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Dynamical analysis of ant colony's collective decision making to explore food from two different resources

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Abstract
A social insect-like Ant lives in a colony. Their collective behavior of food recruitment is a universal character. The mathematical model of the food recruitment of the ant colonies meets different challenges in resolving the nonlinearity due to choosing various food exploration contests between the ants and different environmental parameters. This study possesses several leading parameters to overcome the complexity of the model. Besides, the ants are exploring food sources in diverse trails with the intensity of a chemical substance of pheromone concentration also develops nonlinearity. For generality, this study evaluates two food sources of the ant colony's collective responses concerning environmental conditions without varying the ant's behaviors. The investigation of the ant's food quest in this work concentrates on the dynamical analysis of their food exploration from two different resources described by two ordinary nonlinear differentials equations (ODE). The mean-field model of ODE connected to their dynamical analysis demonstrates that food recruitment varies according to individual variability, supportive communication, and environmental restrictions establishing are connected to the stable or unstable system of the ODE.

Keywords Ant colony; Dynamical analysis; Equilibrium point; Jacobian matrix; Mean-field model.

Paper type Research paper

1. Introduction

Mathematical modeling is the widely used method for the analysis and future prediction of physical phenomena in science and engineering (Afshari & Hajimiri, 2005; Hafez, Roy, Talukder, & Hossain Ali, 2016; Uddin, Hafez, & Iqbal, 2022; Iqbal, Hafez, & Abdul Karim, 2020). Moreover, modeling is a robust method in many other branches of real-world problems. The dynamical analysis (Iqbal, Hafez, Chu, & Park, 2022; Sheikh et al., 2020) initiates the physical model's investigation and forecasting to formulate the portrayed system's mathematical model. Subsequently, it is well defined that dynamical analysis is a mathematical procedure for predicting and

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understanding the general physical phenomena (Iqbal, Hafez, & Uddin, 2022; Iqbal, 2021; Ali, Yilmazer, Yokus, & Bulut, 2020) concerning their previous state corresponded with the governing evaluation rules. Moreover, engineering, chemical, economic, biological, and even social problems can be predicted by understanding their stability and instability investigation by applying dynamical analysis (Marquié, Bilbault, & Remoissenet, 1995).

The dynamical model is classified into two types: a time-dependent system and the time-independent system. This study focuses on the time-independent(nonautonomous) dynamical analysis of the ant colony's food exploration from the two food sources. This work accentuates how collective decision-making leading to two different resources of food exploration strategies may arise for the ant colony, especially Lasius niger (Black Garden ant) (Detrain & Prieur, 2014). Black garden Ant is found all over Europe, North America, and Asia. Without a brain, a genetically preprogrammed Ant colony employs complex, collective behavior to explore food by accessing minimal information (Pino et al., 1985; Dussutour & Nicolis, 2013). Ants communicate with each other using pheromones, sounds, and touch. The pheromone (secreted or excreted chemical factor that triggers a social response in members of the same species) approaches to exploring food. Most of the ants live on the ground. They use the soil surface and leave their pheromone trails which other ants follow. A forager ant quests for trails of food marks to explore food and leave its pheromone on trails on the way to head back with food to the colony; then, other ants reinforce the trail and head back with food to the colony. When the food source is exhausted, no new trails are marked by returning ants, and the scent slowly dissipates. This behavior helps ants deal with changes in their environment (Perna et al., 2012; Baird & Seeley, 1983; Dussutour, Beekman, Nicolis, & Meyer, 2009). For instance, when an obstacle blocks an established path to a food source, the foragers leave the path to explore new routes. If an ant is successful, it leaves a new trail marking the shortest route on its return. More ants follow successful trails, reinforcing better routes and gradually identifying the best path.

Previous work (Baird & Seeley, 1983; Nicolis & Deneubourg, 1999) was focused on amplifying the communications between essential places according to the diverse food sources in various animal societies. For instance, insects and bees are particular kinds of animal society that were studied in many more research works. One of the most rigorous investigations on the food requirements of ants and their process was carried out by Seeley, Camazine, & Sneyd (1991) and Wilson (1962). Their points of view were largely experimental analyses. This work mainly focuses on the well-established mathematical model of ant colony's food explorations dynamical analysis.

The assembling of this study is organized after the introduction's outset; section 2 describes the mathematical model of the ant colony's food exploration connectively, section 3 illustrates the mathematical analysis, section 4 portrays the results and discussions, and lastly, the conclusion concludes this study.

2. Mathematical model

Figure 2 describes the communal choice of two food sources for the ants from the food source A and B. Generally, the ant colonies compete and confront each other to gather foods from diverse food sources.

Figure 1
Lasius niger (Black Garden ant) (Ant species, 2022)

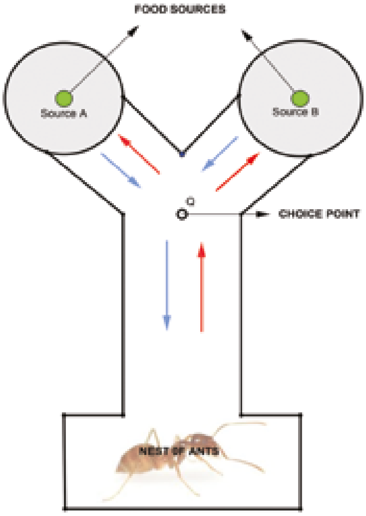

Figure 2
Schematic representation of ant's food choice

In figure 2, Q is the choice between two independent food sources (A and B). There are various facts about the circumstances to confine their trails for the multiple food sources. This article highlights the nature of the traffic between the two trails directing to the two food sources and ignores individual and environmental variability. It is ignored direct contact between the individuals and considers individuals' reactions to their pheromone concentration present in the given trail. The leading variables are pheromone concentrations χ_1 and χ_2 rather than the number of individuals present at a given time on the trails T_1 (Left side for food source A) and T_2 (Left side for food source B). The following model equations can capture the pheromone concentration χ_1 and χ_2 between two food sources.

$$\frac{d\chi_1}{dt} = \Phi q_1 \frac{(k+\chi_1)^\gamma}{(k+\chi_1)^\gamma+(k+\chi_2)^\gamma} - \Gamma\chi_1 \quad (1)$$

$$\frac{d\chi_2}{dt} = \Phi q_2 \frac{(k+\chi_2)^\gamma}{(k+\chi_1)^\gamma+(k+\chi_2)^\gamma} - \Gamma\chi_2 \quad (2)$$

In the equations Eq. (1) and Eq. (2), the first positive term represents the attractiveness of the trail T_1 or T_2 over the others, and the second negative term represents the disappearance of the pheromone on trial T_1 or T_2 . Table I summarize the description of the parameters, which are used in Eq. (1), Eq. (2), and the entire article.

Table 1
Description of the model parameters.

Name of the Parameters	Description
χ_1	Pheromone concentration of in trail T_1
χ_2	Pheromone concentration of in trail T_2
t	Time
Φ	The flux of individuals leaving the nest
q_1	Quantity of the pheromone on the trail T_1
q_2	Quantity of the pheromone on the trail T_2
k	Concentration threshold
Γ	Evaporation rate of pheromone
γ	Measures the sensitivity of the food choice

3. Mathematical analysis

By Scaling the equations Eq. (1) and Eq. (2), one can do by letting $t = \frac{\tau}{\Gamma}$, $\chi_1 = \Xi_1 k, \chi_2 = \Xi_2 k$ and $\Phi = \phi k\Gamma$. Than $d\chi_1 = kd\Xi_1$, $dt = \frac{1}{\Gamma}d\tau$.

Make use of these in Eq. (1) and Eq. (2) to develop:

$$\Gamma k \frac{d\Xi_1}{d\tau} = \phi k \Gamma q_1 \frac{(1+\Xi_1)^{\gamma} k^{\gamma}}{((1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma})k^{\gamma}} - \Gamma k \Xi_1,$$

$$\frac{d\Xi_1}{d\tau} = \phi q_1 \frac{(1+\Xi_1)^{\gamma}}{(1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma}} - \Xi_1, \quad (3)$$

$$\Gamma k \frac{d\Xi_2}{d\tau} = \phi k \Gamma q_2 \frac{(1+\Xi_2)^{\gamma} k^{\gamma}}{((1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma})k^{\gamma}} - \Gamma k \Xi_2,$$

$$\frac{d\Xi_2}{d\tau} = \phi q_2 \frac{(1+\Xi_2)^{\gamma}}{(1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma}} - \Xi_2, \quad (4)$$

Consequently, to calculate the equilibrium points (fixed points), one can obtain by letting the righthand side of Eq. (3) and Eq. (4) be equal to zero for the steady-state solutions:

$$\frac{d\Xi_1}{d\tau} = \phi q_1 \frac{(1+\Xi_1)^{\gamma}}{(1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma}} - \Xi_1 = 0, \quad (5)$$

$$\frac{d\Xi_2}{d\tau} = \phi q_2 \frac{(1+\Xi_2)^{\gamma}}{(1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma}} - \Xi_2 = 0. \quad (6)$$

After generalization the Eq. (5) and Eq. (6),

$$\phi q_1 \frac{(1+\Xi_1)^{\gamma}}{(1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma}} = \Xi_1,$$

$$\phi q_2 \frac{(1+\Xi_2)^{\gamma}}{(1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma}} = \Xi_2,$$

Consequently,

$$\frac{q_1(1+\Xi_1)^{\gamma}}{q_2(1+\Xi_2)^{\gamma}} = \frac{\Xi_1}{\Xi_2}. \quad (7)$$

For instance, one can get from Eq. (7) for $q_1 = q_2$ applying the sensitivity condition, $\gamma = 1$ develop:

$$\frac{(1+\Xi_1)}{(1+\Xi_2)} = \frac{\Xi_1}{\Xi_2},$$

$$\Rightarrow \Xi_1 = \Xi_2 \quad (8)$$

and $\gamma = 2$ create

$$\frac{(1+\Xi_1)^2}{(1+\Xi_2)^2} = \frac{\Xi_1}{\Xi_2},$$

$$\Rightarrow (\Xi_1 - \Xi_2)(1 - \Xi_1\Xi_2) = 0,$$

$$\Rightarrow \Xi_1 = \Xi_2 \quad or \quad \Xi_1 = \frac{1}{\Xi_2}. \quad (9)$$

Subsequently, by choosing the $\gamma = 2$, $q_1 = q_2 = q$ then, from Eq. (5) and Eq. (6), one can obtain: $\phi q \left\{\frac{(1+\Xi)^{\gamma}+(1+\Xi_2)^{\gamma}}{(1+\Xi_1)^{\gamma}+(1+\Xi_2)^{\gamma}}\right\} = \Xi_1 + \Xi_2,$

$$\Rightarrow \phi q = \Xi_1 + \Xi_2. \tag{10}$$

Next, by using $\Xi_1 = \Xi_2$ in Eq. (9) develop:

$$\Xi_2 = \frac{\phi q}{2}, \tag{11}$$

$$\Rightarrow \phi q = \Xi_2 + \frac{1}{\Xi_2},$$

$$\Xi_2^2 - \phi q \Xi_2 + 1 = 0,$$

$$\Xi_2 = \frac{\phi q \pm \sqrt{\phi^2 q^2 - 4}}{2}. \tag{12}$$

Where ϕq ≤-2 and ϕq ≥2.
As a result, the steady-state solutions for $\gamma = 2$, and $q_1 = q_2 = q$ are

$$(\Xi_1, \Xi_2) = (\frac{\phi q_1}{2}, \frac{\phi q_1}{2}), \quad \text{and } (\Xi_1, \Xi_2) = \left(\frac{\phi q \pm \sqrt{\phi^2 q^2 - 4}}{2}, \frac{\phi q \pm \sqrt{\phi^2 q^2 - 4}}{2}\right).$$

Figures 3 to Figure 6 illustrate the phase state profiles of Eq. (3) and Eq. (4) for the values $\phi(\tau) = 0.9, q_1 = 1, q_2 = 2$ and $\gamma = 1$. As a result, the fixed points $\left(\Xi_1(\tau), \Xi_2(\tau)\right) = (0.353, 1.093), (2.546, \ -3.293)$ and the Jacobin matrix Θ_1 are calculated from the Eq.(3a) and Eq.(3b),

$$\Theta_1 = \begin{pmatrix} \frac{0.9}{2 + \Xi_1 + \Xi_2} - \frac{0.9 + 0.9\Xi_1}{(2 + \Xi_1 + \Xi_2)^2} - 1 & -\frac{0.9 + 0.9\Xi_1}{(2 + \Xi_1 + \Xi_2)^2} \\ -\frac{1.8 + 1.8\Xi_2}{(2 + \Xi_1 + \Xi_2)^2} & \frac{1.8}{2 + \Xi_1 + \Xi_2} - \frac{1.8 + 1.8\Xi_2}{(2 + \Xi_1 + \Xi_2)^2} - 1 \end{pmatrix}.$$

At $\Xi_1(\tau) = (0.353, 1.093), \Xi_2(\tau) = (2.546, -3.293)$ the state classifications are in Table II.

Table 2

Phase state classification for $\phi(\tau) = 0.9, q_1 = 1, q_2 = 2$ *and* $\gamma = 1$.

Fixed points associated with Eigenvalues	Fixed points (0.353,1.093)	Fixed points (2.546,-3.293)
Eigenvalues (ν_1, ν_2)	-1.00000000004361, -0.636283742956387	-1.00000000073890, 1.75100333873890
Classification	Stable	Unstable

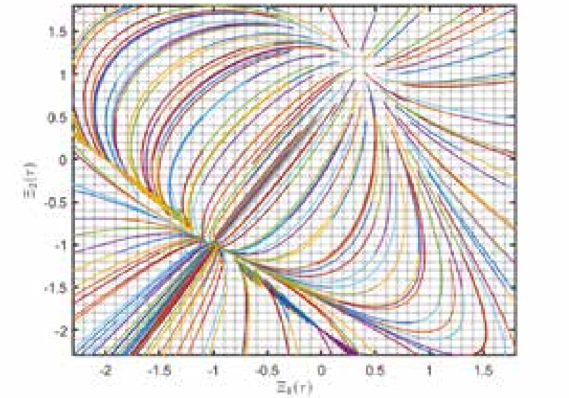

Figure 3

Phase state profile for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9, q_1 = 1,$ $q_2 = 2$ *and* $\gamma = 1.$

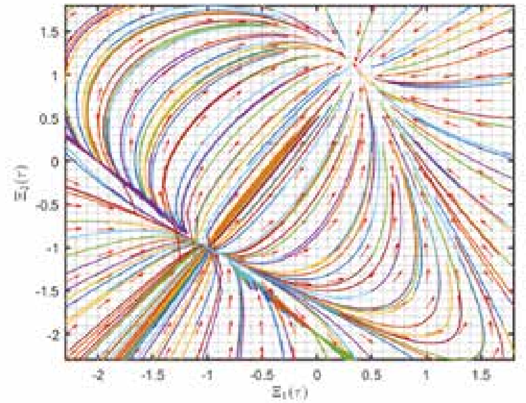

Figure 4

Phase and Vector plot for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9,$ $q_1 = 1, q_2 = 2$ *and* $\gamma = 1.$

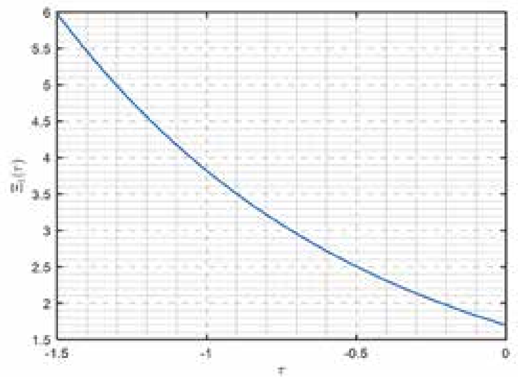

Figure 5

Time series for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9, q_1 = 1,$ $q_2 = 2$ *and* $\gamma = 1.$

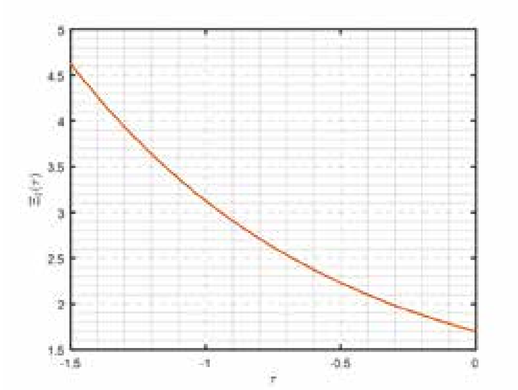

Figure 6

Time series for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9, q_1 = 1, q_2 = 2$ *and* $\gamma = 1$.

Similarly, Figure 5 to Figure 9 illustrate the phase state profiles of Eq. (3) and Eq. (4) for the values $\phi(\tau) = 0.9, q_1 = 1, q_2 = 1$ and $\gamma = 1$, where the quantity of the pheromone is changed from $q_2 = 2$ to $q_2 = 1$. Consequently, the only fixed point $(\Xi_1(\tau), \Xi_2(\tau)) = (0.45, 0.45)$, corresponding Jacobian matrix (Θ_2) and eigenvalues (Table III) are calculated from the Eq.(3a) and Eq.(3b).

$$\Theta_2 = \begin{pmatrix} \frac{0.9}{2 + \Xi_1 + \Xi_2} - \frac{0.9 + 0.9\Xi_1}{(2 + \Xi_1 + \Xi_2)^2} - 1 & -\frac{0.9 + 0.9\Xi_1}{(2 + \Xi_1 + \Xi_2)^2} \\ -\frac{0.9 + 0.9y}{(2 + \Xi_1 + \Xi_2)^2} & \frac{0.9}{2 + \Xi_1 + \Xi_2} - \frac{0.9 + 0.9\Xi_2}{(2 + \Xi_1 + \Xi_2)^2} - 1 \end{pmatrix}$$

Table 3

Phase state classification for $\phi(\tau) = 0.9, q_1 = 1, q_2 = 1$ *and* $\gamma = 1$.

Fixed points associated with Eigenvalues	Fixed points
	(0.45,0.45)
Eigenvalues (ν_1, ν_2)	-0.6896551724,-1
Classification	Stable

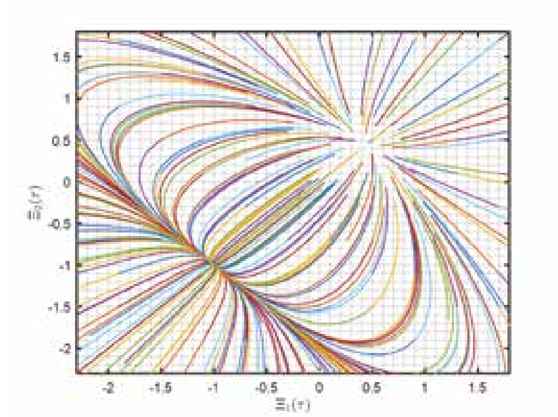

Figure 7
Phase state illustrations for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9$, $q_1 = 1, q_2 = 2$ and $\gamma = 1$.

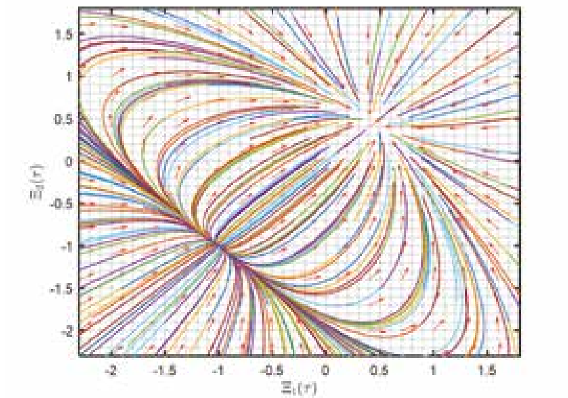

Figure 8
Phase and Vector plot for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9$, $q_1 = 1, q_2 = 2$ and $\gamma = 1$.

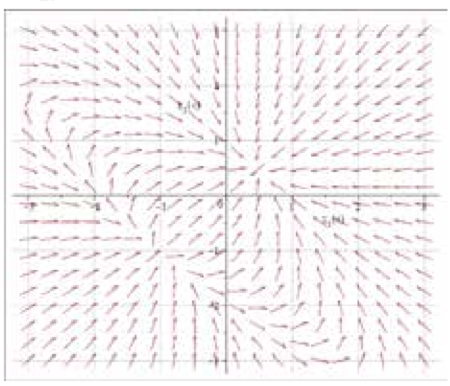

Figure 9
Normalized vector for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9$, $q_1 = 1, q_2 = 2$ and $\gamma = 1$.

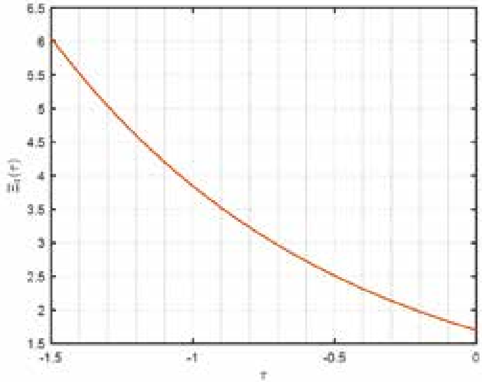

Figure 10

Time series analysis for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9$, $q_1 = 1, q_2 = 2$ *and* $\gamma = 1$.

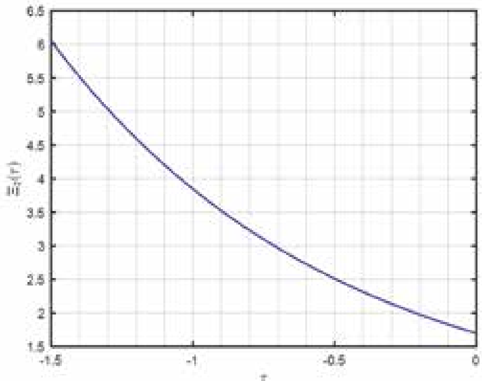

Figure 11

Time series analysis for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9$, $q_1 = 1, q_2 = 2$ *and* $\gamma = 1$.

Likewise, Figure 10 to Figure 13 illustrate the phase state profiles of Eq. for the values $\phi(\tau) = 0.9, q_1 = 1, q_2 = 1$ and $\gamma = 2,$ where the quantity of the choice sensitivity is changed from $\gamma = 1$ to $\gamma = 2.$ Consequently, the equilibrium points $(\Xi_1(\tau), \Xi_2(\tau)) = \{(0.450, 0.450), (0.450 \pm 0.8930285550i)\}$ Jacobian matrix (Θ_3) and corresponding eigenvalues(Table IV) are calculated from the Eq.(3a) and Eq.(3b).

$$\Theta_3 = \begin{pmatrix} \frac{1.8(1+\Xi_1)}{(1+\Xi_1)^2+(1+\Xi_2)^2} - \frac{0.9(1+\Xi_1)^2(2+2\Xi_1)}{((1+\Xi_1)^2+(1+\Xi_2)^2)^2} - 1 & -\frac{0.9(1+\Xi_1)^2(2+2\Xi_2)}{((1+\Xi_1)^2+(1+\Xi_2)^2)^2} \\ -\frac{0.9(1+\Xi_2)^2(2+2\Xi_1)}{((1+\Xi_1)^2+(1+\Xi_2)^2)^2} & \frac{18(1+\Xi_2)}{(1+\Xi_1)^2+(1+\Xi_2)^2} - \frac{0.9(1+\Xi_2)^2(2+2\Xi_2)}{((1+\Xi_1)^2+(1+\Xi_2)^2)^2} - 1 \end{pmatrix}$$

Table 4

Phase state classification for $\phi(\tau) = 0.9, q_1 = 1, q_2 = 1$ *and* $\gamma = 2.$

Fixed points associated with Eigenvalues	Fixed points	
	(0.450,0.450)	
Eigenvalues (ν_1, ν_2)	-0.3793103448, -1	
Classification		Stable

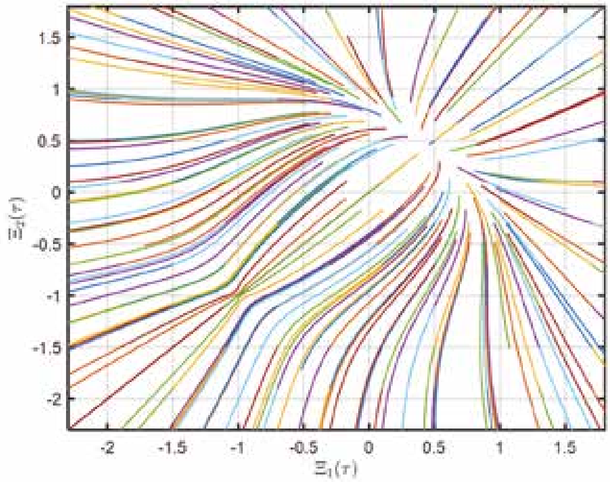

Figure 12

Phase plot for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9, q_1 = 1, q_2 = 1$ *and* $\gamma = 2.$

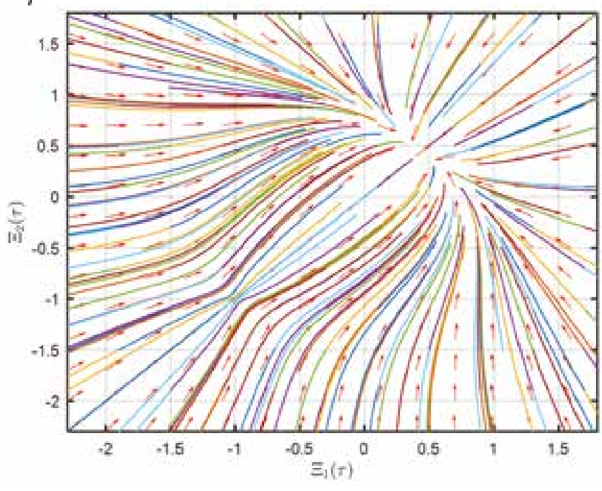

Figure 13

Phase and Vector plot for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9, q_1 = 1, q_2 = 1$ *and* $\gamma = 2.$

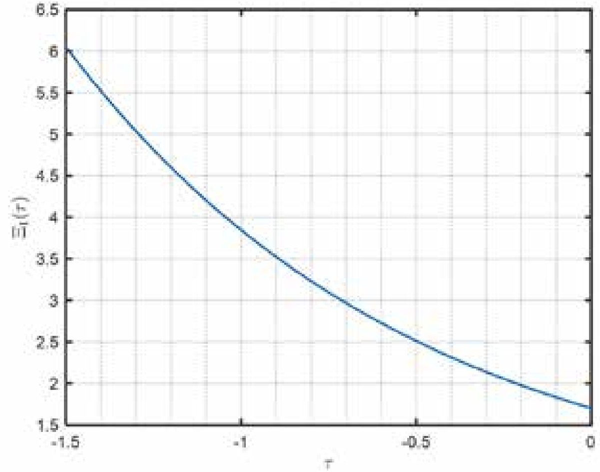

Figure 14

Time profile for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9, q_1 = 1, q_2 = 1$ *and* $\gamma = 2$.

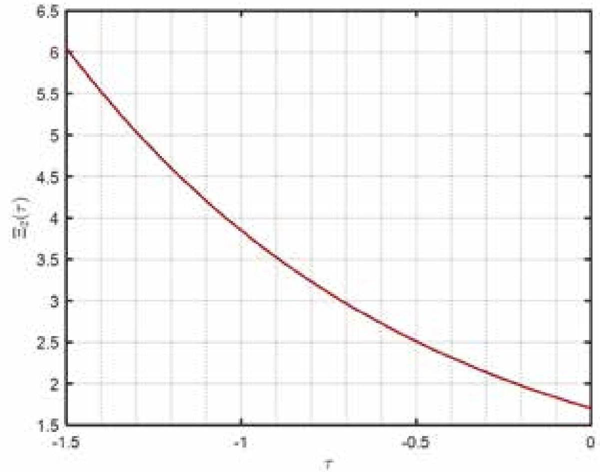

Figure 15

Time series plot for the two food sources ant colony for the value of the parameters $\phi(\tau) = 0.9, q_1 = 1, q_2 = 1$ *and* $\gamma = 2$.

4. Results and discussions

The objective of this study is to analyze the phase-portrait (Iqbal, 2021) compared to the stability and instability of the model equations: Eq. (1) and Eq. (2) of the ant's food exploration from the two different Source-A and Source-B. As a result, the equations: Eq. (3) and Eq. (4) are reduced by applying dimensionless parameters from Eq. (1) and Eq. (2). The following cases investigate the model equations Eq. (3) and Eq. (4).

Case-1:
Applying the arbitrary value of the parameters $\phi(\tau) = 0.9,\ q_1 = 1,\ q_2 = 2,$ and $\gamma = 1$ in Eq. (3) and Eq. (4). Consequently, the two fixed points $(\Xi_1, \Xi_2) = \{(0.353, 1.093), (2.546, -3.293)\}$ associate with the Jacobian matrix Θ_1 and their corresponding eigenvalues (Iqbal, Hafez, & Uddin, 2022) classify the phase state of the model Eq. (3), Eq. (4). The state classification is illustrated in Table II accompanied by Figure 3 to Figure 6. It is observed that, at point $(\Xi_1, \Xi_2) = (0.353, 1.093)$, the eigenvalue $(\nu_1, \nu_2) = (-1.00000000004361, -0.636283742956387)$ represent both the ν_1 and ν_2 are real and negative-signed, developing a stable node at the point (0.353, 1.093). In contrast, compared with another fixed point $(2.546, -3.293)$ has, the eigenvalues $\nu_1 = -1.00000000073890$, and $\nu_2 = 1.75100333873890$ real but opposite in sign, which confirms that the Eq. (3) and Eq. (4) are developing an unstable saddle node. Figure 3 and Figure 4 display the phase portrait and the vector plot of food exploration ants from the two diverse sources $(A \text{ and } B)$. Connectively, Figures 5 and 6 illustrate their corresponding time series of the food explorations.

Case-2:
Likewise, by choosing only the equal concentration of pheromone $q_1 = q_2 = 1,$ compared to Case-1. The only equilibrium point is derived from the Eq. (8), which is $(\Xi_1, \Xi_2) = (0.45, 0.45)$ associated with the Jacobian matrix Θ_2 and their resultant eigenvalues classify the phase state of the model Eq. (3), Eq. (4). Table III depicts the phase state stability at the point $(\Xi_1, \Xi_2) = (0.45, 0.45)$. At $(0.45, 0.45)$ both the eigenvalues $\nu_1 = -0.6896551724,$ and $\nu_{2=} - 1$ are real and negative-signed, which is developing a stable node at the point $(\Xi_1, \Xi_2) = (0.45, 0.45)$ which is clear from Figure 7. The corresponding vector plot in Figure 8 and Figure 9 illustrates the flow of the discovery from the various food sources (A and B). Figure 10 and Figure 11 demonstrate their subsequent time series of the food explorations.

Case-3:
Finally, remaining the arbitrary parameter value $\phi(\tau) = 0.9,\ q_1 = q_2 = 1$ unchanged compared to Case-2. Only the sensitivity measure of the food choice parameter (γ) is changed from $\gamma = 1$ to $\gamma = 2.$ The nonlinearity is observed in Eq. (3) and Eq. (4). Consequently, the equilibrium points are calculated using Eq. (9) to Eq. (12). It is observed that one root is $(\Xi_1, \Xi_2) = \left(\frac{\phi q_1}{2}, \frac{\phi q_1}{2}\right) = (0.45, 0.45),$ which is the only real root for the

values of $\phi(\tau) = 0.9,\ q_1 = q_2 = 1.$ It is analogous that other roots are obtained from the Eq. (12), where the condition for real roots must satisfy the $\phi q \leq -2$ and $\phi q \geq 2.$ However, the arbitrary choosing value $\phi(\tau) = 0.9,\ q_1 = q_2 = 1$ does not meet the condition $\phi q \geq 2$ hence the complex roots $(0.450 \pm 0.8930285550i)$ are obtained. Consequently, the phase plot, along with the connected vector plot, are displayed in Figure 12 and Figure 13 for the real equilibrium point $(\Xi_1, \Xi_2) = (0.45, 0.45)$. The state point $(0.45, 0.45)$ is a stable node, since its eigenvalues (Table-IV) $(\nu_1, \nu_2) = (-0.3793103448,\ -1)$ are in the same negative-signed. The related time series of Figure 14 and Figure 15 also illustrate the stability of the fixed point $(0.45,\ 0.45)$.

5. Conclusion

This study works on the food recruitment (Dussutour, Beekman, Nicolis, & Meyer, 2009) of ant colonies for the two trials, T_1, and T_2. Where the arbitrary number of parameter values are considered to analyze the model equations Eq.(1) and Eq.(2), the analysis is based on both sources (A and B). The food sources are identical by assessing the rate of concentration (q_1 and q_2) of each trail's phenomenon and the sensibility (γ) of choosing the trails. It is noted that, at the equilibrium points (Ξ_1, Ξ_2). Coordinate in negative-signed indicates the pheromone concentration is decreasing rapidly. Table II demonstrates that one pair of real-equilibrium points, where the point $(\Xi_1, \Xi_2) = (2.546, -3.293)$ has negative-signed and point $(\Xi_1, \Xi_2) = (0.353, 1.093)$ is positive-signed, define ants exploring foods from the trails going $(2.546, -3.293)$ point to a stable point $(0.353, 1.093)$. The flow is illustrated in Figure 3 and Figure 4. As a result, point $(0.353, 1.093)$ is saddle-node unstable, and point $(2.546, -3.293)$ is a stable node. In comparison, when the coordinates of the equilibrium points $(\Xi_1 = \Xi_2 = 0.45)$ coordinates) are equal. It demonstrates the pheromone concentration of both trails T_1, and T_2 is identical; consequently, ants choose two foods trails equally to form one equilibrium point. It is portrayed in Figure 7, Figure 8, Figure 9, Figure 12 and Figure 13. Two food sources (A and B) have identical or dissimilar qualities. In the case of equal sources, it is observed that there is equivalent exploitation for the small amount of pheromone deposition (Nicolis & Deneubourg, 1999) on the trail of the two food sources. The system switches to a chosen advantage of one or another food source if a small amount changes the threshold (for instance, ϕ, γ) (Nicolis & Deneubourg, 1999) value for the species Lasius niger. In the same tone, using a state diagram, we can represent other techniques for food recruitment from more than two food sources. This research can be beneficial for biological developments or artificial systems and control the conduct of the population in the agriculture field.

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