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Approximate Shortest Distance and Direction between two Places on the Spherical Earth and the Oblate Spherical Earth

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Abstract**—** Differential geometry is an important aspect of current physics and the study of any planar surface. Quantum mechanics and modern science are developing with the help of differential Geometry in daily life. In this paper, we discuss two mathematical formulas named Haversine formula for spherical earth and Lambert's formula for oblate spherical earth. These formulas are used to determine the shortest distance between two points on Earth, as well as the sine formula for determining the direction of various locations towards the North Pole. Some examples of different locations are illustrated in this literature which gives a better overview.

Index Terms - Differential Geometry, Oblate sphere, Latitude, Longitude, Haversine formula, Lambert's formula.

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I. INTRODUCTION

EARTH is a real-life approximation of a sphere. But the earth is not a perfect sphere, it has an extensive surface [1]. Since the earth is slightly flatted at the ends of its axis of rotation it is called 'oblate spheroid'. Humans need to calculate the distance between two locations on earth for many reasons and the calculation is not very easy. An algorithm named the Haversine algorithm can be used to calculate the shortest distance between two coordinates that are on the surface of the earth [2], [3], [4]. Latitude and longitude are common things in every place on the earth. These comprehend a coordinate scheme that can detect or recognize geographic positions on the surfaces of planets like earth. Latitude can be defined concerning the equatorial reference plane which passes through the center of the earth, and also contains the great circle representing the equator [5]. Longitude can be defined in terms of meridians which are considered as half-circles running from pole to pole. Longitudes are defined by a reference meridian named prime meridian [6]. Haversine algorithm is used to calculate the shortest distance by applying those and with the help of spherical geometry which also estimates the value for travel the shortest distant traveling [7], [8].

With the progress of technology, data and information in cyberspace are becoming more and more accessible through regular handheld gadgets like a smart phone without complication. Knowing those two points on the earth is sufficient to measure the distance and hence find ways to destination with the help of specialized calculating tools [9].

There are different formulas for determining the distance between two locations on the flat and round planes [10], [11]. But, spherically curved planes like earth require the involvement of trigonometry which makes these types of problems sophisticated [12]. This formula is one such tool to approach solving these types of problems.

Determining the best route to the destination requires an intelligent route search. The suitable selection of branches is also a problem in route searching. Using this single algorithm can help determine the shortest distance to the destination.

In section II, some basic ideas and theories will be discussed. In section III, the Haversine formula for spherical earth distance, Lambert's formula for oblate spherical earth distance, and Sine formula for direction between two points on earth towards the North Pole are discussed. Section IV represents some illustrative examples, and section V represents the result and discussion. Finally, the paper concludes in section VI.

II. Preliminaries

A. Great Circle

The intersection of the sphere's surface with a plane crossing through the sphere's center C generates a great circle. A great circle's radius equals the radius of a sphere. The large circle's center aligns with the sphere's center. A great circle divides a spherical into two halves. A smaller circle is cut off by an intersecting plane that does not pass through the center of the sphere (has a smaller radius).

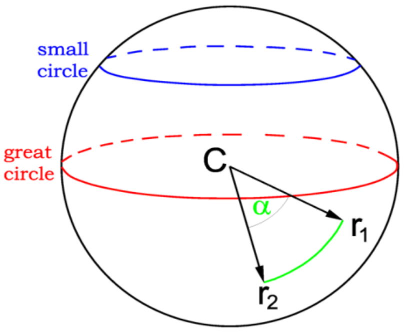

***Fig 1**:* Great Circle and Small Circle

It is feasible to show that a great circle passing between two points is a geodesic, or that the shortest distance line connecting the two points is a geodesic [13]. The image illustrates a plane that is bounded by two vectors (r_1) and (r_2) that intersect at C. As a result, the green arc forms part of the great circle that connects the two locations and symbolizes the shortest distance between them.

B. 3D Surface

A 3D surface model is a computer representation of features in three dimensional space, either physical or virtual. It is often created by sampling point, line, or polygon data and converting it to a digital 3D surface using specifically developed algorithms [14].

A landscape, an urban corridor, underground gas deposits, and a network of good depths to calculate water table depth are all examples of 3D surfaces. Although these are all instances of genuine characteristics, surfaces can be derived or imagined. The contamination levels of a specific bacteria in each well are an example of a derived surface. Those contaminants could also be mapped as a three-dimensional surface. The types of 3D surfaces shown in the film are frequently imaginative.

C. Sphere:

A sphere is a solid shape in which every point on its surface is equally far from the sphere's center, which is a fixed point within it.

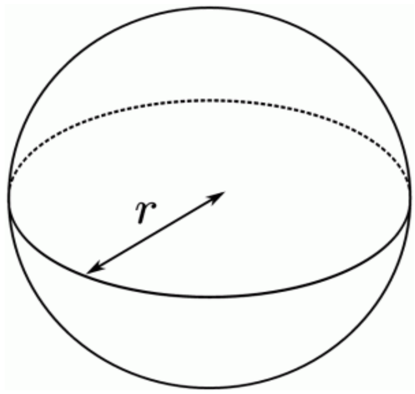

***Fig 2**:* Sphere with radius r

The radius of the sphere is any straight line connecting the center of the sphere to any point on the surface, while the diameter of the sphere is the straight line traced through the center and ended both ways by the sphere. The collection of all points in three-dimensional space that are all the same distance apart from the center.

D. Latitude

Latitude lines are east-west and parallel to one another. Latitude values grow as you travel north. Finally, latitude (Y-values) varies from -90 to +90 degrees [7].

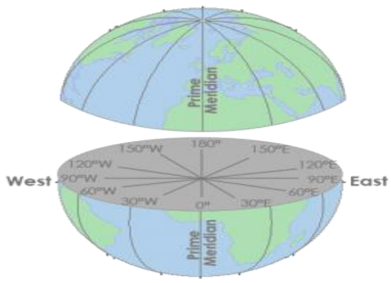

***Fig 3**:* Latitude (Horizontal Circles)

Longitude lines have X-coordinates from -180 to +180 degrees, as seen in the graph below

E. Longitude

Longitude lines are drawn from north to south. At the poles, they converge. Its X-coordinates range from -180 to +180 degrees. [7]. Lines of latitude, on the other hand, have Y-values ranging from -90 to +90 degrees.

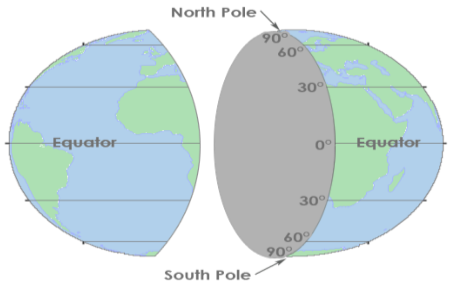

Fig 4: Longitude (Vertical Circles)

F. Geodesic

The Geodetic Reference System 1967 will be superseded by the Geodetic Reference System 1980, which will be based on the theory of the geocentric equipotential ellipsoid and described by the following conventional constants:

- Equatorial radius of the Earth:
  $a = 6378137\ m$
- Polar radius of the Earth:
  $b = 6356752.314\ m$
- Flattering for the Earth:
  $f = \dfrac{a-b}{a}$
  $= 0.003352810703188$
- Mean radius of the Earth:
  $r = 6\ 371\ 008.7714\ m$

G. Oblate Sphere

The equatorial distance is higher than the distance between the poles in this planetary configuration. The Earth is a planet with an oblate shape. An oblate spheroid is a well-known shape. It is the form of the Earth and a few other planets. It looks like a sphere that has been squished from the top such that the diameter around the poles is less than the circumference around the equator [7]. This sort of shape is known as an ellipsoid. The flatter the oblate spheroid, the faster the spin.

It has the form of an oblate spheroid. This simply implies that it is flatter towards the poles and wider at the equator. Because of centrifugal force during rotation, the Earth bulges near the equator. The bulk pushes outwards and flattens out along the axis of rotation, much to spinning a pizza on, the 5th of January, 2021.

A well-known shape is an oblate spheroid. The Earth and some other planets have this form. It's like a sphere that's been crushed from the top so that the circumference around the poles is less than the equator's circumference. This type of shape is known as an ellipsoid.

III. Formulation

Theorem 1.0

If the arc $s = AB$ of the circle r subtends an angle θ at the center, then $s = r\theta$, where θ is measured in radians.

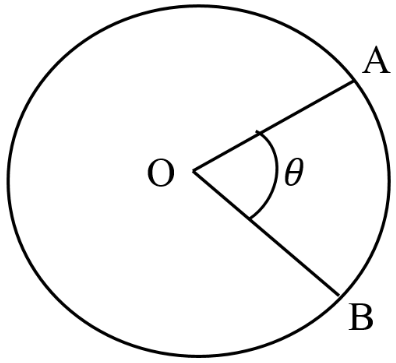

Fig 5: Circle subtends an angle θ at the centre

A. Haversine's Formula for Spherical Earth Distances

Calculate the central angle θ in radians between two points (φ_1, λ_1) and (φ_2, λ_2)on a sphere using the Great-circle distance method (law of Haversine formula)

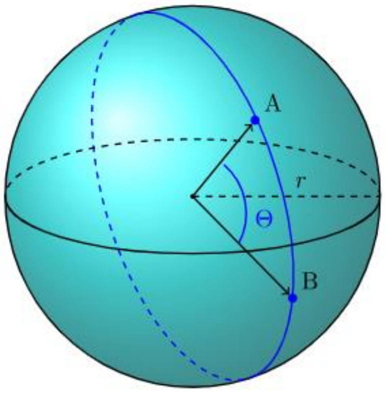

Fig 6: Two points A(φ_1, λ_1) and B(φ_2, λ_2) on Spherical Earth

Central angle θ:

The Haversine formula follows the Haversine of θ (that is, $\mathrm{hav}(\theta)$) to be computed directly from the latitude (represented by φ) and longitude (represented by λ) of the two points:

$$\mathrm{hav}(\theta) = \mathrm{hav}(\varphi_2 - \varphi_1) + \cos(\varphi_1)\cos(\varphi_2)\,\mathrm{hav}(\lambda_2 - \lambda_1)$$

One can prove the formula by transforming the points given by their latitude and longitude into Cartesian coordinates, then taking their dot product.

where,

- φ_1: latitude of point A
  φ_2: latitude of point B
- λ_1: longitude of point A
  λ_2: longitude of point B

Although these representations resemble spherical coordinates, latitude is measured as an angle from the equator rather than the North Pole. In Cartesian coordinates, these points have the following representations:

$$A = (\cos\lambda_1 \cos\varphi_1, \sin\lambda_1 \cos\varphi_1, \sin\varphi_1)$$
$$B = (\cos\lambda_2 \cos\varphi_2, \sin\lambda_2 \cos\varphi_2, \sin\varphi_2)$$

We could attempt to calculate the dot product and proceed from here, but the formulas become significantly simpler when we consider the following fact: the distance between the two points does not change when the sphere is rotated along the z-axis. This will in effect add a constant to λ_1, λ_2. By choosing our constant to be $-\lambda_1$, and setting $\lambda' = \lambda_2 - \lambda_1$, our new points become:

$$A' = (\cos 0 \cos\varphi_1, \sin 0 \cos\varphi_1, \sin\varphi_1)$$
$$= (\cos\varphi_1, 0, \sin\varphi_1)$$
$$B' = (\cos\lambda' \cos\varphi_2, \sin\lambda' \cos\varphi_2, \sin\varphi_2)$$

With θ denoting the angle between A and B. we now have that:

$$\cos\theta = \langle A, B\rangle = \langle A', B'\rangle$$
$$= \cos\lambda' \cos\varphi_1 \cos\varphi_2 + \sin\varphi_1 \sin\varphi_2$$
$$= \sin\varphi_2 \sin\varphi_1 + \cos\varphi_2 \cos\varphi_1 - \cos\varphi_2 \cos\varphi_1 + \cos\lambda' \cos\varphi_2 \cos\varphi_1$$
$$= \cos(\varphi_2 - \varphi_1) + \cos\varphi_2 \cos\varphi_1 (\cos\lambda' - 1)$$
$$= hav(\theta) = hav(\varphi_2 - \varphi_1) + \cos\varphi_2 \cos\varphi_1\, hav(\lambda_2 - \lambda_1)$$

$$\therefore \sin^2\left(\frac{\theta}{2}\right) = \sin^2\left(\frac{\varphi_2 - \varphi_1}{2}\right) + \cos(\varphi_1)\cos(\varphi_2)\sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)$$

$$\boldsymbol{\theta = 2\sin^{-1}\sqrt{\sin^2\left(\frac{\varphi_2 - \varphi_1}{2}\right) + \cos(\varphi_1)\cos(\varphi_2)\sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)}}$$

Approximate shortest distance $= r \times \theta$

B. Lambert's Formula for Oblate Spherical Earth Distances [15].

First convert the latitudes φ_1, φ_2 of the two points to reduced latitudes β_1, β_2.

$\tan\beta = (1 - f)\tan\varphi$; where f is flatting.

$$\therefore\ \beta = \tan^{-1}\left((1 - f)\tan\varphi\right)$$

Then calculate the central angle θ in radians between two points (β_1, λ_1) and (β_2, λ_2) on a sphere using the Great-circle distance method (law of Haversine formula), with longitudes λ_1 and λ_2 being the same on the sphere as on the spheroid.

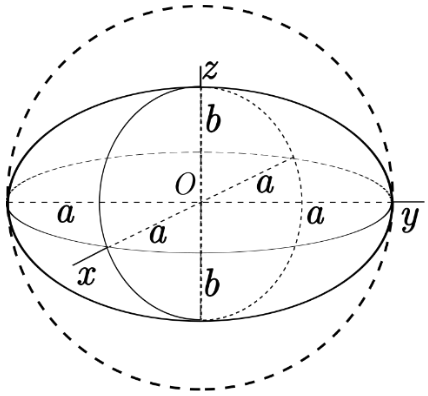

Fig 7: Oblate Sphere with Equatorial radius a and Polar radius b

Central angle θ:

$$hav(\theta) = hav(\beta_2 - \beta_1) + \cos(\beta_1)\cos(\beta_2)\, hav(\lambda_2 - \lambda_1)$$

$$\sin^2\left(\frac{\theta}{2}\right) = \sin^2\left(\frac{\beta_2 - \beta_1}{2}\right) + \cos(\beta_1)\cos(\beta_2)\sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)$$

$$\boldsymbol{\theta = 2\sin^{-1}\sqrt{\sin^2\left(\frac{\beta_2 - \beta_1}{2}\right) + \cos(\beta_1)\cos(\beta_2)\sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)}}$$

$$P = \frac{\beta_1 + \beta_2}{2}; \qquad Q = \frac{\beta_2 - \beta_1}{2}$$

$$X = (\theta - \sin(\theta))\frac{\sin^2(P)\cos^2(Q)}{\sin^2\left(\frac{\theta}{2}\right)}$$

$$Y = (\theta + \sin(\theta))\frac{\sin^2(Q)\cos^2(P)}{\sin^2\left(\frac{\theta}{2}\right)}$$

Approximate shortest distance

$$= a\left(\theta - \frac{f}{2}(X + Y)\right)$$

C. Enumeration of bearing angle between A and B towards the North Pole

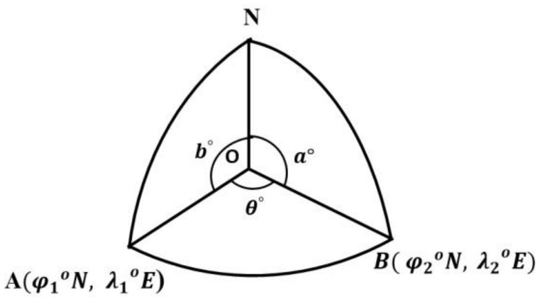

Fig 8: Spherical Triangle

Let NAB be the spherical triangle where A and B be the two places on the earth.

$A \equiv (\varphi_1{}^o N,\ \lambda_1{}^o E),\ B \equiv (\varphi_2{}^o N,\ \lambda_2{}^o E)$. Let N be the North Pole, O be the center of the spherical earth. Let $\angle AOB = \theta^\circ$, $\angle BON = a^\circ$, $\angle AON = b^\circ$
We have to find out the direction of the two places towards the North Pole.

From spherical triangle using sign rule

$$\frac{\sin A}{\sin a^\circ} = \frac{\sin B}{\sin b^\circ} = \frac{\sin N}{\sin \theta^\circ}$$

$$\sin A = \frac{\sin a^\circ \sin N}{\sin \theta^\circ}$$

$$= \frac{\sin(90^\circ - \varphi_2{}^\circ)\sin(\lambda_2{}^\circ - \lambda_1{}^\circ)}{\sin \theta^\circ}$$

$$= \frac{\cos \varphi_2{}^\circ \sin(\lambda_2{}^\circ - \lambda_1{}^\circ)}{\sin \theta^\circ}$$

Similarly,

$$\sin B = \frac{\cos \varphi_1{}^\circ \sin(\lambda_2{}^\circ - \lambda_1{}^\circ)}{\sin \theta^\circ}$$

We can simply discover the direction of any point in relation to another point in the direction of the North Pole using these two equations.

Remark: '-ve' sign will be added for South latitude and West longitude.

IV. Illustrative Examples

Example 1
Shortest distance between *Dhaka* and *Chandpur* [16], [17]:

A ≡ *Dhaka*: $\varphi_1 = 23.8103^o$N, $\lambda_1 = 90.4125^o$E
B ≡ *Chandpur*: $\varphi_2 = 23.2321^o$N, $\lambda_2 = 90.6631^o$E

$\theta = 0.622183714819177^\circ$
$\theta = 0.010859154375884\ rad.$

Approximate shortest distance between *Dhaka* and *Chandpur*

$$= r \times \theta$$
$$= 69.1837677787\ km$$

And the estimated shortest distance by using Google earth is 68.98931 km

∴ The bearing angle of *Dhaka* in respect of *Chandpur* towards the North Pole is

$$\sin A = \frac{\cos \varphi_2{}^\circ \sin(\lambda_2{}^\circ - \lambda_1{}^\circ)}{\sin \theta^\circ}$$

$$\angle A = 21.7231298^\circ$$

***Fig 9**:* Map View of *Dhaka* (23.8103°N, 90.4125°E) and *Chandpur* (23.2321°N, 90.6631°E) [Image is created by using Google earth]

The bearing angle of *Chandpur* in respect of *Dhaka* towards the North Pole is

$$\sin B = \frac{\cos \varphi_1{}^\circ \sin(\lambda_2{}^\circ - \lambda_1{}^\circ)}{\sin \theta^\circ}$$

$$\angle B = 21.62311685^\circ$$

Example 2
Shortest distance between *City Campus (CC)* and *Permanent Campus (PC)* (Green University of Bangladesh) [18], [19]:
A ≡ *CC*: $\varphi_1 = 23.7869^o$N, $\lambda_1 = 90.3774^o$E
B ≡ *PC*: $\varphi_2 = 23.8295^o$N, $\lambda_2 = 90.5663^o$E
$\theta = 0.177997819004589^\circ$
$\theta = 0.003106648002999\ rad.$

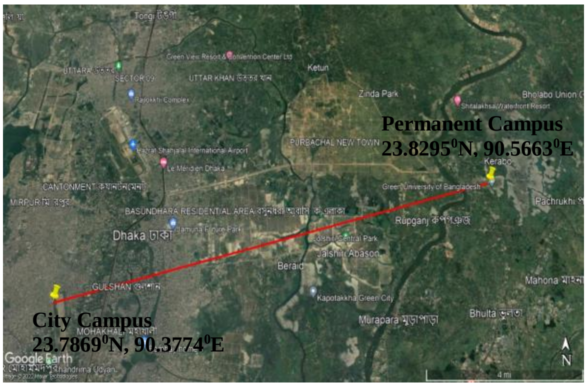

***Fig 10**:* Map View of *City Campus* (23.7869°N, 90.3774°E) and *Permanent Campus* (23.8295°N, 90.5663°E) [Image is created by using Google earth]

Approximate shortest distance between *CC* and *PC*

$$= r \times \theta$$
$$= 19.792481676759\ km$$

And the estimated shortest distance by using Google earth is 19.81914 km
∴ The bearing angle of *CC* in respect of *PC* towards the North Pole is

$$\sin A = \frac{\cos\varphi_2^{\circ}\sin(\lambda_2^{\circ} - \lambda_1^{\circ})}{\sin\theta^{\circ}}$$
$$\angle A = 76.114959057130463^{\circ}$$

The bearing angle of *PC* in respect of *CC* towards the North Pole is

$$\sin B = \frac{\cos\varphi_1^{\circ}\sin\left(\lambda_2^{\circ} - \lambda_1^{\circ}\right)}{\sin\theta^{\circ}}$$
$$\angle B = 76.191213561669045^{\circ}$$

Example 3

Shortest distance between *Dhaka* and *Mecca* [16], [20]:

A ≡*Mecca*: $\varphi_1 = 21.3891^{o}$N, $\lambda_1 = 39.8579^{o}$E

B ≡*Dhaka*: $\varphi_2 = 23.8103^{o}$N, $\lambda_2 = 90.4125^{o}$E

$\beta_1 = 21.323836760155913^{\circ}$
$\beta_2 = 23.739308623554674^{\circ}$
$\theta = 46.515279955665946^{\circ}$
$\theta = 0.811844787713293\ rad.$
$P = -0.021078968502796\ rad.$
$Q = 0.021078968502796\ rad.$

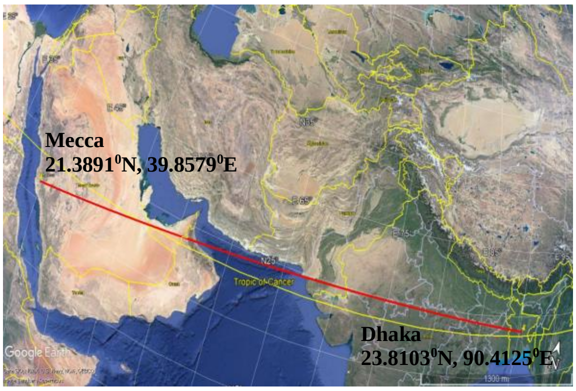

***Fig 11**:* Map View of *Dhaka* (23.8103ºN, 90.4125ºE) and *Mecca* (21.3891ºN, 39.8579ºE) [Image is created by using Google earth]

$$X = (\theta - \sin(\theta))\frac{\sin^2(P)\cos^2(Q)}{\sin^2\left(\frac{\theta}{2}\right)}$$
$$= 0.000045394393501886$$
$$Y = (\theta + \sin(\theta))\frac{\sin^2(Q)\cos^2(P)}{\sin^2\left(\frac{\theta}{2}\right)}$$
$$= 0.004378534324726$$

Approximate shortest distance between *Dhaka* and *Mecca*

$$= a(\theta - \frac{f}{2}(X+Y))$$
$$= 5178.009976608\ km$$

And the estimated shortest distance by using Google earth is $5177.98928\ km$

∴ The bearing angle of *Mecca* in respect of *Dhaka* towards the North Pole is

$$\sin A = \frac{\cos\beta_2^{\circ}\sin(\lambda_2^{\circ} - \lambda_1^{\circ})}{\sin\theta^{\circ}}$$
$$\angle A = 76.974552769^{\circ}$$

The bearing angle of *Dhaka* in respect of *Mecca* towards the North Pole is

$$\sin B = \frac{\cos\beta_1^{\circ}\sin\left(\lambda_2^{\circ} - \lambda_1^{\circ}\right)}{\sin\theta^{\circ}}$$
$$\angle B = 82.50777145^{\circ}$$

Remark: All Muslims peoples of *Dhaka (Bangladesh)* should pray to ALLAH to the direction of *Mecca* in 82.50777145° towards the North Pole.

Example 4

Shortest distance between *Michigan* and *Mecca* [20], [21]:

A ≡*Michigan*: $\varphi_1 = 44.3148^{o}$N, $\lambda_1 = -85.6024^{o}$W

B≡*Mecca*: $\varphi_2 = 21.3891^{o}$N, $\lambda_2 = 39.8579^{o}$E

$\beta_1 = 44.218619229599575^{\circ}$
$\beta_2 = 21.323836760155913^{\circ}$
$\theta = 97.684097267975247^{\circ}$
$\theta = 1.704909124164566\ rad.$
$P = 0.199794667809835\ rad.$
$Q = -0.199794667809835\ rad.$

$$X = (\theta - \sin(\theta))\frac{\sin^2(P)\cos^2(Q)}{\sin^2\left(\frac{\theta}{2}\right)}$$

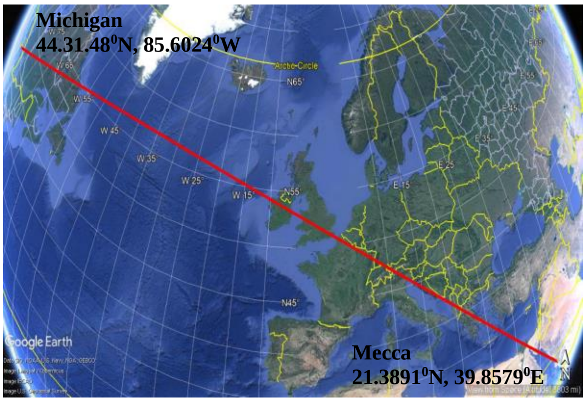

***Fig 12**:* Map View of *Michigan* (44.3148ºN, 85.6024ºW) and *Mecca* (21.3891ºN, 39.8579ºE) [Image is created by using Google earth]

$$X = 0.062362924631916$$
$$Y = (\theta + \sin(\theta))\frac{\sin^2(Q)\cos^2(P)}{\sin^2\left(\frac{\theta}{2}\right)}$$
$$= 0.179955354923535$$

Approximate shortest distance between *Michigan* and *Mecca*

$$= a(\theta - \frac{f}{2}(X+Y))$$
$$= 10871.5530163\ km$$

And the estimated shortest distance by using Google earth is $10871.55712\ km$

∴ The bearing angle of *Michigan* in respect of *Mecca* towards the North Pole is

$$\sin A = \frac{\cos \beta_2^{\circ} \sin(\lambda_2^{\circ} - \lambda_1^{\circ})}{\sin \theta^{\circ}}$$
$$\angle A = 49.96315462574^{\circ}$$

The bearing angle of *Mecca* in respect of *Michigan* towards the North Pole is

$$\sin B = \frac{\cos \beta_1^{\circ} \sin(\lambda_2^{\circ} - \lambda_1^{\circ})}{\sin \theta^{\circ}}$$
$$\angle B = 36.0889976943^{\circ}$$

Remark: All Muslims peoples of *Michigan (U.S.A)* should pray to ALLAH to the direction of *Mecca* in 49.96315462574° towards the North Pole.

V. Result and Discussion

Because the Earth is not flat due to its rotation, wobbling motion, and other forces, all calculations must be made with caution. For $||\lambda_1| - |\lambda_2||\ or\ ||\varphi_1| - |\varphi_2|| < 5^{\circ}$, assuming the earth curvature like spherical with a Mean radius and we have calculated the approximate distance ((small or medium)) and direction across the surface.

For $||\lambda_1| - |\lambda_2||\ or\ ||\varphi_1| - |\varphi_2|| \geq 5^{\circ}$, assuming the earth curvature like oblate spheroid with a Polar radius and an Equatorial radius and we have calculated the approximate distance (large) and direction. Our calculated results from the proposed methods and the results from Google earth are identical with a maximum of 0.1% error which indicates very effeteness of the proposed methods.

VI. Conclusion

The planet Earth is a close approximation to a sphere in real life, but it is not a perfect spherical. More accurately the earth is called an ‘oblate spheroid’ because it is slightly flatted at the ends of its axis of rotation, the North and South Poles. The purpose of this paper is to calculate the small shortest distance considering the earth as spherical and the longest shortest distance considering the earth as oblate spherical. We demonstrated some ideas which is related with the navigation system, networking range and locating the direction of religious place like *Mecca* to other places.

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