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ON INTUITIONISTIC FUZZY $T_0 -$ SPACES

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ABSTRACT

In this paper, intuitionistic fuzzy T_0 I spaces were studied. The authors investigated some relations among them and also investigated the relationship between intuitionistic fuzzy topological spaces and intuitionistic topological spaces.

Key words: Intuitionistic topological space, Intuitionistic fuzzy topological space, Intuitionistic fuzzy T_0 I spaces

INTRODUCTION

The fundamental concept of a fuzzy set was introduced by Zadeh (1965). Chang (1968) introduced the concept of a fuzzy topological space by using fuzzy sets. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov (1986, 1988). Coker (1997, 2000) and his colleagues (Bayhan 2001, 2003) introduced intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In this paper, the authors investigated the properties of T_0 I spaces.

Definition 1.1. An intuitionistic set A is an object having the form $A = (x, A_1, A_2)$ where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A while A_2 is called the set of non-members of A. Throughout this paper, we use the simpler notation $A = (A_1, A_2)$ for an intuitionistic set (Coker 1996).

Remark 1.2. Every subset A on a nonempty set X may obviously be regarded as an intuitionistic set having the form $A' = (A, A^c)$, where $A^c = X \setminus A$ is the complement of A in X (Coker 1996).

Definition 1.3. Let the intuitionistic sets A and B on X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$, respectively. Furthermore, let $\{A_j : j \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_j = \left(A_j^{(1)}, A_j^{(2)}\right)$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

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(c) $\bar{A} = (A_2, A_1)$ denotes the complement of A.

(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$.

(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$.

(f) $\phi_\sim = (\phi, X)$ and $X_\sim = (X, \phi)$ (Coker 1996).

Definition: 1.4. An intuitionistic topology on a set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(1) $\phi_\sim, X_\sim \in \tau$.

(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.

(3) $\cup G_i \in \tau$ for any arbitrary family $G_i \in \tau$.

In this case, the pair (X, τ) is called an intuitionistic topological space (ITS in short) and any intuitionistic set in τ is known as an intuitionistic open set (IOS in short) in X (Coker 2000).

Definition 1.5. Let X be a non empty set and *I* be the unit interval $[0, 1]$. An intuitionistic fuzzy set A (IFS in short) in X is an object having the form

$$A = \{(x, \mu_A(x), \nu_A(x)), x \in X\},$$

where $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership and the degree of non- membership respectively, and $\mu_A(x) + \nu_A(x) \leq 1$.

Let $I(X)$ denote the set of all intuitionistic fuzzy sets in X. Obviously every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form ($\mu_A, 1 - \mu_A$).

Throughout this paper, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$ (Atanassov 1986).

Definition 1.6. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.

(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = (\nu_A, \mu_A)$.

(4) $A \cap B = (\mu_A \cap \mu_B; \nu_A \cup \nu_B)$.

(5) $A \cup B = (\mu_A \cup \mu_B; \nu_A \cap \nu_B)$.

(6) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$ (Atanassov 1986).

Definition 1.7. An intuitionistic fuzzy topology (IFT in short) on X is a family t of

IFSs in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in t$.

(2) if $A_1, A_2 \in t$, then $A_1 \cap A_2 \in t$.

(3) if $A_i \in t$ for each i, then $\cup A_i \in t$.

The pair (X, t) is called an intuitionistic fuzzy topological space (IFTS in short).

Let (X, t) be an IFTS. Then any member of t is called an intuitionistic fuzzy open set (IFOS in short) in X. The complement of an IFOS in X is called an intuitionistic fuzzy closed set (IFCS in short) in X (Coker 1997).

2. Intuitionistic fuzzy T_0-spaces

Theorem 2.1. Let (X, τ) be an intuitionistic topological space and let $t = \{1_1 A : A \in \tau\}$, $1_{(A_1, A_2)} = (1_{A_1}, 1_{A_2})$, then (X, t) is an intuitionistic fuzzy topological space.

Proof: (i) $\phi_\sim = (\phi, X) \in \tau \Rightarrow 1_1(\phi_1{\sim}) = (1_1\phi, 1_1 X) = (0^\uparrow{\sim}, 1^\uparrow{\sim}) = 0_\sim \in t.$

Hence $\phi \in \tau \Leftrightarrow 0_\sim \in t.$

Now $X_\sim = (X, \phi) \in \tau \Rightarrow 1_{X_\sim} = (1_1 X, 1_1\phi) = (1^\uparrow{\sim}, 0^\uparrow{\sim}) = 1_\sim \in t.$

Hence $X_\sim \in \tau \Leftrightarrow 1_\sim \in t.$

(ii) Let $G_1, G_2 \in \tau$, then $G_i = (G_i^1, G_i^2) \in \tau$ (i =1, 2)

Now $1_{G_i} = (1_1(G_i{}^\uparrow 1), 1_1(G_i{}^\uparrow 2))$

Hence $G_i \in \tau \Leftrightarrow 1_{G_i} \in t$ (i =1, 2)

Now $G_1 \cap G_2 \in \tau \Leftrightarrow (G_1^1 \cap G_2^1, G_1^2 \cup G_2^2) \in \tau$

And $1_{G_1 \cap G_2} = (1_{G_1^1 \cap G_2^1}, 1_{G_1^2 \cup G_2^2}) = 1_{G_1} \cap 1_{G_2}$

Hence $G_1 \cap G_2 \in \tau \Leftrightarrow 1_{G_1 \cap G_2} \in t$

(iii) Let $G_i \in \tau \Leftrightarrow \cup_i G_i = (\cup_i G_i^1, \cap_i G_i^2) \in \tau.$ (i= 1,2,3…,)

And $1_{\cup_i G_i} = (1_{\cup_i G_i^1}, 1_{\cap_i G_i^2}) = \cup_i 1_{G_i}$

Hence $\cup_i G_i \in \tau \Leftrightarrow 1_{\cup_i G_i} \in t$. Therefore, (X, t) is an intuitionistic fuzzy topological space.

Definition 2.2. As defined in theorem 2.1 (X, t) is called the intuitionistic fuzzy topological space to the corresponding intuitionistic topological space (X, τ) .

Definition 2.3. An intuitionistic fuzzy topological space (X, t) is called

(1) IF $-T_0$(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that

$\mu_A(x) = 1, \nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) = 1$

or $\mu_A(y) = 1, \nu_A(y) = 0;\ \mu_A(x) = 0, \nu_A(x) = 1$.

(2) IF$-T_0$(ii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that

$\mu_A(x) = 1, \nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) > 0$

or $\mu_A(y) = 1, \nu_A(y) = 0;\ \mu_A(x) = 0, \nu_A(x) > 0$.

(3) IF$-T_0$(iii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that

$\mu_A(x) > 0$, $\nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) = 1$

or $\mu_A(y) > 0$, $\nu_A(y) = 0;\ \mu_A(x) = 0, \nu_A(x) = 1$.

(4) IF$-T_0$(iv) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that

$\mu_A(x) > 0$, $\nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) > 0$

or $\mu_A(y) > 0$, $\nu_A(y) = 0;\ \mu_A(x) = 0, \nu_A(x) > 0$.

Definition 2.4. Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) $\alpha -$ IF$-T_o$(i) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that

$\mu_A(x) = 1, \nu_A(x) = 0;\ \mu_A(y) = 0$, $\nu_A(y) \geq \alpha$

or $\mu_1 A(y) = 1, \nu_A(y) = 0;\ \mu_A(x) = 0$, $\nu_A(x) \geq \alpha$.

(b) $\alpha -$ IF$-T_o$(ii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that

$\mu_A(x) \geq \alpha$, $\nu_A(x) = 0;\ \mu_A(y) = 0$, $\nu_A(y) \geq \alpha$

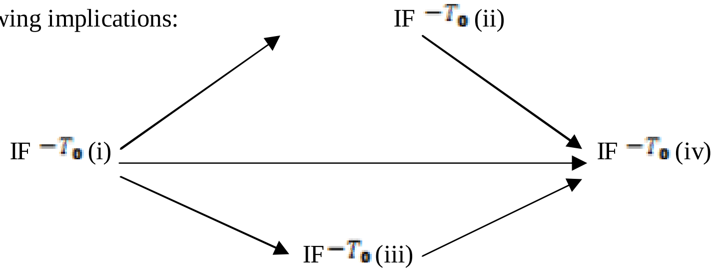
or $\mu_A(y) \geq \alpha$, $\nu_A(y) = 0;\ \mu_A(x) = 0$, $\nu_A(x) \geq \alpha$.

(c) $\alpha -$ IF$-T_o$(iii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that

$\mu_A(x) > 0$, $\nu_A(x) = 0;\ \mu_A(y) = 0$, $\nu_A(y) \geq \alpha$

or $\mu_1 A(y) > 0$, $\nu_A(y) = 0;\ \mu_A(x) = 0$, $\nu_A(x) \geq \alpha$.

Theorem 2.5. Let $(X, t$) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is an IF$-T_0$(i) space. We shall prove that (X, t) is an IF$-T_0$ (ii).

Since (X, t) is an IF$-T_0$(i) space, then for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A , \nu_A) \in t$ such that

μ_A (x) =1, ν_A (x) = 0; μ_A (y) = 0, ν_A (y) =1

$\Rightarrow$ μ_A (x) =1, ν_A (x) = 0; μ_A (y) = 0, ν_A (y)> 0

which is IF$-T_0$ (ii). Hence IF$-T_0$(i) $\Rightarrow$ IF $-T_0$(ii) .

Again, suppose (X, t) is an IF$-T_0$(i) space. We shall prove that (X, t) is an IF$-T_0$(iii).

Since (X, t) is an IF$-T_0$(i) space, then for all $x, y \in X,$ $x \neq y$ there exists A =$(\mu_A , \nu_A) \in t$ such that

μ_A (x) = 1, ν_A (x) = 0; μ_A (y) = 0, ν_A (y) =1

$\Rightarrow$ μ_A (x)> 0 , ν_A (x) = 0; μ_A (y) = 0, ν_A (y)$= 1$.

which is IF$-T_0$(iii). Hence IF$-T_0$(i)$\Rightarrow$ IF$-T_0$(iii) . Furthermore, One can be easily verified that IF$-T_1 0$ (i) $\Rightarrow$ IF$-T_1 0$ (iv), IF$-T_1 0$ (ii) $\Rightarrow$ IF$-T_0$ (iv) and IF$-T_1 0$ (iii)$\Rightarrow$ IF$-T_1 0$ (iv).

None of the reverse implications is true in general as can be seen from the following examples.

Example 2.5.1. Let $X = \{x, y$ } and let t be the intuitionistic fuzzy topology on X generated by {A }, where A = {(x , 1, 0),(y , 0, 0.3)}. We see that the IFTS (X, t) is IF$-T_1 0$ (ii) but not IF$-T_1 0$ (i).

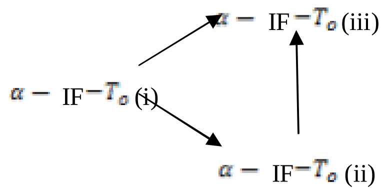
Example 2.5.2. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by {A }, where A = {(x , 0.4, 0), (y , 0, 1)}. We see that the IFTS (X, t) is IF$-T_0$ (iii) but not IF$-T_0$ (i).

Example 2.5.3. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X

generated by {A }, where A ={(x , 1, 0), (y , 0, 0,5)}. The authors saw that the IFTS (X, t) is IF$-T_0$ (ii) but not IF$-T_0$ (iii).

Example 2.5.4. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by {A }, where A = {(x , 0.6, 0), (y , 0, 1)}. The authors noted that the IFTS (X, t) is IF$-T_0$ (iii) but not IF$-T_0$ (ii).

Theorem 2.6. Let (X, t) be an intuitionistic fuzzy topological space. Then the authors made the following implications:



Proof: Let $\alpha \in$ (0, 1). Suppose (X, t) is an $\alpha -$ IF$-T_o$ (i) space. The authors shall prove that (X, t) is $\alpha -$ IF$-T_o$ (ii). Since (X, t) is $\alpha -$ IF$-T_o$ (i) space, then for all $x, y \in X,$ $x \neq y$ there exists $A = (\mu_A , \nu_A) \in t$ such that

μ_A (x) =1, ν_A (x) = 0; μ_A (y)= 0 , ν_A (y)$\geq \alpha$

$\Rightarrow$ μ_A (x)$\geq \alpha$, ν_A (x) = 0; μ_A (y) = 0 , ν_A (y)$\geq \alpha$ for any $\alpha \in$ (0, 1).

which is $\alpha -$ IF$-T_o$ (ii). Hence $\alpha -$ IF$-T_o$ (i) $\Rightarrow$ $\alpha -$ IF$-T_o$ (ii).

Again, let $\alpha \in$ (0, 1). Suppose (X, t) is $\alpha -$ IF$-T_o$ (ii) space. The authors shall prove that (X, t) is $\alpha -$ IF$-T_o$ (iii). Since (X, t) is $\alpha -$ IF$-T_o$ (ii) space, then for all $x, y \in X,$ $x \neq y$ there exists $A = (\mu_A , \nu_A) \in t$ such that

μ_A (x)$\geq \alpha$, ν_A (x) = 0; μ_A (y) = $0,$ ν_A (y)$\geq \alpha$

$\Rightarrow$ μ_A (x)> 0 , ν_A (x) = 0; μ_A (y) = $0,$ ν_A (y)$\geq \alpha$ for any $\alpha \in$ (0, 1).

which is $\alpha -$ IF$-T_o$ (iii). Hence $\alpha -$ IF$-T_o$ (ii) $\Rightarrow$ $\alpha -$ IF$-T_o$ (iii).

Furthermore, One can easily verify that $\alpha -$ IF$-T_o$ (i) $\Rightarrow$ $\alpha -$ IF$-T_o$ (iii) .

None of the reverse implications is true in general as can be seen from the following examples.

Example 2.6.1. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by {A }, where A = {(x , 0.5, 0), (y , 0, 0.5)}. For $\alpha = 0.3,$ the authors notes that the IFTS (X, t) is $\alpha -$ IF$-T_0$ (ii) but not $\alpha -$ IF$-T_10$ (i).

Example 2.6.2. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$, where $A = \{(x, 0.3, 0), (y, 0, 0.7)\}$. For $\alpha = 0.5$, the authors saw that the IFTS (X, t) is

$\alpha -$ IF$-T_0$(iii) but not $\alpha -$ IF$-T_0$(i) nor $\alpha -$ IF$-T_0$(ii).

Theorem 2.7. Let (X, t) be an intuitionistic fuzzy topological space and $0 < \alpha \le \beta < 1$, then

(a) $\beta -$ IF$-T_o$(i) $\Rightarrow$ $\alpha -$ IF$-T_o$(i).

(b) $\beta -$ IF$-T_o$(ii) $\Rightarrow$ $\alpha -$ IF$-T_o$(ii).

(c) $\beta -$ IF$-T_o$(iii) $\Rightarrow$ $\alpha -$ IF$-T_o$(iii).

Proof (a): Suppose the intuitionistic fuzzy topological space (X, t) is $\beta -$ IF$-T_o$(i). The authors shall prove that (X, t) is $\alpha -$ IF$-T_o$(i).

Since (X, t) is $\beta -$ IF$-T_o$(i), then for all $x, y \in X,\ x \ne y$ with $\beta \in (0, 1)$ there exists

$A = (\mu_A, \nu_A) \in t$ such that

$$\mu_A(x) = 1,\ \nu_A(x) = 0;\ \mu_A(y) = 0,\ \nu_A(y) \ge \beta$$

$$\Rightarrow \quad \mu_A(x) = 1,\ \nu_A(x) = 0;\ \mu_A(y) = 0,\ \nu_A(y) \ge \alpha \quad \text{as } 0 < \alpha \le \beta < 1.$$

Which is $\alpha -$ IF$-T_o$(i). Hence $\beta -$ IF$-T_o$(i) $\Rightarrow$ $\alpha -$ IF$-T_o$(i).

The proofs that $\beta -$ IF$-T_o$(ii) $\Rightarrow$ $\alpha -$ IF$-T_o$(ii) and $\beta -$ IF$-T_o$(iii) $\Rightarrow$ $\alpha -$ IF$-T_o$(iii) are similar.

None of the reverse implications is true in general as can be seen from the following examples.

Example 2.7.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 1, 0), (y, 0, 0.5)\}$. For $\alpha = 0.3$ and $\beta = 0.8$, it is clear that the IFTS (X, t) is $\alpha -$ IF$-T_0$(i) but not $\beta -$ IF$-T_0$(i).

Example 2.7.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 0.4, 0), (y, 0, 0.4)\}$. For $\alpha = 0.3$ and $\beta = 0.5$, it is clear that the IFTS (X, t) is $\alpha -$ IF$-T_0$(ii) but not $\beta -$ IF$-T_0$(ii).

Example 2.7.3. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 0.2, 0), (y, 0, 0.6)\}$. For $\alpha = 0.5$ and $\beta = 0.7$, it is clear that the IFTS (X, t) is $\alpha -$ IF$-T_0$(iii) but not $\beta -$ IF$-T_0$(iii).

Theorem 2.8. Let (X, t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and

$t_U = \{ A/_U : A \in t \}$ and $\alpha \in (0, 1)$, then

(a) (X, t) is IF$-T_0$(i) $\Rightarrow (U, t_1U)$ is IF$-T_0$(i).

(b) (X, t) is IF$-T_0$ (ii) $\Rightarrow (U, t_1U)$ is IF$-T_0$ (ii).

(c) (X, t) is IF$-T_0$(iii) $\Rightarrow (U, t_1U)$ is IF$-T_0$ (iii).

(d) (X, t) is IF$-T_0$ (iv) $\Rightarrow (U, t_1U)$ is IF$-T_0$ (iv).

(e) (X, t) is $\alpha -$ IF$-T_0$ (i) $\Rightarrow (U, t_1U)$ is $\alpha -$ IF$-T_0$ (i).

(f) (X, t) is $\alpha -$ IF$-T_0$(ii) $\Rightarrow (U, t_1U)$ is $\alpha -$ IF$-T_0$(ii).

(g) (X, t) is $\alpha -$ IF$-T_0$ (iii) $\Rightarrow (U, t_1U)$ is $\alpha -$ IF$-T_0$(iii).

The proofs (a), (b), (c), (d), (e), (f), (g) are similar. As an example the authors proved (e).

Proof(e): Suppose (X, t) is an intuitionistic fuzzy topological space and is also $\alpha -$ IF$-T_0$(i).

We shall prove that (U, t_1U) is $\alpha -$ IF$-T_0$ (i).

Let $x, y \in U$ with $x \neq y$, then $x, y \in X$ with $x \neq y$ as $U \subseteq X$. Since (X, t) is $\alpha -$ IF$-T_0$ (i), then there exists $B = (\mu_B, \nu_B) \in t$ such that

$$\mu_B(x) = 1,\ \nu_B(x) = 0;\ \mu_B(y) = 0,\ \nu_B(y) \geq \alpha$$

$$\Rightarrow \quad \mu_1 B\,|U\,(x) = 1,\ \nu_1 B\,|U\,(x) = 0;\ \ \mu_1 B\,|U\,(y) = 0,\ \nu_1 B\,|U\,(y) \geq \alpha\ .$$

Since $B|U = (\mu_1 B\,|U, \nu_B|U) \in t_U$

Hence the intuitionistic fuzzy topological space (U, t_1U) is $\alpha -$ IF$-T_0$ (i).

Theorem 2.9. Every intuitionistic topological space corresponds to an intuitionistic fuzzy topological space but the converse is not true in general.

Proof: First part has been proved in theorem 2.1.

Conversely, let $X = \{x, y, z\}$ and let t be the intuitionistic fuzzy topology on X where

$t = \{ 1_\sim, 0_\sim, (\mu_1 1, \mu_1^c), (\mu_1 2, \mu_2^c), (\mu_1 3, \mu_3^c) \}$ and

$\mu_1(x) = 1$, $\quad \mu_2(x) = 0.9$, $\quad \mu_3(x) = 0.8$

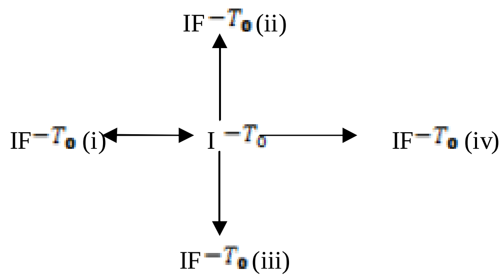
$\mu_1(y) = 0.9$, $\quad \mu_2(y) = 0.8$, $\quad \mu_3(y) = 0.7$

$\mu_1(z) = 0.8$, $\quad \mu_2(z) = 0.7$, $\quad \mu_3(z) = 0.6$

Then (X, t) is an intuitionistic fuzzy topological space but there is no intuitionistic topological space which corresponds to (X, t).

Definition 2.10. An intuitionistic topological space (ITS in short) (X, τ) is called intuitionistic T_0 $-$ space (I $-T_0$ space) if for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, \quad C_2) \in \tau$ such that ($x \in C_1$ and $y \in C_2$) or ($y \in C_1$ and x $\in C_2$)

Theorem 2.11. Let (X, τ) be an intuitionistic topological space and let (X, t) be the intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, τ) is an I $-T_0$ space. We shall prove that (X, t) is an IF$-T_1 0$ (i) .

Since (X, τ) is an I $-T_0$, then for all $x, y \in X$, $x \neq y$ there exists $C = (C_1, \quad C_2) \in \tau$ such that

$x \in C_1$ and $y \in C_2$

$\Rightarrow$ $1_{C_1}(x) = 1$, $1_{C_2}(y) = 1$.

Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, then

μ_A (x) =1, υ_A (x) =0; μ_A (y) =0, υ_A (y) =1 .

Since (μ_A, υ_A) $\in t$ $\Rightarrow$ (X, t) is IF$-T_1 0$ (i) .

Hence I $-T_0$ $\Rightarrow$ IF$-T_1 0$ (i) .

Conversely, suppose (X, t) is an IF$-T_1 0$ (i) . We shall show that (X, τ) is I $-T_0$.

Since (X, t) is an IF$-T_1 0$ (i) , then for all $x, y \in X$, $x \neq y$ $\exists$ $A = (\mu_1 A$, υ_A) $\in t$ such that

μ_A (x) =1, υ_A (x) =0; μ_A (y) =0, υ_A (y) =1

Let C_1 = μ_A^{-1} {1}, C_2 = υ_A^{-1} {1}, then

$x \in C_1$ and $y \in C_2$.

$(C_1 1, \quad C_1 2) \in \tau \Rightarrow (X, \tau)$ is $I - T_0$.

Hence IF$-T_1 0$ (i) $\Rightarrow$ $I - T_0$. Therefore $I - T_0 \Leftrightarrow$ IF$-T_1 0$ (i) . Furthermore, it can be easily shown that $I - T_0 \Rightarrow$ IF$-T_o$ (ii), $I - T_0 \Rightarrow$ IF$-T_o$ (iii) and $I - T_0 \Rightarrow$ IF$-T_o$ (iv).

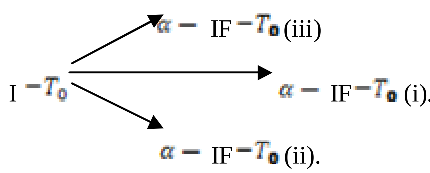
None of the reverse implications is true in general as can be seen from the following examples.

Example 2.11.1. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 1, 0), (y, 0, 0.3)\}$. It is clear that the IFTS (X, t) is IF$-T_1 0$ (ii) but not $I - T_0$.

Example 2.11.2. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 0.5, 0), (y, 0, 1)\}$. It is clear that the IFTS (X, t) is IF$-T_0$ (iii) but not $I - T_0$.

Example 2.11.3. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A = \{(x, 0.5, 0), (y, 0, 0.4)\}$. It is clear that the IFTS (X, t) is IF$-T_0$ (iv) but not $I - T_0$.

Theorem 2.12. Let (X, τ) be an intuitionistic topological space and let (X, t) be the intuitionistic fuzzy topological space. Then the authors have the following implications:



Proof: Let $\alpha \in (0, 1)$. Suppose (X, τ) is an $I - T_0$ space. The authors shall prove that (X, t) is $\alpha -$ IF$-T_1 0$ (i) . Since (X, τ) is $I - T_0$, then for all $x, y \in X$, $x \neq y$ there exists

$C = (C_1, \quad C_2) \in \tau$ such that $x \in C_1$ and $y \in C_2$

$\Rightarrow \quad 1_{C_1}(x) = 1, \; 1_{C_2}(y) = 1$

Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, then

$\mu_A(x) = 1, \upsilon_A(x) = 0; \mu_A(y) = 0, \upsilon_A(y) = 1$

$\Rightarrow \quad \mu_A(x) = 1, \upsilon_A(x) = 0; \mu_A(y) = 0, \upsilon_A(y) \geq \alpha$ for any $\alpha \in (0, 1)$.

Since $(\mu_A, \quad \upsilon_A) \in t \Rightarrow (X, t)$ is $\alpha -$ IF$-T_1 0$ (i) . Hence

I $-T_0 \Rightarrow \alpha-$ IF $-T_0$ (i) .

Furthermore, it can be easily verified that I $-T_0 \Rightarrow \alpha-$ IF $-T_0$ (ii), I $-T_0 \Rightarrow \alpha-$ IF $-T_0$ (iii).

None of the reverse implications is true in general as can be seen from the following examples.

Example 2.12.1. Let $X=\{x,y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A=\{(x,1,0),(y,0,0.5)\}$. For $\alpha=$ 0.4, it is clear that the IFTS (X,t) is

$\alpha-$ IF $-T_0$ (i) but not the corresponding I $-T_0$ space.

Example 2.12.2. Let $X=\{x,y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A=\{(x,0.7,0),(y,0,0.7)\}$. For $\alpha=$ 0.6, it is clear that the IFTS (X,t) is

$\alpha-$ IF $-T_0$ (ii) but not the corresponding I $-T_0$ space.

Example 2.12.3. Let $X=\{x,y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A\}$ where $A=\{(x,0.2,0),(y,0,0.5)\}$. For $\alpha=$ 0.4, it is clear that the IFTS (X,t) is

$\alpha-$ IF $-T_0$ (iii) but not the corresponding I $-T_0$ space.

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