

# Mathematical Modeling for Launch Vessel Operators at Internal Waterways Transportation in Bangladesh

Tanzila Yeasmin Nilu, Hashnayne Ahmed and Shek Ahmed

**Abstract**— Launch based water vessel transportation has been growing in Bangladesh as an easier and cheaper transportation system. The shipping liners are introducing more launches in different routes as the growing demands of passengers in spite of having much difficulties with respect to uncertainty. However, this trade sector is facing imbalance in terms of profits in several service categories but they have to fight with uncertainties, a common issue faced by the liner operators. In this paper, we develop a mathematical model for the launch vessel operators to compare the profit considering different scenarios which give a broad idea about meeting the customer's demand over time, and maximize their profit by considering different stochastic parameters. It is necessary to make decision about building launches as per demand categories, and also to maximize the profit. We show how scenario-based stochastic optimization could be applied in this perspective and discuss some solving techniques with modification as required. Finally, we conduct a numerical case study to evaluate the value of the stochastic scenario-based model over some uncertain issues according to the model.

**Index Terms**— Uncertainty Modelling, Stochastic Programming, Scenarios, Transportation.

## I. INTRODUCTION

**T**HE ECONOMIC growth of a country depends largely on the transportation system in that country. And the transportation system could be classified according to the transportation of goods, public transport, etc. in a faster and easiest way at the cheaper cost.

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Bangladesh is a developing country and recently it has become one of the fastest economically growing countries in the world. The transportation system in this country has faced a dramatic change and it directly affects the export-based economy of this country. For internal goods transportation and public transport, the launch vessel is one of the most famous waterway-based vehicles in this country. This medium of transportation is very famous among the people of the country as it is cheaper than any other available transportation system.

It also gives the most comfortable way of traffic less journey to a section of peoples of the country. Currently, about 12% of people in the country (about 20 million) depends on launch vessel transportation like public transport and goods transportation. It is predicted that there will be a slight increase in the size of launch vessel transport business. For example, one major vessel operator on the internal waterway, who provides day-by-day launch service with a single one now serves more than 6 launches with larger and luxurious vessels

This paper focuses in developing a stochastic model that can assist launch vessel operators to minimize their total costs while maximizing their profit by deciding on the space distribution for cabin passengers, goods, and deck passengers in their upcoming vessels. It will also give a rear view of costs in different weather situations as very bad weather, bad, and normal weather situations. The core of this model is the number of possible cabin passengers, goods unit, and deck passengers, which have been taken as random variables. We then propose a numerical study and possible solution techniques for the real-life scenario-based problem. We illustrate how the cost fluctuates in different periods annually and by taking weather situations into account.

The literature of this paper could be characterized into two categories: modeling a stochastic model for launch

vessel operators and solving the models for making comparisons according to different possible situations. In stochastic programs, observing the uncertain parameters gives the decision values. The common objective is to optimize the objective function and expected future values with the current decision. The expected future values are known as the recourse functions, depend on the current decision. Solution methods for stochastic programs could be of three kinds: non-linear approximation, stochastic quasi-gradient methods, and scenario methods. In scenario methods, we use some fixed samples to approximate probability space and then solve the large-scale problem with decomposition techniques. Finally, through numerical tests, we compare the behavior of stochasticity occurred in different scenarios. [3, 8-13]

## II. PRELIMINARIES

### A. Stochastic Programming (SP)

Stochastic programming is the study of mathematical optimization (maximizing profit or minimizing cost) that deals with stochasticity. Stochasticity means randomness, not exactly uncertainty. We may also call it uncertainty with a possible probability distribution. Stochastic programming aims to find an optimal decision with random data. A stochastic programming problem can be expressed as the following canonical form:

$$\begin{aligned} & \text{Optimize (Maximize or Minimize)} z = c^T x + \\ & \quad E_{\xi} Q(x, \xi) \\ & \text{Subject to, } Ax(\geq, \text{ or } =, \text{ or } \leq) b \\ & \quad \text{and } x \geq 0 \end{aligned}$$

where  $x$  is the decision variables and corresponding coefficients are  $c$  and  $b$ . The second term in the objective function is for uncertain data where  $E_{\xi}$  is referred to as the mathematical expectation concerning some random vector  $\xi$ . [1]

### B. Scenario-based Stochastic Programming

There are many types of stochastic programming problems: expected value problems, chance constraints problems, scenario-based problems, etc. The scenario optimization is a technique for obtaining solutions to problems based on a sample of the constraints. Where a scenario is a sequence of events constructed to focus on casual processes and decision points.

For scenario-based optimization we may rewrite the stochastic programming problem as follows:

$$\begin{aligned} & \text{Optimize (Maximize or Minimize)} z = c_{\omega}^T x_{\omega} \\ & \text{Subject to, } Ax_{\omega}(\geq, \text{ or } =, \text{ or } \leq) b \\ & \quad \text{and } x_{\omega} \geq 0 \end{aligned}$$

where  $x_{\omega}$  is the stochastic decision variables and  $c_{\omega}$  is the corresponding coefficients with random experiments  $\omega$ .

After considering probabilities from experience, we may choose the mathematical formulation as follows:

$$\begin{aligned} & \text{Optimize (Maximize or Minimize)} z = p_{\omega} c_{\omega}^T x_{\omega} \\ & \text{Subject to, } Ax_{\omega}(\geq, \text{ or } =, \text{ or } \leq) b \\ & \quad \text{and } x_{\omega} \geq 0 \end{aligned}$$

where  $p_{\omega}$  is the corresponding probability distribution for scenarios  $\omega$ . [2, 19, 23]

## III. A MULTI-PERIOD STOCHASTIC MODEL

### A. Problem Definition

Bangladesh is a riverine country. A large number of people in this country depend on rivers for their transportation. Indeed, it is comparatively cheaper as well as advantageous than other existing transportation facilities. River-based transportation is not only used for travelling but also for goods and other necessary things for daily life. Dhaka is the capital of Bangladesh and the economy is mainly based on capital city of this country. We consider the divisional city Barishal, which is mainly surrounded by rivers clearly maximum transportation of this region is river-based. Launch transportation is the most popular transport from Barishal to the capital city, Dhaka. We considered a launch ‘‘Marine Vessel Manami’’ of the renowned navigation company on this lane called ‘‘Salam Shipping Line Ltd.’’ for our problem. They operate two launches every day in our considered riverway: one for Dhaka to Barishal and another one for Barishal to Dhaka and we will study the problems as a combined issue for the launch vessel company. Their target is to get the maximum profit under the issues they face; but some random parameters like weather, the effect of other launches, fluctuation of deck ticket prices, amount of goods, etc. reduces their profit. In this mathematical model, we aim to discuss their profit situations under the variation of weather statistics and make the decision variables easier to take.

For launch operators, we may divide the season of this region as the following three time periods considering the weather situation: summer (April, May, June, July), rainy season (July, August, September, and October), winter (November, December, January, February). We may also consider three scenarios based on the weather occurs in these periods: very bad weather condition, bad weather condition, and normal weather condition. For launch transportation, we may divide the transportation into three categories: cabin passengers, goods transport, and deck passengers. There are a lot of uncertain issues that arise considering the categories.

### B. Labeling

Let us define the following notations for the considered categories: (single trip and daily)

(Index Sets)

- ❖  $t$ , number of periods.
- ❖  $\omega$ , the number of scenarios.

(Cabin Passengers)

For the cabin passengers, there are different categories: single cabin ( $a$ ), double cabin ( $b$ ), semi-VIP cabin ( $c$ ), VIP cabin ( $d$ ), family cabin ( $e$ ), etc. For the omission of complexity, we will convert the number of total cabins, cost per cabin, number of passengers, and prices per cabin into a single cabin unit according to the cost.

- ❖  $x_i^T$ , the number of paid cabin unit for periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (stochastic decision variable)
- ❖  $a_i^T$ , profit per cabin units for periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (uncertain parameter)
- ❖  $p_i^T = \alpha_i^T \beta_i^T$ , the probability for cabin passengers where  $\alpha_i^T$ , is the probability for the number of paid cabin and  $\beta_i^T$ , is the probability for profit per cabin for periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (uncertain parameter)
- ❖  $m_c$ , average rented cabin units considering all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (deterministic parameter)
- ❖  $\sum_{i=1}^{\omega} x_i^T = b_c$ , the sum of total cabin units for all scenarios ( $i = 1, 2, \dots, \omega$ ) in a single period where  $x' = a + b + c + d + e + \dots$  is the total cabin units. (deterministic parameter)

(Goods Transportation)

- ❖  $y_i^T$ , amount of total goods unit for periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (stochastic decision variable)
- ❖  $b_i^T$ , profit per unit goods transportation for periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (uncertain parameter)
- ❖  $q_i^T = \gamma_i^T \lambda_i^T$ , the probability for goods transportation where  $\gamma_i^T$ , is the probability for the amount of total goods unit and  $\lambda_i^T$ , is the probability for profit per unit goods transportation for periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (uncertain parameter)
- ❖  $m_g$ , average transportation goods units considering all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (deterministic parameter)
- ❖  $\sum_{i=1}^{\omega} y_i^T = b_g$ , the sum of the total amount of goods units for all scenarios ( $i = 1, 2, \dots, \omega$ ) in a single period where  $y'$ , is the total possible goods unit for transportation. (deterministic parameter)

(Deck Passengers)

- ❖  $z_i^T$ , the number of total deck passengers for periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (stochastic decision variable)
- ❖  $c_i^T$ , the profit per head deck passengers for periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (uncertain parameter)
- ❖  $r_i^T = \phi_i^T \psi_i^T$ , the probability for deck passengers where  $\phi_i^T$ , is the probability for the number of total deck passengers and  $\psi_i^T$ , is the probability for profit per head deck passengers for periods  $T =$

$1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (uncertain parameter)

- ❖  $m_d$ , average boarding deck passengers considering all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ . (deterministic parameter)
- ❖  $\sum_{i=1}^{\omega} z_i^T = b_d$ , the sum of total deck passengers for all scenarios ( $i = 1, 2, \dots, \omega$ ) in a single period where  $z'$ , is the total possible goods unit for transportation. (deterministic parameter)

(Notice that the parameters and the variables are defined if corresponding launch schedule exists and it is for a daily basis single trip. We can use the model for the double trip too, in that case, no major changes should have been made. There is some other influencer to the stochastic decision variables: effects of other launches, special days, fluctuation of deck ticket prices, and the face value of the company to the passengers. We concluded these in the stochastic decision variable as we collected the needed data from experience.

Also, the per-unit profit has been determined based on experience. Since it depends on weather conditions and other uncertain factors, the profit has been taken as stochastic. It estimates after considering all the launch expenses like engine cost, manufacturing cost, manpower cost, etc. and corresponding incomes.)

### C. Formulation of the Stochastic Model

Our objective is to maximize the total profit satisfying the criteria assumptions for all periods  $T = 1, 2, \dots, t$ , and scenarios  $i = 1, 2, \dots, \omega$ . Therefore, the stochastic model is given by,

$$\text{Maximize } z = \sum_{i=1}^{\omega} \sum_{T=1}^t p_i^T a_i^T x_i^T + \sum_{i=1}^{\omega} \sum_{T=1}^t q_i^T b_i^T y_i^T + \sum_{i=1}^{\omega} \sum_{T=1}^t r_i^T c_i^T z_i^T$$

Subject to

- $\sum_{i=1}^{\omega} x_i^T \leq b_c$ , for all periods  $T = 1, 2, \dots, t$ .
- $\alpha_i^T x_i^T < m_c$ , for all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ .
- $\sum_{i=1}^{\omega} y_i^T \leq b_g$ , for all periods  $T = 1, 2, \dots, t$ .
- $\gamma_i^T y_i^T < m_g$ , for all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ .
- $\sum_{i=1}^{\omega} z_i^T \leq b_d$ , for all periods  $T = 1, 2, \dots, t$ .
- $\phi_i^T z_i^T < m_d$ , for all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ .

and  
 $x_i^T, y_i^T, z_i^T \geq 0$ ; where  $T = 1, 2, \dots, t$  and  $i = 1, 2, \dots, \omega$ .

(Notice that we may consider the model for any scenarios ( $i = 1, 2, \dots, \omega$ ) as required, and any periods ( $T = 1, 2, \dots, t$ ) could be chosen as required.) [2, 7]

## IV. NUMERICAL CASE STUDY

### A. Physical Problem Description

“Marine Vessel Manami” is one of the renowned navigation companies of Bangladesh. They operate launch from Dhaka to Barishal and Barishal to Dhaka every day. However, they want to maximize their profit by minimizing their transportation cost but their profit can change due to some uncertainty issues such as weather conditions, political haphazard, lack of raw materials, demand of customers faced by the liner operators. We collected data from the company’s experience on an average basis.

In this model, we divided the whole year into three time periods ( $t = 3$ ) and considered three scenarios ( $\omega = 3$ ) for each period. For a day basis assumption, there are a total of 275 (single cabin - 71 (1000 Taka per), double cabin - 57 (2000 Taka per), family cabin -

10 (2500 Taka per), semi-VIP cabin - 6 (3500 Taka per), VIP - 4 (8000 Taka per), sofa - 20 (600 Taka per) cabin units in the launch, total amounts of goods capacity are 200 units (tons), and deck passenger capacity is 1000. For normal weather conditions, on average, about 261 cabin units (1000 Taka per), 150 tons goods (1000 Taka per), 700 deck passengers (200 Taka per head) have been rented. The vessel’s per day total cost is 3,90,000 Taka (Oil Cost - 250,000 Taka, Maintenance & Building - 80,000 Taka, Manpower Cost - 60,000 Taka) where the cost for per unit cabin is 728 Taka, per ton goods is 667 Taka, per head deck is 143 Taka. The probability table for the model is given below:

**Table 1.** Probability Table.

Periods	Scenario's	$\alpha_i^T$	$\beta_i^T$	$p_i^T$	$\gamma_i^T$	$\lambda_i^T$	$q_i^T$	$\phi_i^T$	$\psi_i^T$	$r_i^T$
Summer	Very bad	0.963	0.9	0.867	0.8	0.85	0.68	0.76	1	0.76
	Bad	1.05	1.1	1.155	0.95	1	0.95	1	1.1	1.1
	Normal	1.15	1.2	1.38	1	1.2	1.2	1.1	1.3	1.573
Rainy Season	Very Bad	0.96	0.88	0.845	1	0.7	0.7	0.85	0.87	0.74
	Bad	1.1	0.9	0.99	1.05	0.96	1.008	1.05	0.9	0.945
	Normal	1.2	1	1.2	1.1	1	1.1	1.1	0.95	1.045
Winter	Very Bad	0.96	0.85	0.816	0.78	0.88	0.686	0.73	0.9	0.657
	Bad	1.05	0.98	1.029	1.1	0.98	1.078	0.95	1.1	1.045
	Normal	1.1	1.05	1.155	1.2	1.1	1.32	1.15	1.2	1.38

**B. Solving Techniques**

There are many techniques for solving stochastic programming problems. Some of them are shortly described below:

**B.1. L-Shaped Method**

The main idea of the L-shaped algorithm is to approximate the non-linear term in the objective function. It was developed by exploiting the dual structure by Dantzig Wolfe decomposition, or a Benders Decomposition of the primal problem. The method has been extended in stochastic programming to take care of feasibility questions by Van Slyke and Wets. The main drawback of this method is, when the number of scenarios is too much, it is very time consuming to solve the problem by this method as of large iteration procedures. [4-6]

**Step 1.** Since the non-linear objective term involves a solution of all second stage recourse linear programs, it is obvious to avoid the evaluation of numerous functions. Therefore, step 1 decomposes the whole problem into a master problem, and the recourse function as a sub-problem.

**Step 2.** Now for the second stage recourse function, the algorithm works in an iterative procedure with a piecewise linear concave function, working as like cutting plane algorithm

**B.2. Modified Mamer & McBride’s Algorithm**

Mamer and McBride introduced the decomposition-based pricing process to solve large-scale problems in 2000 which becomes familiar as a DBP procedure. The strategy behind the decomposition can be summarized as *divide & relax-to-conquer*. It works by decomposing the whole large-scale problem into subproblem and master problem.

**Step 1.** Relax the complicating constraints of the whole problem by subtracting them from the objective function using any Lagrange multipliers  $\lambda^k \geq$  as the dual value of the relaxed constraints.

$$\text{Maximize } z = \sum_{i=1}^{\omega} \sum_{T=1}^t p_i^T a_i^T x_i^T - \lambda^k \left( \sum_{i=1}^{\omega} x_i^1 \leq b_c \right)$$

**Step 2.** Decompose the original problem where the sub-problem includes the relaxed objective function with the rest of the constraints.

$$\text{Maximize } z = \sum_{i=1}^{\omega} \sum_{T=1}^t p_i^T a_i^T x_i^T - \lambda^k \left( \sum_{i=1}^{\omega} x_i^1 \leq b_c \right)$$

Subject to,  $\sum_{i=1}^{\omega} x_i^T \leq b_c$ , for all periods  $T = 2, \dots, t$ .

$\alpha_i^T x_i^T < m_c$ , for all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ .

$x_i^T \geq 0$ , for all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ .

**Step 3.** If the relaxed sub-problem gives an optimal solution then generate the master problem as the original problem. Otherwise, generate a master problem by deleting any non-negative variables from the original problem makes a subset of the original column also called the “column restricted” version.

$$\text{Maximize } z = \sum_{i=1}^{\omega} \sum_{T=1}^t p_i^T a_i^T x_i^T$$

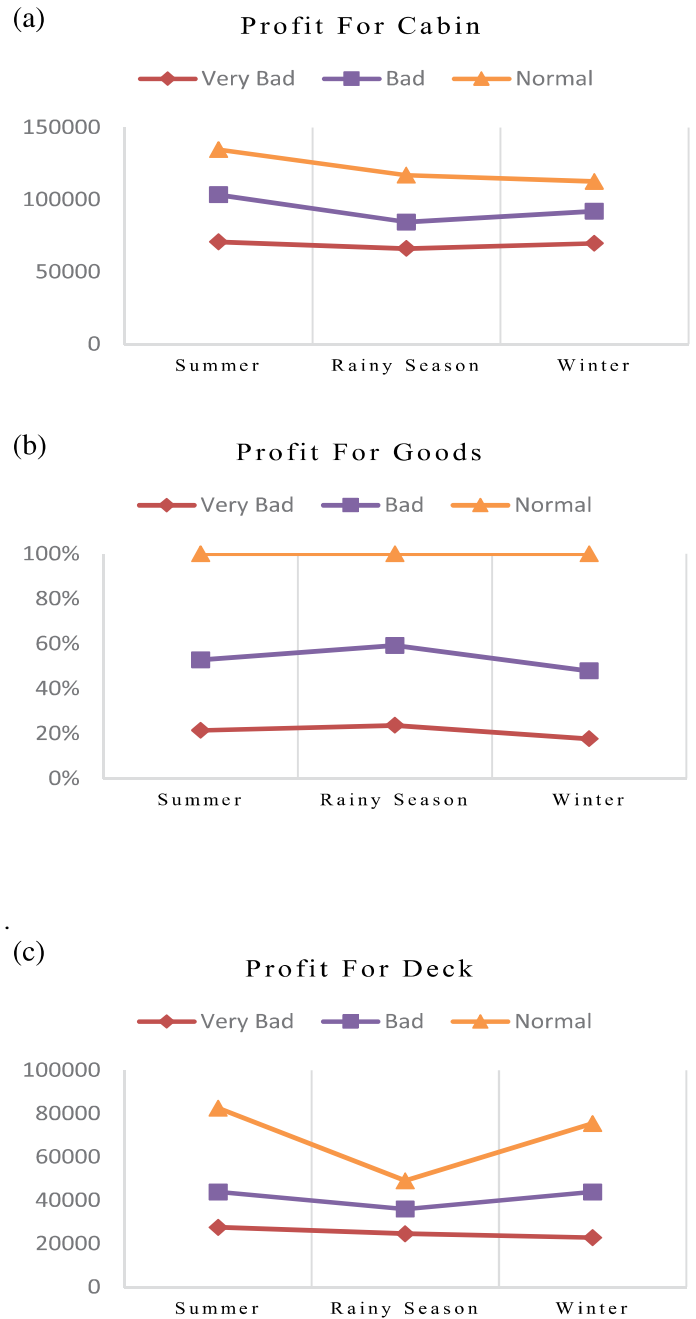
Subject to,  $\sum_{i=1}^{\omega} x_i^T \leq b_c$ , for all periods  $T = 2, \dots, t$ .  
 $\alpha_i^T x_i^T < m_c$ , for all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ .

$x_i^T \geq 0$ , for all periods  $T = 1, 2, \dots, t$  and scenarios  $i = 1, 2, \dots, \omega$ .

**Step 4.** If the sub-problem value equals the master problem value or no new variables come in the master problem, then stop. Otherwise, go to **step 1**. [2, 5, 14]

*C. Stochastic Scenario's on Total Profits*

For the above physical problem with the existing solving techniques, we get the following total profit comparison for three time periods (summer, rainy season, winter) with three scenarios (very bad, bad, normal): [8, 14-22]



**Figure 1.** Profit comparisons of (a) cabin units, (b) goods, and (c) deck passengers where their scenarios have differed from each other marked with three colors dark red (diamond points), orange (triangular points) and purple (square points).

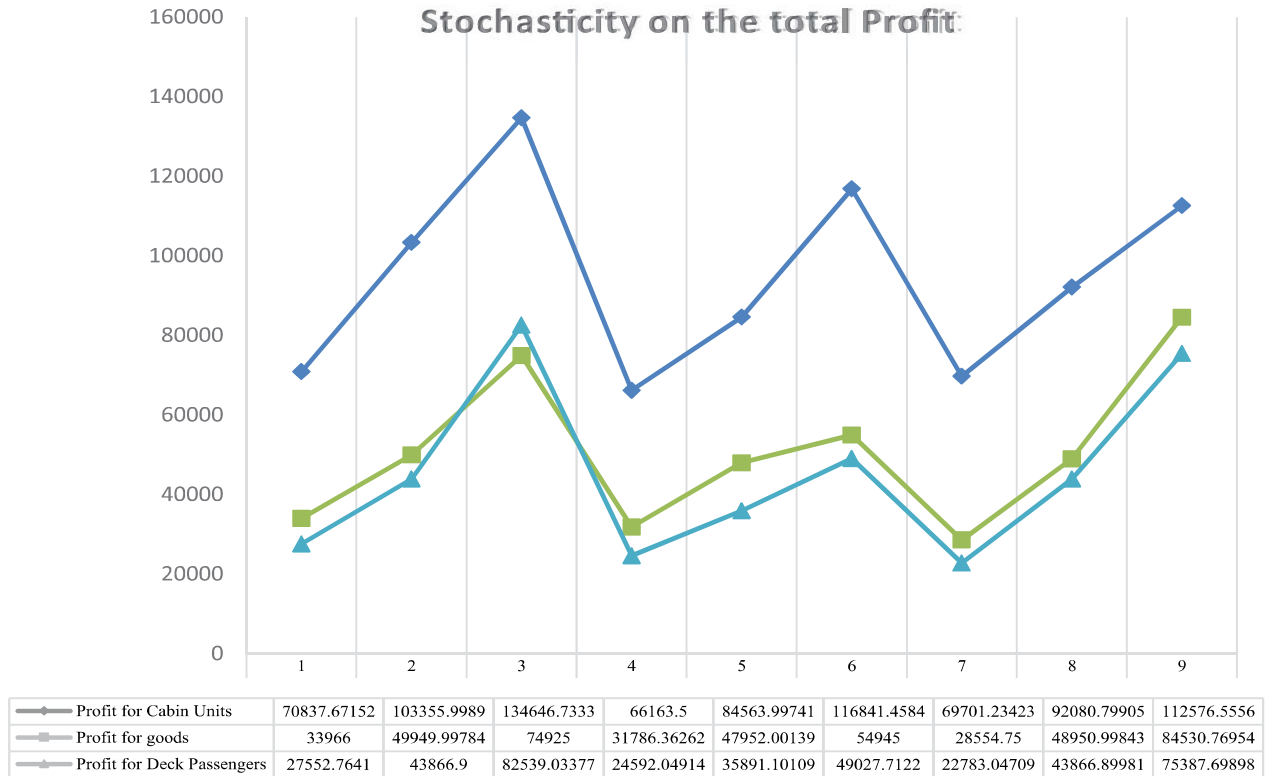


Figure 2. Stochasticity on the total profit

Here, Figure 1(a) represents the profit comparison for cabin units where their scenarios differed each other with three colors dark red, orange, and purple. In the same manner Figure 1(b), and 1(c) represents the profit comparisons for goods and deck passengers respectively. In Figure 2, column 1, 2, and 3 in the chart represents summer profit where column 1, column 2, and column 3 are for the scenarios very bad, bad, and normal respectively. Similarly, column 4, 5, and 6 is for the rainy season and column 7, 8, and 9 are for winter. Three different colors have been used to present the three different types of profit sectors: cabin units, goods, and deck passengers respectively. Again, in every column profit for cabin units, profit for goods transportation, and profit for cabin passengers have been colored differently. The profit has been calculated in Taka. It is clear that, in every period, normal weather condition has the most impact on maximizing profit, later bad weather conditions, and very bad weather conditions give poor profit in every case.

## V. CONCLUSION

In this paper, we had discussed the procedure of maximizing profit with some uncertain issues faced by shipping liners by formulating a stochastic scenario model. The decision could be made faster and more profitable as per the model as well as its numerical case study. A comprehensive idea was developed according to the weather conditions about boarding passengers and goods in several categories. The decision-based pricing solution technique had modified for this scenario-based model. We wish that this study can help to capture the issues for scenarios and periods as per arises. Finally, the numerical case study showed the comparison of profit which could be helpful for the liner operators to make decisions.

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