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JORDAN DERIVATIONS ON 2-TORSION FREE SEMIPRIME Γ-RINGS

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ABSTRACT

The objective of this paper was to study Jordan derivations on semiprime Γ-ring. Let M be a 2-torsion free semiprime Γ-ring satisfying the condition $a\alpha b\beta c = a\beta b\alpha c$ for all $a,b,c \in M$ and $\alpha, \beta \in \Gamma$. The authors proved that every Jordan derivation of M is a derivation of M.

Key words: Derivation, Jordan derivation, Two torsion, Semiprime Γ-ring

INTRODUCTION

Let M and Γ be additive abelian groups. If there is a mapping $M \times \Gamma \times M \rightarrow M$ sending (x, α, y) into $x\alpha y$ such that the conditions

(i) $(x + y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha + \beta)y = x\alpha y + x\beta y$, $x\alpha(y + z) = x\alpha y + x\alpha z$ and
(ii) $(x\alpha y)\beta z = x\alpha(y\beta z)$

are satisfied for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, then M is called a Γ-ring.

This definition is due to Barnes (1966). A Γ-ring M is 2-torsion free if $2a = 0$ implies $a = 0$, for all $a \in M$. M is called a semiprime Γ-ring if $a\Gamma M\Gamma a = 0$ (with $a \in M$) implies $a = 0$. M is called completely semiprime if $a\Gamma a = 0$ (with $a \in M$) implies $a = 0$. It was noted that every completely semiprime Γ-ring was a semiprime Γ-ring. The authors defined $[a, b]_\alpha$ by $a\alpha b - b\alpha a$ which is known as a commutator of a and b with respect to α. Let M be a Γ-ring. An additive mapping $d : M \rightarrow M$ is called a derivation if $d(a\alpha b) = d(a)\alpha b + a\alpha d(b)$ for all $a, b \in M$ and $\alpha \in \Gamma$. $d: M \rightarrow M$ is called a Jordan derivation if $d(a\alpha a) = d(a)\alpha a + a\alpha d(a)$ for all $a \in M$ and $\alpha \in \Gamma$. Throughout the article, the authors used the condition $a\alpha b\beta c = a\beta b\alpha c$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$ and this is denoted by (*).

The concepts of derivation and Jordan derivation of a Γ-ring were introduced by Sapanci and Nakajima (1997). For classical ring theory, Herstien (1966) proved that every Jordan derivation in a 2-torsion free prime ring is a derivation. Bresar (1988) proved this result in semiprime rings. Sapanci and Nakajima (1997) proved the same result for completely prime Γ-rings. Haetinger (2002) worked on higher derivations on prime rings and extended this result to Lie ideals in a prime ring.

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In this article, it has been shown that every Jordan derivation of a 2-torsion free semiprime Γ-ring with the condition $a\alpha b\beta c = a\beta b\alpha c$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, is a derivation of M.

Some Consequences of Jordan Derivations on Semiprime Γ-rings

In this section, the authors developed some useful consequences regarding the Jordan derivations of a 2-torsion free semiprime Γ-ring which are needed for proving the main result.

Lemma 2.1: Let M be a Γ-ring and let d be a Jordan derivation of M. Then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, the following statements hold:

(i) $d(a\alpha b + b\alpha a) = d(a)\alpha b + d(b)\alpha a + a\alpha d(b) + b\alpha d(a)$.

(ii) $d(a\alpha b\beta a + a\beta b\alpha a) = d(a)\alpha b\beta a + d(a)\beta b\alpha a + a\alpha d(b)\beta a$

$+ a\beta d(b)\alpha a + a\alpha b\beta d(a) + a\beta b\alpha d(a)$.

In particular, if M is 2-torsion free and M satisfies the condition (*), then

(iii) $d(a\alpha b\beta a) = d(a)\alpha b\beta a + a\alpha d(b)\beta a + a\alpha b\beta d(a)$.

(iv) $d(a\alpha b\beta c + c\alpha b\beta a) = d(a)\alpha b\beta c + d(c)\alpha b\beta a + a\alpha d(b)\beta c$

$+ c\alpha d(b)\beta a + a\alpha b\beta d(c) + c\alpha b\beta d(a)$.

Definition 1: Let d be a Jordan derivation of a Γ-ring M. Then for all $a, b \in M$ and $\alpha \in \Gamma$, define $G_\alpha(a, b) = d(a\alpha b) - d(a)\alpha b - a\alpha d(b)$. Thus $G_\alpha(b, a) = d(b\alpha a) - d(b)\alpha a - b\alpha d(a)$.

Lemma 2.2: Let d be a Jordan derivation of a Γ-ring M. Then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, the following statements hold:

(i) $G_\alpha(a, b) + G_\alpha(b, a) = 0$;

(ii) $G_\alpha(a + b, c) = G_\alpha(a, c) + G_\alpha(b, c)$;

(iii) $G_\alpha(a, b + c) = G_\alpha(a, b) + G_\alpha(a, c)$;

(iv) $G_{\alpha+\beta}(a, b) = G_\alpha(a, b) + G_\beta(a, b)$.

Remark: d is a derivation of a Γ-ring M if and only if $G_\alpha(a, b) = 0$ for all $a, b \in M$ and $\alpha \in \Gamma$.

Lemma 2.3: Let M be a 2-torsion free Γ-ring satisfying the condition (*), and let d be a Jordan derivation of M. Then

(i) $G_\alpha(a, b)\beta m\gamma [a, b]_\alpha + [a, b]_\alpha\beta m\gamma G_\alpha(a, b) = 0$ for all $a, b, m \in M$ and $\alpha, \beta, \gamma \in \Gamma$;

(ii) $G_\alpha(a, b)\alpha m\alpha[a, b]_\alpha + [a, b]\alpha a m\alpha G_\alpha(a, b) = 0$ for all $a, b, m \in M$ and $\alpha \in \Gamma$;

(iii) $G_\alpha(a, b)\beta m\beta[a, b]_\alpha + [a, b]_\alpha\beta m\beta G_\alpha(a, b) = 0$ for all $a, b, m \in M$ and $\alpha, \beta \in \Gamma$.

Proof: (i) For any $a, b, m \in M$ and $\alpha, \beta, \gamma \in \Gamma$ by using Lemma 2 .1 (iv)

$$W = d(a\alpha b\beta m\gamma b\alpha a + b\alpha a\beta m\gamma a\alpha b)$$

$$= d((a\alpha b)\beta m\gamma(b\alpha a) + (b\alpha a)\beta m\gamma(a\alpha b))$$

$$= d(a\alpha b)\beta m\gamma b\alpha a + a\alpha b\beta d(m)\gamma b\alpha a + a\alpha b\beta m\gamma d(b\alpha a)$$

$$+ d(b\alpha a)\beta m\gamma a\alpha b + b\alpha a\beta d(m)\gamma a\alpha b + b\alpha a\alpha m\gamma d(a\alpha b).$$

On the other hand by using Lemma 2 .1 (iii)

$$W = d(a\alpha(b\beta m\gamma b)\alpha a + b\alpha(a\beta m\gamma a)\alpha b)$$

$$= d(a\alpha(b\beta m\gamma b)\alpha a) + d(b\alpha(a\beta m\gamma a)\alpha b)$$

$$= d(a)\alpha b\beta m\gamma b\alpha a + a\alpha d(b\beta m\gamma b)\alpha a + a\alpha b\beta m\gamma b\alpha d(a)$$

$$+ d(b)\alpha a\beta m\gamma a\alpha b + b\alpha d(a\beta m\gamma a)\alpha b + b\alpha a\beta m\gamma a\alpha d(b)$$

$$= d(a)\alpha b\beta m\gamma b\alpha a + a\alpha d(b)\beta m\gamma b\alpha a + a\alpha b\beta d(m)\gamma b\alpha a + a\alpha b\beta m\gamma d(b)\alpha a$$

$$+ a\alpha b\beta m\gamma b\alpha d(a) + d(b)\alpha a\beta m\gamma a\alpha b + b\alpha d(a)\beta m\gamma a\alpha b + b\alpha a\beta d(m)\gamma a\alpha b$$

$$+ b\alpha a\beta m\gamma d(a)\alpha b + b\alpha a\beta m\gamma a\alpha d(b).$$

Equating two expressions for W and cancelling the like terms from both sides, one gets

$$d(a\alpha b)\beta m\gamma b\alpha a + a\alpha b\beta m\gamma d(b\alpha a) + d(b\alpha a)\beta m\gamma a\alpha b + b\alpha a\beta m\gamma d(a\alpha b)$$

$$= d(a)\alpha b\beta m\gamma b\alpha a + a\alpha d(b)\beta m\gamma b\alpha a + a\alpha b\beta m\gamma d(b)\alpha a + a\alpha b\beta m\gamma b\alpha d(a)$$

$$+ d(b)\alpha a\beta m\gamma a\alpha b + b\alpha d(a)\beta m\gamma a\alpha b + b\alpha a\beta m\gamma d(a)\alpha b + b\alpha a\beta m\gamma a\alpha d(b).$$

This gives, $d(a\alpha b)\beta m\gamma b\alpha a - d(a)\alpha b\beta m\gamma b\alpha a - a\alpha d(b)\beta m\gamma b\alpha a + d(b\alpha a)\beta m\gamma a\alpha b$
$- d(b)\alpha a\beta m\gamma a\alpha b - b\beta d(a)\beta m\gamma a\alpha b + a\alpha b\beta m\gamma d(b\alpha a) - a\alpha b\beta m\gamma d(b)\alpha a$
$- a\alpha b\beta m\gamma b\alpha d(a) + b\alpha a\alpha m\gamma d(a\alpha b) - b\alpha a\beta m\gamma d(a)\alpha b - b\alpha a\beta m\gamma a\alpha d(b) = 0.$

This implies, $(d(a\alpha b) - d(a)\alpha b - a\alpha d(b))\alpha m\gamma b\alpha a + (d(b\alpha a) - d(b)\alpha a - b\alpha d(a))\alpha m\gamma a\alpha b$

$$+ a\alpha b\alpha m\gamma(d(b\alpha a) - d(b)\alpha a - b\alpha d(a)) + b\alpha a\alpha m\gamma(d(a\alpha b) - d(a)\alpha b - a\alpha d(b)) = 0.$$

Using Definition 1, $G_\alpha(a, b)\beta m\gamma b\alpha a + G_\alpha(b, a)\beta m\gamma a\alpha b + a\alpha b\beta m\gamma G_\alpha(b, a) + b\alpha a\beta m\gamma G_\alpha(a, b) = 0$.

This implies, $G_\alpha(a, b)\,\beta m\gamma\,[a, b]_\alpha + [a, b]_\alpha\beta m\gamma G_\alpha(a, b) = 0$ for all $a, b, m \in M, \alpha, \beta, \gamma \in \Gamma$.

By considering $W = d(a\alpha b\alpha m\alpha b\alpha a + b\alpha a\alpha m\alpha a\alpha b)$ and $W = d(a\alpha b\beta m\beta b\alpha a + b\alpha a\beta m\beta a\alpha b)$ for (ii) and (iii), respectively and proceeding in the same way as in the proof of (i) by the similar arguments, one gets (ii) and (iii).

Lemma 2.4: Let M be a 2-torsion free semiprime Γ-ring and let $a, b, m \in M$.

If $a\alpha m\beta b + b\alpha m\beta a = 0$, for all $m \in M, \alpha, \beta \in \Gamma$, then $a\alpha m\beta b = 0 = b\alpha m\beta a$.

Proof : Let $x \in M$ and $\gamma, \delta \in \Gamma$ be any elements.

Using the relation $a\alpha m\beta b + b\alpha m\beta a = 0$ for all $m \in M$ and $\alpha, \beta \in \Gamma$ repeatedly

$$\begin{aligned}(a\alpha m\beta b)\gamma x\delta(a\alpha m\beta b) &= -(b\alpha m\beta a)\gamma x\delta(a\alpha m\beta b)\\ &= -(b\alpha(m\beta a\gamma x)\delta a)\alpha m\beta b = (a\alpha(m\beta a\gamma x)\delta b)\alpha m\beta b\\ &= a\alpha m\beta(a\gamma x\delta b)\alpha m\beta b = -a\alpha m\beta(b\gamma x\delta a)\alpha m\beta b\\ &= -(a\alpha m\beta b)\gamma x\delta(a\alpha m\beta b).\end{aligned}$$

This implies, $2\,((a\alpha m\beta b)\gamma x\delta(a\alpha m\beta b)) = 0$.

Since M is 2-torsion free, $(a\alpha m\beta b)\,\gamma x\delta\,(a\alpha m\beta b) = 0$. Therefore, $(a\alpha m\beta b)\,\Gamma M\Gamma\,(a\alpha m\beta b) = 0$.

By the semiprimeness of M, $a\alpha m\beta b = 0$. Similarly, it can be shown that $b\alpha m\beta a = 0$.

Corollary 2.1: Let M be a 2-torsion free semiprime Γ-ring satisfying the condition (*), and let d be a Jordan derivation of M. Then for all $a, b, m \in M$ and $\alpha, \beta, \gamma \in \Gamma$:

(i) $G_\alpha(a, b)\beta m\gamma[a, b]_\alpha = 0$; (ii) $[a, b]_\alpha\beta m\gamma G_\alpha(a, b) = 0$;

(iii) $G_\alpha(a, b)\alpha m\alpha[a, b]_\alpha = 0$; (iv) $[a, b]_\alpha\alpha m\alpha G_\alpha(a, b) = 0$;

(v) $G_\alpha(a, b)\beta m\beta[a, b]_\alpha = 0$; (vi) $[a, b]_\alpha\beta m\beta G_\alpha(a, b) = 0$.

Proof: Applying the result of Lemma 2.4 in that of Lemma 2.3, the authors obtained these results.

Lemma 2.5: Let M be a 2-torsion free semiprime Γ-ring satisfying the condition (*), and let d be a Jordan derivation of M. Then for all $a, b, x, y, m \in M$ and $\alpha, \beta, \gamma \in \Gamma$:

(i) $G_\alpha(a, b)\beta m\beta[x, y]_\alpha = 0$; (ii) $[x, y]_\alpha\beta m\beta G_\alpha(a, b) = 0$

(iii) $G_\alpha(a, b)\beta m\beta[x, y]_\gamma = 0$; (iv) $[x, y]_\gamma\beta m\beta G_\alpha(a, b) = 0$.

Proof: (i) If substitute $a + x$ for a in the Corollary 2 .1 (v), $G_\alpha(a+x, b)\beta m\beta[a+x, b]_\alpha = 0$ is obtained.

Thus $G_\alpha(a, b)\beta m\beta[a, b]_\alpha + G_\alpha(a, b)\beta m\beta[x, b]_\alpha + G_\alpha(x, b)\beta m\beta[a, b]_\alpha + G_\alpha(x, b)\beta m\beta[x, b]_\alpha = 0$.

By using Corollary 2 .1 (v), $G_\alpha(a, b)\beta m\beta[x, b]_\alpha + G_\alpha(x, b)\beta m\beta[a, b]_\alpha = 0$ is obtained.

Thus, $(G_\alpha(a, b)\beta m\beta[x, b]_\alpha)\beta m\beta(G_\alpha(a, b)\beta m\beta[x, b]_\alpha) = -G_\alpha(a, b)\beta m\beta[x, b]_\alpha \beta m\beta G_\alpha(x, b)\beta m\beta [a, b]_\alpha = 0$ is obtained.

Hence, by the semiprimeness of M, $G_\alpha(a, b)\beta m\beta[x, b]_\alpha = 0$.

Similarly, by replacing $b + y$ for b in this result $G_\alpha(a, b)\beta m\beta[x, y]_\alpha = 0$.

(ii) Proceeding in the same way as described above by the similar replacements successively in Corollary 2 .1 (vi), $[x, y]_\gamma \beta m\beta G_\alpha(a, b) = 0$ for all $a, b, x, y, m \in M, \alpha, \beta \in \Gamma$ is obtained.

(iii) Replacing $\alpha + \gamma$ for α in (i), $G_{\alpha+\gamma}(a, b)\beta m\beta[x, y]_{\alpha+\gamma} = 0$.

This implies, $(G_\alpha(a, b) + G_\gamma(a, b)_\alpha)\beta m\beta([x, y]_\alpha + [x, y]_\gamma) = 0$.

Therefore, $G_\alpha(a, b)\beta m\beta[x, y]_\alpha G_\alpha(a, b)\beta m\beta[x, y]_\gamma + G_\gamma(a, b)\beta m\beta[x, y]_\alpha + G_\gamma(a, b)\beta m\beta[x, y]_\gamma = 0$.

Thus by using Corollary 2 .1 (vi), $G_\alpha(a, b)\beta m\beta[x, y]_\gamma + G_\gamma(a, b)\beta m\beta[x, y]_\alpha = 0$.

Thus, $(G_\alpha(a, b)\beta m\beta[x, y]_\gamma)\beta m\beta(G_\alpha(a, b)\alpha m\beta[x, y]_\gamma) = -G_\alpha(a, b)\beta m\beta[x, y]_\gamma \beta m\beta G_\gamma(a, b)\beta m\beta [x, y]_\alpha = 0$ is obtained.

Hence, by the semiprimeness of M, $G_\alpha(a, b)\beta m\beta[x, y]_\gamma = 0$.

(iv) As in the proof of (iii), the similar replacement in (ii) produces (iv).

Lemma 2 .6: Every semiprime Γ-ring contains no nonzero nilpotent ideal.

Corollary 2.2: Semiprime Γ-ring has no nonzero nilpotent element.

Lemma 2.7: The center of a semiprime Γ-ring does not contain any nonzero nilpotent element.

Jordan Derivations on Semiprime Γ-rings

The authors proved main result as follows:

Theorem 3.1: Let M be a 2-torsion free semiprime Γ-ring satisfying the condition (*), and let d be a Jordan derivation of M. Then d is a derivation of M.

Proof: Let d be a Jordan derivation of a 2-torsion free semiprime Γ-ring M, and let $a, b, y, m \in M$ and $\alpha, \beta \in \Gamma$. Then by Lemma 2 .5 (iii),

$[G_\alpha(a, b), y]_\beta \beta m\beta[G_\alpha(a, b), y]_\beta = (G_\alpha(a, b)\beta y - y\beta G_\alpha(a, b))\beta m\beta[G_\alpha(a, b), y]_\beta$

$= G_\alpha(a, b)\beta y\beta m\beta[G_\alpha(a, b), y]_\beta - y\beta G_\alpha(a, b)\beta m\beta[G_\alpha(a, b), y]_\beta = 0$.

Since $y\beta m \in M$ and $G_\alpha(a, b) \in M$ for all $a, b, y, m \in M$ and $\alpha, \beta \in \Gamma$.

By the semiprimeness of M, $[G_\alpha(a, b), y]_\beta = 0$, where $G_\alpha(a, b) \in M$ for all $a, b, y \in M$ and $\alpha, \beta \in \Gamma$. Therefore, $G_\alpha(a, b) \in Z(M)$, the center of M.

Now, let $\gamma, \delta \in \Gamma$. By Lemma 2 .5 (ii), $G_\alpha(a, b)[x, y]_\alpha \delta m \delta G_\alpha(a, b)\gamma[x, y]_\alpha = 0$.

Since M is semiprime, so $G_\alpha(a, b)\gamma[x, y]_\alpha = 0$. (1)

Also, by Lemma 2 .5 (i), $[x, y]_\alpha \gamma G_\alpha(a, b) \delta m \delta [x, y]_\alpha \gamma G_\alpha(a, b) = 0$.

Hence by the semiprimeness of M, $[x, y]_\alpha \gamma G_\alpha(a, b) = 0$. (2)

Similarly, by Lemma 2 .5 (iv), $G_\alpha(a, b)\gamma[x, y]_\beta \delta m \delta G_\alpha(a, b) \gamma[x, y]_\beta = 0$.

Since M is semiprime, it follows that $G_\alpha(a, b)\gamma[x, y]_\beta = 0$. (3)

Also, by Lemma 2.5 (iii), $[x, y]_\beta \gamma G_\alpha(a, b)\delta m \delta [x, y]_\beta \gamma G_\alpha(a, b) = 0$.

Hence by the semiprimeness of M, $[x, y]_\beta \gamma G_\alpha(a, b) = 0$. (4)

Thus, $2 G_\alpha(a, b)\gamma G_\alpha(a, b) = G_\alpha(a, b)\gamma(G_\alpha(a, b + G_\alpha(a, b))$

$$= G_\alpha(a, b) \gamma(G_\alpha(a, b) - G_\alpha(b, a))$$

$$= G_\alpha(a, b)\gamma(d(a\alpha b) - d(a)\alpha b - a\alpha d(b) - d(b\alpha a) + d(b)\alpha a + b\alpha d(a))$$

$$= G_\alpha(a, b)\gamma(d(a\alpha b - b\alpha a) + (b\alpha d(a) - d(a)\alpha b) + (d(b)\alpha a - a\alpha d(b)))$$

$$= G_\alpha(a, b)\gamma(d([a, b]_\alpha) + [b, d(a)]_\alpha + [d(b), a]_\alpha)$$

$$= G_\alpha(a, b)\gamma d([a, b]_\alpha) - G_\alpha(a, b)\gamma[d(a), b]_\alpha - G_\alpha(a, b)\gamma[a, d(b)]_\alpha.$$

Since $d(a), d(b) \in M$ by using (1), $G_\alpha(a, b)\gamma[d(a), b]_\alpha = G_\alpha(a, b)\gamma[a, d(b)]_\alpha = 0$.

Thus $2G_\alpha(a, b) \gamma G_\alpha(a, b) = G_\alpha(a, b) \gamma d([a, b]_\alpha)$ (5)

Adding (3) and (4), $G_\alpha(a, b) \gamma[x, y]_\beta + [x, y]_\beta \gamma G_\alpha(a, b) = 0$, is obtained.

Then by Lemma 2 .1 (i) with the use of (3),

$0 = d(G_\alpha(a, b) \gamma[x, y]_\beta + [x, y]_\beta \gamma G_\alpha(a, b))$

$= d(G_\alpha(a, b))\gamma[x, y]_\beta + d([x, y]_\beta)\gamma G_\alpha(a, b) + G_\alpha(a, b)\gamma d([x, y]_\beta) + [x, y]_\beta \gamma d(G_\alpha(a, b))$

$= d(G_\alpha(a, b)) \gamma[x, y]_\beta + 2G_\alpha(a, b) \gamma d([x, y]_\beta) + [x, y]_\beta \gamma d(G_\alpha(a, b))$.

Since $G_\alpha(a, b) \in Z(M)$ and therefore $d([x, y]_\beta)G_\alpha(a, b) = G_\alpha(a, b)d([x, y]_\beta)$.

Hence, $2 G_\alpha(a, b)\gamma d([x, y]_\beta) = -d(G_\alpha(a, b)) \gamma[x, y]_\beta - [x, y]_\beta \gamma d(G_\alpha (a, b))$. (6)

Then from (5) and (6)

$4G_\alpha(a, b) \gamma G_\alpha (a, b) = 2 G_\alpha (a, b) \gamma d([a, b]_\alpha) = -d(G_\alpha (a, b)) \gamma[a, b]_\alpha - [a, b]_\alpha \gamma d(G_\alpha(a, b))$

So, $4G_\alpha(a, b)\gamma G_\alpha(a, b)\gamma G_\alpha(a, b) = -d(G_\alpha(a, b))\gamma[a, b]_\alpha\gamma G_\alpha(a, b) - [a, b]_\alpha\gamma d(G_\alpha(a, b))\gamma G_\alpha(a, b)$.

Here, by using (4) $d(G_\alpha(a, b))\gamma[a, b]_\alpha\gamma G_\alpha(a, b) = 0$ and also, by Corollary 2 .1 (vi), $[a, b]_\alpha\gamma d(G_\alpha(a, b))\gamma G_\alpha(a, b) = 0$.

Since $d(G_\alpha(a, b)) \in M$ for all $a, b \in M$ and $\alpha \in \Gamma$. So, $4G_\alpha(a, b)\gamma G_\alpha(a, b)\gamma G_\alpha(a, b) = 0$.

Therefore, $4(G_\alpha(a, b)\gamma)^2 G_\alpha(a, b) = 0$. Since M is 2-torsion free, so $(G_\alpha(a, b)\gamma)^2 G_\alpha(a, b) = 0$. But, it follows that $G_\alpha(a, b)$ is a nilpotent element of the Γ-ring M.

Since by Lemma 2.7, the center of a semiprime Γ-ring does not contain any nonzero nilpotent element, so $G_\alpha(a, b) = 0$ for all $a, b \in M$ and $\alpha \in \Gamma$. It means d is a derivation of M.

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