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SOME FEATURES OF α-T_0 SPACES IN SUPRA FUZZY TOPOLOGY

M. F. HOQUE, M.S. HOSSAIN[1] AND D. M. ALI[1]

Department of Mathematics, Pabna Science and Technology University, Pabna-6600, Bangladesh

ABSTRACT

Four concepts of T_0 supra fuzzy topological spaces are introduced and studied in this paper. The workers also established some relationships among them and studied some other properties of these spaces.

Key words: Fuzzy topology, Supra fuzzy topology

INTRODUCTION

The fundamental concept of a fuzzy set was introduced by Zadeh (1965) to provide a foundation for the development of many areas of knowledge. Chang (1968) and Lowen (1976) developed the theory of fuzzy topological spaces using fuzzy sets. Mashhour *et al.* (1983) introduced supra topological spaces and studied s-continuous functions and s^*-continuous functions. They also gave the concept of α-T_0 fuzzy topological spaces. In 1987, Abd EL-Monsef *et al.* introduced the fuzzy supra topological spaces and studied fuzzy supra continuous functions and characterized a number of basic concepts. Ali (1993) made some remarks on α-T_0, α-T_1 and α-T_2 fuzzy topological spaces. In this paper, the present workers studies some features of α-T_0 spaces and obtained certain characterizations in supra fuzzy topological spaces. As usual I = [0, 1] and I_1 = [0, 1).

Definition: For a set X, a function $u: X \to [0,1]$ is called a fuzzy set in X. For every $x \in X$, u(x) represents the grade of membership of x in the fuzzy set u. Some authors say that u is a fuzzy subset of X. Thus a usual subset of X, is a special type of a fuzzy set in which the range of the function is {0, 1} (Zadeh 1965).

Definition: Let X be a nonempty set and A be a subset of X. The function $1_A : X \to [0,1]$ {0, 1} defined by $1_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A \end{cases}$ is called the characteristic function of A. The present authors also write 1_x for the characteristic function of {x}. The characteristic functions of subsets of a set X are referred to as the crisp sets in X (Zadeh 1965).

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[1] Department of Mathematics, Rajshahi University, Rajshai-6205, Bangladesh.

Example : Suppose X is real number R and the fuzzy set of real numbers much greater than 5 in X that could be defined by the continuous function $U : X \to [0,1]$ such that

$$u(x) = \begin{cases} 0 \ \ if \ \ x \le 5 \\ \dfrac{x-5}{50} \ \ if \ \ 5 < x < 55 \\ 1 \ \ if \ \ x \ge 55 \end{cases}.$$

Definition: Let X be a non empty set and t be the collection of fuzzy sets in I^X. Then t is called a fuzzy topology on X if it satisfies the following conditions:

(i) $1, 0 \in t$,

(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\cup_{i\in\Lambda} u_i \in t$.

(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.

If t is a fuzzy topology on X, then the pair (X, t) is called a fuzzy topological space (fts, in short) and members of t are called t- open (or simply open) fuzzy sets. If u is open fuzzy set, then the fuzzy sets of the form 1-u are called t- closed (or simply closed) fuzzy sets (Chang 1968).

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(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.

(iv) All constant fuzzy sets in X belong to t.

Then t is called a fuzzy topology on X (Lowen 1976).

Definition: Let X be a non empty set. A subfamily t^* of I^X is said to be a supra topology on X if and only if

(i) $1, 0 \in t^*$,

(ii) If $u_i \in t^*$ for each $i \in \Lambda$, then $\cup_{i\in\Lambda} u_i \in t^*$.

Then the pair (X, t^*) is called a supra fuzzy topological spaces. The elements of t^* are called supra open sets in (X, t^*) and complement of supra open set is called supra closed set (Mashhour *et al.* 1983).

Example: Let X = {x, y} and $u, v \in I^X$ are defined by u(x) = .8, u(y) = .6 and v(x) = .6, v(y) = .8. Then we have $w(x) = (u \cup v)(x) = .8$, $w(y) = (u \cap v)(y) = .8$ and k(x) =

$(u \cap v)(x) = .6$, $k(y) = (u \cap v)(y) = .6$. If we consider t^* on X generated by {0, u, v, w, 1}, then t^* is supra fuzzy topology on X but t^* is not fuzzy topology. Thus we see that every fuzzy topology is supra fuzzy topology but the converse is not always true.

Definition: Let (X, t) and (X, s) be two topological spaces. Let t^* and s^* are associated supra topologies with t and s. respectively and $f : (X, t^*) \rightarrow (Y, s^*)$ be a function. Then the function f is a supra fuzzy continuous if the inverse image of each i.e., if for any $v \in s^*$, $f^{-1}(v) \in t^*$. The function f is called supra fuzzy homeomeophic if and only if f is supra bijective and both f and f^{-1} are supra fuzzy continuous (Mashhour *et al.* 1983).

Definition: Let (X, t^*) and (Y, s^*) be two supra topological spaces. If u_1 and u_2 are two supra fuzzy subsets of X and Y respectively, then the Cartesian product $u_1 \times u_2$ is a supra fuzzy subset of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min [u_1(x), u_2(y)]$, for each pair $(x, y) \in X \times Y$ (Azad 1981).

Definition: Suppose $\{ X_i , i \in \Lambda \}$, be any collection of sets and X denoted the Cartesian product of these sets, i.e., $X = \Pi_{i\in\Lambda} X_i$. Here X consists of all points $p = < a_i, i \in \Lambda >$, where $a_i \in X_i$. For each $j_o \in \Lambda$, the authors defined the projection $\pi_{jo} : X \rightarrow X_{jo}$ by $\pi_{jo} (< a_i : i \in \Lambda >) = a_{jo}$. These projections are used to define the product supra topology (Wong 1974).

Definition: Let $\{X_\alpha\}_{\alpha \in \Lambda}$ be a family of nonempty sets. Let $X = \Pi_{\alpha\in\Lambda} X_\alpha$ be the usual product of X_α 's and let $\pi_\alpha : X \rightarrow X_\alpha$ be the projection. Further, assume that each X_α is a supra fuzzy topological space with supra fuzzy topology t^*_α. Now the supra fuzzy topology generated by $\{\pi_\alpha^{-1}(b_\alpha) : b_\alpha \in t^*_\alpha, \alpha \in \Lambda\}$ as a sub basis, is called the product supra fuzzy topology on X. Thus if w is a basis element in the product, then there exist $\alpha_1, \alpha_2, \ldots. \alpha_n \in \Lambda$ such that $w(x) = \min \{b_\alpha (x_\alpha) : \alpha = 1, 2, 3, ... n\}$, where $x = (x_\alpha)_{\alpha\in\Lambda} \in X$ (Wong 1974).

Definition: Let (X, T) be a topological space and T^* be associated supra topology with T. Then a function $f : X \rightarrow R$ is lower semi continuous if and only if $\{x \in X : f(x) > \alpha\}$ is open for all $\alpha \in R$ (Abd EL-Monsef *et al.* 1987).

Definition: Let (X, T) be a topological space and T^* be associated supra topology with T. Then the lower semi continuous topology on X associated with T^* is $\omega(T^*) = \{\mu : X \rightarrow [0,1], \mu \text{ is supra lsc}\}$. If $\omega(T^*) : (X, T^*) \rightarrow [0, 1]$ be the set of all lower semi continuous (lsc) functions. We can easily show that $\omega(T^*)$ is a supra fuzzy topology on X (Ming *et al.* 1980).

Let P be the property of a supra topological space (X, T^*) and FP be its supra fuzzy topological analogue. Then FP is called a 'good extension' of P "if and only if the statement (X, T^*) has P if and only if $(X, \omega(T^*))$ has FP" holds good for every supra topological space (X, T^*).

Definition: A fuzzy topological space (X, t) is said to be fuzzy T_0 if and only if (i) for all x, y $\in$ X with x $\neq$ y, there exists u $\in$ t such that u(x) = 1, u(y) = 0 or u(x) = 0, u(y) = 1, (ii) for all x, y $\in$ X with x $\neq$ y, there exists u $\in$ t such that u(x) < u(y) or u(y) < u(x) (Ali 1987).

α-T_0(I), α-T_0(II), α-T_0(III) AND T_0(IV) SPACES IN SUPRA FUZZY TOPOLOGY

Definition: Let (X, t) be a fuzzy topological space and t^* be associated supra topology with t and $\alpha \in I_1$. Then

(a) (X, t^*) is an α - T_0 (i) space if and only if for all distinct elements x, y $\in$ X, there exists u $\in t^*$ such that u(x) = 1, u(y) $\leq \alpha$ or there exists v$\in t^*$ such that v(x) $\leq \alpha$, v(y) = 1.

(b) (X, t^*) is an α - T_0 (ii) space if and only if for all distinct elements x, y $\in$ X, there exists u$\in t^*$ such that u(x) = 0, u(y) > α or there exists v$\in t^*$ such that v(x)>α , v(y) = 0.

(c) (X, t^*) is an α - T_0 (iii) space if and only if for all distinct elements x, y $\in$ X, there exists u$\in t^*$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$ or there exists v $\in t^*$ such that $0 \leq v(y) \leq \alpha < v(x) \leq 1$.

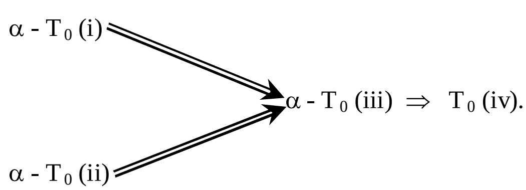
(d) (X, t^*) is a T_0 (iv) space if and only if for all distinct elements x, y $\in$ X, there exists u $\in t^*$ such that u(x) $\neq$ u(y) .

Lemma: Suppose (X, t) is a topological space and t^* is associated supra topology with t and $\alpha \in I_1$. Then the following implications are true:

(a) (X, t^*) is α - T_0 (i) implies (X, t^*) is α - T_0 (iii) implies (X, t^*) is T_0 (iv) .

(b) (X, t^*) is α - T_0 (ii) implies (X, t^*) is α - T_0 (iii) implies (X, t^*) is T_0 (iv) .

Also, these can be shown in a diagram as follows:



Proof: Suppose that (X, t^*) is α - T_0 (i). Let x and y be any two distinct elements in X. Since (X, t^*) is α - T_0 (i) , for $\alpha \in I_1$, by definition, there exists u $\in t^*$ such that u(x) = 1, u(y) $\leq \alpha$ which shows that $0 \leq u(y) \leq \alpha < u(x) \leq 1$. Hence by definition (c), (X, t^*) is α - T_0(iii).

Suppose (X, t^*) is α - T_0 (iii). Then, for x, y $\in$ X with x $\neq$ y, there exist u $\in t^*$ such that $0 \leq u(x) \leq \alpha < u(y) \leq 1$, i.e., u(x) $\neq$ u(y), hence by definition, (X, t^*) is α - T_0(iv).

Let (X, t^*) is α - T_0 (ii). Then, for x, y $\in$X with $x \neq y$, there exists $u \in t^*$ such that u(x) = 0 and u(y) > α, which implies $0 \leq u(x) \leq \alpha < u(y) \leq 1$. Hence, by definition, (X, t^*) is α - T_0(iii) and hence (X, t^*) is α - T_0(iv). Therefore, the proof is complete.

The non-implications among α - T_0 (i), α - T_0 (ii), α - T_0 (iii) and T_0 (iv) are shown in the following examples, i.e., the following examples show that:

(a) T_0 (iv) does not imply α - T_0 (iii), so, not imply α - T_0 (i) and α - T_0 (ii).

(b) α - T_0 (iii) does not imply α - T_0 (i) and α - T_0 (ii).

(c) α - T_0 (i) does not imply α - T_0 (ii).

(d) α - T_0 (ii) does not imply α - T_0 (i).

Example: Let X = {x, y} and $u \in I^X$ is defined by u(x) = 0.4, u(y) = 0.7. Let the supra fuzzy topology t^* on X generated by {0, u, 1, Constants}. Then for α = 0.8, we can easily show that (X, t^*) is T_0(iv) but (X, t^*) is not α - T_0(iii), so, not α - T_0 (i) and α - T_0 (ii).

Example: Let X = {x, y} and $u \in I^X$ be defined by u(x) = 0.5, u(y) = 0.9. Let the supra fuzzy topology t^* on X generated by {0, u, 1, Constants}. For α = 0.7, we have $0 \leq u(x) \leq 0.7 < u(y) \leq 1$. Thus according to the definition, (X, t^*) is α - T_0(iii) but (X, t^*) is not α - T_0(i). Also it can be easily shown that (X, t^*) is not α -T_0(ii).

Example: Let X = {x, y} and $u \in I^X$ be defined by u(x) = 1, u(y) = 0.5. Consider the supra fuzzy topology t^* on X generated by {0, u, 1, Constants}. For α = 0.7, we have $u(x) = 1$ and $u(y) \leq \alpha$. Thus according to the definition (X, t^*) is α -T_0(i) but (X, t^*) is not α -T_0(ii).

Example: Let X = {x, y} and $u \in I^X$ be defined by u(x) = 0, u(y) = 0.8. Let the supra fuzzy supra topology t^* on X generated by {0, u, 1, Constants}. For α = 0.4, it can easily show that (X, t^*) is α - T_0(ii) but (X, t*) is not α -T_0(i). This completes the proof.

Lemma: Let (X, t^*) be a supra fuzzy topological space and $\alpha, \beta \in t^*$ with $0 \leq \alpha \leq \beta < 1$, then

(a) (X, t^*) is α - T_0 (i) implies (X, t^*) is β -T_0 (i).

(b) (X, t^*) is β- T_0(ii) implies (X, t^*) is α -T_0(ii).

(c) (X, t^*) is 0 – T_0(ii) if and only if (X, t^*) is 0 – T_0(iii).

Proof: Suppose that (X, t^*) is a supra fuzzy topological space and (X, t^*) is α - T_0(i). We have to show that (X, t^*) is β - T_0(i). Let any two distinct elements $x, y \in X$. Since (X, t^*) is α - T_0 (i), for $\alpha \in I_1$, there is $u \in t^*$ such that u(x) = 1, and u(y) $\leq \alpha$. This implies that u(x) = 1, and u(y) $\leq \beta$, since $0 \leq \alpha \leq \beta < 1$. Hence by definition, (X, t^*) is β - T_0 (i).

Suppose that (X, t^*) is β - T_0 (ii). Then, for $x, y \in X$ with $x \neq y$, there exist $u \in t^*$ such that $u(x) = 0$ and $u(y) > \beta$, which implies $u(x) = 0$ and $u(y) > \alpha$, since $0 \leq \alpha \leq \beta < 1$. Hence we have (X, t^*) is α -T_0 (ii).

Example: Let $X = \{x, y\}$ and $u \in I^X$ be defined by $u(x) = 1$, $u(y) = 0.6$. Let the supra fuzzy topology t^* on X generated by {0, u, 1, Constants}. Then by definition, for $\alpha = 0.5$ and $\beta = 0.8$; (X, t^*) is β -T_0 (i) but (X, t^*) is not α -T_0 (i).

Example: Let $X = \{x, y\}$ and $u \in I^X$ be defined by $u(x) = 0$, $u(y) = 0.65$. Let the supra fuzzy topology t^* on X generated by {0, u, 1, Constants}. Then by definition, for $\alpha = 0.45$ and $\beta = 0.75$; (X, t^*) is α -T_0 (ii) but (X, t^*) is not β -T_0 (ii).

In the same way, it can be proved that (X, t^*) is $0 - T_0$ (ii) if and only if (X, t^*) is $0 - T_0$ (iii).

Theorem: Let (X, T) be a topological space and T^* be associated supra topology with T and $\alpha \in I_1$. Suppose that the following statements:

(1) (X, T^*) be a T_0 – space.
(2) $(X, \omega (T^*))$ be an α- T_0 (i) space.
(3) $(X, \omega (T^*))$ be an α- T_0 (ii) space.
(4) $(X, \omega (T^*))$ be an α- T_0 (iii) space.
(5) $(X, \omega (T^*))$ be a T_0 (iv) space.

Then the following implications are true:

(a) $(1) \Rightarrow (2) \Rightarrow (4) \Rightarrow (5) \Rightarrow (1)$.

(b) $(1) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5) \Rightarrow (1)$.

Proof: Suppose (X, T^*) is a T_0 – topological space. We have to prove that $(X, \omega (T^*))$ is α - T_0 (i) space. Suppose x and y are two distinct elements in X. Since (X, T^*) is T_0, there is $U \in T^*$ such that $x \in U$, $y \notin U$. By the definition of lsc, we have $1_U \in \omega(T^*)$ and $1_U(x) = 1$, $1_U(y) = 0$. Hence we have $(X, \omega (T^*))$ is α -T_0 (i) space. Also we have $(X, \omega (T^*))$ is α - T_0 (ii) space. Further, it is easy to show that $(2) \Rightarrow (4)$, $(3) \Rightarrow (4)$ and $(4) \Rightarrow (5)$. We therefore prove that $(4) \Rightarrow (1)$.

Suppose $(X, \omega (T^*))$ be a T_0 (iv) space. We have to prove that (X, T^*) is T_0 – space. Let $x, y \in X$ with $x \neq y$. Since $(X, \omega(T^*))$ is T_0 (iv), there is $u \in \omega(T^*)$ such that $u(x) < u(y)$ or $u(x) > u(y)$. Suppose $u(x) < u(y)$. Then for $r \in I_1$, such that $u(x) < r < u(y)$. We observe that $x \notin u^{-1}(r, 1]$, $y \in u^{-1}(r, 1]$, and by definition of lsc, $u^{-1}(r, 1] \in T^*$. Hence (X, T^*) is T_0 – space.

Thus it is seen that $\alpha - T_0$ (p) is a good extension of its topological counter part (p = i, ii, iii, iv).

Theorem: Let (X, t^*) be a supra fuzzy topological space, $\alpha \in I_1$ and $I_\alpha(t^*) = \{u^{-1}(\alpha, 1) : u \in t^*\}$, then

(a) (X, t^*) is an α -T_0(i) implies $(X, I_\alpha(t^*))$ is T_0.
(b) (X, t^*) is an α -T_0(ii) implies $(X, I_\alpha(t^*))$ is T_0.
(c) (X, t^*) is an α -T_0(iii) if and only if $(X, I_\alpha(t^*))$ is T_0.

Proof: (a) Let (X, t^*) be a supra fuzzy topological space and (X, t^*) be α - T_0 (i). Suppose x and y be any two distinct elements in X. Then, for $\alpha \in I_1$, there exists $u \in t^*$ such that $u(x) = 1$, $u(y) \le \alpha$. Since $u^{-1}(\alpha, 1] \in I_\alpha(t^*)$, $y \notin u^{-1}(\alpha, 1]$ and $x \in u^{-1}(\alpha, 1]$, we have that $(X, I_\alpha(t^*))$ is T_0 – space. Similarly, (b) can be proved.

(c) Suppose that (X, t^*) is α - T_0 (iii). Let $x, y \in X$ with $x \neq y$, then for $\alpha \in I_1$, there exists $u \in t^*$ such that $0 \le u(x) \le \alpha < u(y) \le 1$. Since $u^{-1}(\alpha, 1] \in I_\alpha(t^*)$, $x \notin u^{-1}(\alpha, 1]$ and $y \in u^{-1}(\alpha, 1]$, so these implies $(X, I_\alpha(t^*))$ is T_0 – space.

Conversely, suppose that $(X, I_\alpha(t^*))$ be T_0 – space. Let $x, y \in X$ with $x \neq y$, then there exists $u^{-1}(\alpha, 1] \in I_\alpha(t^*)$ such that $x \in u^{-1}(\alpha, 1]$ and $y \notin u^{-1}(\alpha, 1]$, where $u \in t^*$. Thus, we have $u(x) > \alpha$, $u(y) \le \alpha$, i.e., $0 \le u(y) \le \alpha < u(x) \le 1$, and hence by definition, (X, t^*) is α - T_0 (iii) space.

Example: Let $X = \{x, y\}$ and $u \in I^X$ be defined by $u(x) = 0.7$, $u(y) = 0$. Suppose the supra fuzzy topology t^* on X generated by {0, u, 1, Constants}. Then by definition, for $\alpha = 0.5$, (X, t^*) is not α-T_0(i) and (X, t^*) is not α - T_0 (ii). Now $I_\alpha(t^*) = \{X, \phi, \{x\}\}$. Then we see that $I_\alpha(t^*)$ is a supra topology on X and $(X, I_\alpha(t^*))$ is a T_0 -space. This completes the proof.

Theorem: Let (X, t^*) be a supra fuzzy topological space, $A \subseteq X$, and $t^*_A = \{ u/A : u \in t^*\}$, then

(a) (X, t^*) is α - T_0 (i) implies (A, t_A) is α - T_0 (i).
(b) (X, t^*) is α - T_0 (ii) implies (A, t^*_A) is α - T_0 (ii).
(c) (X, t^*) is α - T_0 (iii) implies (A, t^*_A) is α - T_0 (iii).
(d) (X, t^*) is T_0 (iv) implies (A, t^*_A) is T_0 (iv).

Proof: (c) Suppose that (X, t^*) is α - T_0 (iii). Let $x, y \in A$ with $x \neq y$, so that $x, y \in X$, as $A \subseteq X$. Then, for $\alpha \in I_1$, there exists $u \in t^*$ such that $0 \le u(x) \le \alpha < u(y) \le 1$. For $A \subseteq X$, we have $u / A \in t^*_A$ and $0 \le (u/A)(x) \le \alpha < (u/A)(y) \le 1$ as $x, y \in A$. Hence, by definition, (A, t^*_A) is α -T_0 (iii).

Similarly, we can prove (a), (b) and (d).

Theorem: Suppose that (X_i, t^*_i), $i \in \Lambda$ be supra fuzzy topological spaces and $X = \Pi_{i\in\Lambda} X_i$ and t^* be the product supra fuzzy topology on X, then

(a) $\forall$ i$\in \Lambda$, (X_i, t^*_i) is α - T_0 (i) if and only if (X, t^*) is α - T_0 (i).
(b) $\forall$ i$\in \Lambda$, (X_i, t^*_i) is α - T_0 (ii) if and only if (X, t^*) is α - T_0 (ii).
(c) $\forall$ i$\in \Lambda$, (X_i, t^*_i) is α - T_0 (iii) if and only if (X, t^*) is α - T_0 (iii).
(d) $\forall$ i$\in \Lambda$, (X_i, t^*_i) is $_0$ (iv) if and only if (X, t^*) is T_0 (iv).

Proof: (a) Suppose that $\forall$ i $\in \Lambda$, (X_i, t^*_i) is α - T_0 (i). Let x, y$\in$ X with x $\neq$ y, then $x_i \neq y_i$, for some i $\in \Lambda$. Then for $\alpha \in I_1$, there exists $u_i \in t^*_i$, i $\in \Lambda$ such that u_i (x_i) = 1 and ui $(y_i) \leq \alpha$. But we have π_i (x) = x_i, and π_i (y) = y_i. Thus $u_i(\pi_i(x))$ =1 and $u_i(\pi_i (y)) \leq \alpha$, i.e., $(u_i$ o $\pi_i)$ (x) = 1 and $(u_i$ o $\pi_i)$ (y) $\leq \alpha$. It follows that there exists (u_i o π_i) $\in t^*$ such that $(u_i$ o $\pi_i)(x)$ = 1, $(u_i$ o $\pi_i)$ (y) $\leq \alpha$. Hence by definition, (X, t^*) is α - T_0 (i).

Conversely, suppose that (X, t^*) is α - T $_0$ (i) space. We have to show that (X_i, t^*_i), i $\in \Lambda$ is α - T_0 (i). Let a_i be a fixed element in X_i and Ai = {x $\in$ X = $\Pi_{i\in\Lambda} X_i$: $x_j = a_j$ for some i $\neq$ j}. Thus A_i is a subset of X and hence (A_i, t^*_{Ai}) is also a subspace of (X, t^*). Since (X, t^*) is α - T_0 (i), (A_i, t^*_{Ai}) is also α- T_0 (i). Now we have A_i is homeomorphic image of X_i. Thus, we have (X_i, t^*_i), i$\in\Lambda$ is α - T_0 (i) .

Similarly, (b), (c) and (d) can be proved.

Theorem: Let (X, t^*) and (Y, s^*) be two supra fuzzy topological spaces and f : X $\rightarrow$ Y be a one-one , onto and open map, then

(a) (X, t^*) is α -T_0 (i) implies (Y, s^*) is α- T_0 (i).
(b) (X, t^*) is α -T_0 (ii) implies (Y, s^*) is α- T_0 (ii).
(c) (X, t^*) is α -T_0 (iii) implies (Y, s^*) is α- T_0 (iii).
(d) (X, t^*) is T_0 (iv) implies (Y, s^*) is T_0 (iv).

Proof: (b) Suppose (X, t^*) be α -T_0 (ii). Then for y_1, $y_2 \in$ Y with y1 $\neq y_2$, there exist x_1, $x_2 \in$ X with $f(x_1) = y_1$, $f(x_2) = y_2$, since f is onto, and thus $x_1 \neq x_2$ as f is one-one. Again, since (X, t^*) is α-T_0 (ii), for $\alpha \in I_1$, there exists u$\in t^*$ such that u(x) = 0, u(y) > α.

Now, f(u) (y_1) = { Sup $u(x_1)$: $f(x_1) = y_1$ }

= 0, otherwise.

f(u) (y_2) = { Sup $u(x_2)$: $f(x_2) = y_2$ }

> α, otherwise.

Since f is open, f (u) $\in s^*$ as u $\in t^*$. We observe that there exists f (u) $\in s^*$ such that f (u) (y_1) = 0, f(u) $(y_2) > \alpha$. Hence by definition, (Y, s^*) is α - T_0 (ii).

Similarly, (a), (c) and (d) can be proved.

Theorem: Let (X, t^*) and (Y, s^*) be two supra fuzzy topological spaces and f : X $\rightarrow$ Y be continuous and one-one map, then

(a) (Y, s^*) is α- T_0(i) implies (X, t^*) is α - T_0(i).

(b) (Y, s^*) is α- T_0(ii) implies (X, t^*) is α - T_0(ii).

(c) (Y, s^*) is α- T_0(iii) implies (X, t^*) is α - T_0(iii).

(d) (Y, s^*) is T_0(iv) implies (X, t^*) is T_0(iv).

Proof: (c) Suppose (Y, s^*) be α - T_0(iii). Then, for $x_1, x_2 \in X$ with $x_1 \neq x_2$, we have, $f(x_1) \neq f(x_2)$ in Y, since f is one-one. Also, since (Y, s^*) is α-T_0(iii), for $\alpha \in I_1$, there exists $u \in s^*$ such that $0 \leq u(f(x_1)) \leq \alpha < u (f(x_2)) \leq 1$. This implies that $0 \leq f^{-1}(u) (x_1) \leq \alpha < f^{-1}(u) (x_2) \leq 1$, since $u \in s^*$ and f is continuous, then $f^{-1}(u) \in t^*$. Thus, there is an $f^{-1}(u) \in t^*$ such that $0 \leq f^{-1}(u) (x_1) \leq \alpha < f^{-1}(u) (x_2) \leq 1$. Hence, by definition, (X, t^*) is α - T_0(iii).

Similarly, (a), (b) and (d) can be proved.

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