduce them to a set of linear equations:

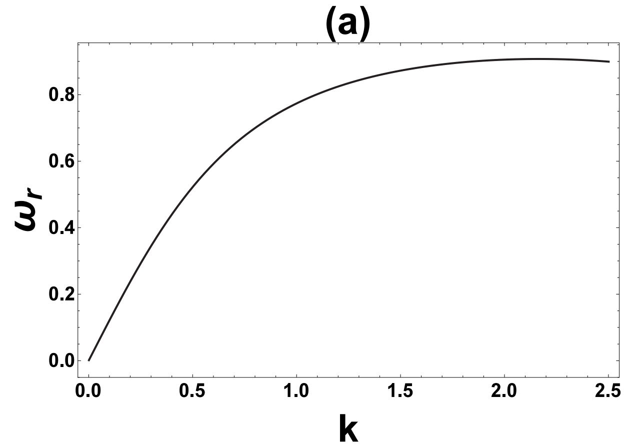

Fig. 1: The dispersion curve of the DPDMNA waves for $\alpha = 0.9$, $\gamma = 4/3$ and $\eta = 0$.

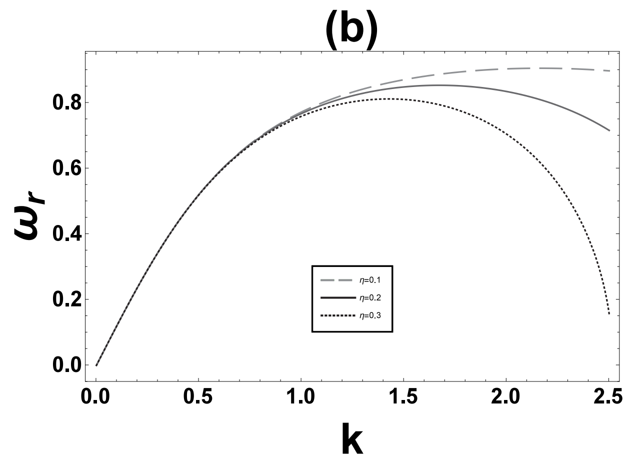

Fig. 2: The dispersion curves of the DPDMNA waves for $\alpha = 0.9$, $\gamma = 4/3$ and $\eta = 0.1$ (dashed curve); $\gamma = 5/3$ and $\eta = 0.2$ (solid curve); $\gamma = 5/3$ and $\eta = 0.3$ (dotted curve).

$$\frac{\partial\tilde{n}_l}{\partial t} + \frac{\partial}{\partial x}(n_{l0}\tilde{u}_l) = 0, \tag{9}$$

$$\frac{\partial\tilde{u}_l}{\partial t} = -\frac{Z_l e}{m_l}\frac{\partial\tilde{\phi}}{\partial x} + \eta_l\frac{\partial^2\tilde{u}_l}{\partial x^2}, \tag{10}$$

$$\frac{\partial^2\tilde{\phi}}{\partial x^2} = 4\pi e\left[\left(\frac{n_{e0}}{\gamma}\right)\frac{e\tilde{\phi}}{\mathcal{E}_{e0}} - Z_l\tilde{n}_l\right], \tag{11}$$

where $\tilde{n}_l$, $\tilde{u}_l$, and $\tilde{\phi}$ are the perturbed parts of the quantities involved. It is now assumed that all of these perturbed quantities are directly proportional to $\exp(-i\omega t + ikx)$, where k is the propagation constant of the DPDMNA waves. This assumption reduces (9)−(11) to the linear dispersion relation for the DPDMNA waves:

$$\tilde{\omega}^2(1+\alpha\gamma\tilde{k}^2) + i\tilde{\omega}\tilde{\eta}(1+\alpha\gamma\tilde{k}^2) - \tilde{k}^2\alpha\gamma = 0, \tag{12}$$

where $\alpha = Z_l n_{l0}/n_{e0}$ with $\mu = 1-\alpha$. Here, ω, k and η_l are normalized by ω_{pl}, L_q^{-1} and $\omega_{pl}L_q^2$ respectively,

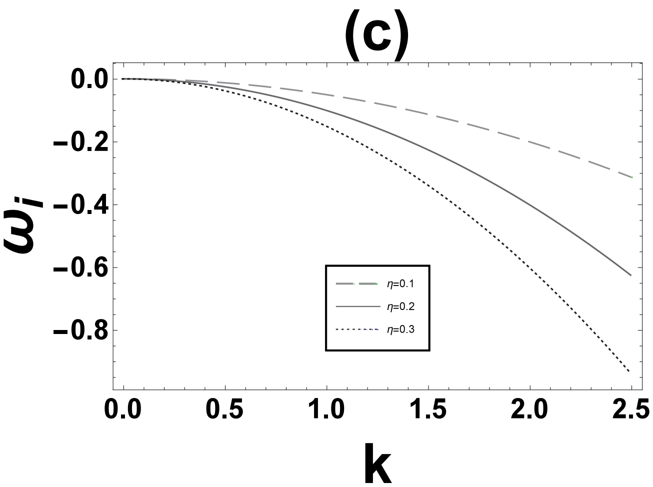

Fig. 3: The dispersion curves of the DPDMNA waves for $\gamma = 5/3$, $\alpha = 0.9$ and $\eta = 0.1$ (dashed curve), $\eta = 0.2$ (solid curve) and $\eta = 0.3$ (dotted curve).

where $L_q = (\mathcal{E}_{e0}/4\pi Z_l n_{l0})^{1/2}$ is the modified DQP screening length; $\omega_{pl} = (4\pi n_{l0} Z_[^2 n_{l0}/m_l)^{1/2}$ is the light nucleus plasma frequency.

III. Results & Discussions

It is obvious that (i) in the DPDMNA waves the inertia is provided by the mass density of the LNS, while the restoring force is provided by the degenerate pressure of the DES; (ii) the presence of positive HNS significantly modifies the dispersion properties of the DPDMNA waves; (iii) The phase speed V_p of the DPNA decreases with the rise of the charge number density of the stationary HNS in both non-relativistically and ultra-relativistically DES. This is due to the fact that as the charge number density of the stationary HNS increases, the repulsive force (acting between positive LNS and HNS) reduces the frequency of compression and rarefaction of the LNS.

When we apply that $\eta = 0$, the result becomes exactly equal to the outcome of [8] as shown in fig 1, in his paper which ensures our accuracy of this coreresponding report.

The numerical results are depicted in figures 2 and 3, which indicate that (i) the nature of the dispersion curves for the new DPDMNA waves is similar to that for the well known ion-acoustic (IA) waves; (ii) the UR DES and HNS have insignificant effect on the dispersion curves for both short and long wavelength limits ($kL_q \ll 1$ and $kL_q \gg 1$), but they have significant effects on the dispersion curves in between these two limits.

To conclude, the new results, which have been found from this investigation, can be pinpointed as follows:

(i) The existence of a new kind of nucleus-acoustic waves (named here DPDMNA waves) with viscous force propagating in DQP systems is predicted for the first time. (ii) The DPDMNA waves are new not only from the view of the restoring force (which is essential for the existence of any kind of waves), but also from the view of their phase speed and length scale as shown in Table I.

TABLE I: The basic differences between the new DPDMNA waves and the well-known IA waves.

Properties	DPDMNA Waves	IA Waves
Restoring force	Degenerate pressure	Thermal pressure
Phase speed	$\sqrt{\frac{\mathcal{E}_{e0}}{m_l}}$	$\sqrt{\frac{k_B T_e}{m_l}}$
Existence at $T_e = 0$	Possible	Not possible
Length scale	$\sqrt{\frac{\mathcal{E}_{e0}}{4\pi n_{e0} e^2}}$	$\sqrt{\frac{k_B T_e}{4\pi n_{e0} e^2}}$

(iii) The DPDMNA waves completely disappear if the degenerate pressure associated with DES is neglected.

(iv) The phase speed V_p of the DPDMNA waves decreases with the rise of the charge number density of the stationary HNS in both non-relativistic and ultra-relativistic situations of DES. This is due to the fact as the charge number density of the stationary HNS increases, the repulsive force (acting between positive LNS and HNS) reduces frequency of the compression and rarefaction of the LNS. We finally hope that the DPDMNA waves and their dispersion properties with viscous force (identified here for the first time) should be useful in understanding the basic features of the electrostatic perturbation mode in space [3, 4, 5, 6] and laboratory [9, 10, 11] DQP systems.

Author contribution statement

Both of the authors contributed equally to the paper.

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