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ON T_0 FUZZY BITOPOLOGICAL SPACES

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ABSTRACT

In this paper, the authors introduced two notions of fuzzy pairwise-T_0 bitopological spaces and compared them with other such concepts. The authors also studied some other properties of these spaces.

Key words: Fuzzy Bitopological spaces, Quasi-coincidence, Q-neighbourhood, Fuzzy pairwise-T_0 bitopological spaces

INTRODUCTION

Kelly (1963) introduced bitopological spaces first time in 1963. The concept of fuzzy sets was introduced by Zadeh (1965). Chang (1968) and Lowen (1976) developed the theory of fuzzy topological spaces based on Zadeh's concept. Fuzzy pairwise-T_0 (in short FPT_i, i = 0, 1, 2) bitopological spaces have been introduced earlier by Kandil and El-Shafee (1991). Fuzzy pairwise-T_0 separation axioms have also been introduced by Abu Sufiya *et al.* (1994) and Nouh (1996). Here the present authors introduced two definitions of fuzzy pairwise-T_0 bitopological spaces and obtained several of their properties.

PRELIMINARIES

Definition: A function μ from X into the closed unit interval I is called a fuzzy set in X. For every $x \in X$, $\mu(x) \in I$ is called the grade of membership of x. The class of all fuzzy sets from X into the closed unit interval I will be denoted by I^X (Zadeh 1965).

Definition: A fuzzy set μ in a set X is called a fuzzy singleton iff $\mu(x) = r$, $(0 < r \leq 1)$ for a certain $x \in X$ and $\mu(y) = 0$ for all points y of X except x. The fuzzy singleton is denoted by x_r and x is its support. The class of all fuzzy singletons in X will be denoted by $S(X)$. If $\mu \in I^X$ and $x_r \in S(X)$, then we say that $x_r \in \mu$ iff $r \leq \mu(x)$ (Pau-Ming and Ying Ming1980).

Definition: A fuzzy set μ in a set X is called a fuzzy point iff $\mu(x) = r$, $(0 < r < 1)$ for a certain $x_r \in X$ and $\mu(y) = 0$ for all points y of X except x. The fuzzy point is denoted by x_r and x is its support (Wong 1974).

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Definition: Let $I = [0, 1]$, x be a non-empty set, and I^x be the collection of all fuzzy sets in X. A fuzzy topology on X is defined as a family t of members of I^x satisfying the following conditions:

(i) $0, 1 \in t$;

(ii) If $\mu i \in t$ for each $i \in A$, then $\bigcup_{i\in\Lambda} \mu i \in t$;

(iii) If $\mu_1, \mu_2 \in t$ then $\mu_1 \cap \mu_2 \in t$.

The pair (X, t) is called a fuzzy topological space (fts in short) and members of t are called t-open (or simply open) fuzzy sets. A fuzzy set μ is called a t-closed (or simply closed) fuzzy set if $1 - \mu \in t$ (Chang 1968).

Definition: Let f be a real valued function on a topological space. If $\{x : f(x) > a\}$ is open for every real $\alpha \in I_1$, then f is called lower semicontinuous function (Rudin 1974).

Definition: Let x be a non-empty set and T be a topology on X. Let $t = \omega(T)$ be the set of all lower semicontinuous (lsc in short) functions from (X, T) to I (with usual topology). Thus $\omega(T) = \{\mu \in I^X : \mu^{-1}(\alpha, 1] \in T\}$ for each $\alpha \in I_1$. It can be shown that $\omega(T)$ is a fuzzy topology on X (Lowen 1976).

Definition: A fuzzy singleton x_r is said to be quasi-coincident with μ, denoted by $x_r q\mu$ iff $r + \mu(x) > 1$.If x_r is not quasi-coincident with μ, we write (Kandil and El-Shafee 1991).

Definition: A fuzzy set u of (X, t) is called quasi-neighborhood (Q-nbd in short) of x_r iff there exists $v \in t$ such that $x_r qv$ and $v \subset u$.

If x_r is a fuzzy point, then $N(x_r, t) = \{\mu \in t : x_r \in u\}$ is the family of all fuzzy t-open neighborhoods (t-nbds) of x_r. Similarly, if x_r is a fuzzy point, then

$N_Q(x_r, t) = \{\mu \in t : x_r q\ u\}$ is the family of all Q-neighborhoods (Q-nbd) of x_r (Nouh 1996).

Definition: A system (X, t_1, t_2) consisting of a set x with two fuzzy topologies t_1 and t_2 on X is called a fuzzy bitopological space (fbts in short) (Kandil *et al.* 1999).

Fuzzy Pairwise T_0-Spaces

Definition: An fbts (X, t_1, t_2) is called

(a) $FPT_0(i)$ iff for every pair of fuzzy singletons $x_p, y_r \in S(X)$ with $x \neq y$, there is a t_1-open fuzzy set or a t_2-open fuzzy set which contains one of the fuzzy singletons and not quasi-coincident with the other.

(b) $FPT_0(ii)$ iff for every pair of fuzzy singletons $x_p, y_r \in S(X)$ with $x \neq y$, there exists a fuzzy set $\mu \in t_1 \cup t_2$ such that $(x_p q\mu, y_1 r \cap \mu = 0)$ or $(y_r q\mu, x_1 p \cap \mu = 0)$.

(c) FPT_0(*iii*) iff for any two distinct fuzzy points p, q in X there exists a fuzzy set $\mu \in t_1 \cup t_2$ such that ($p \in \mu$. $q \cap \mu = 0$) or ($q \in \mu$. $p \cap \mu = 0$) (Abu *et al.* 1994).

(d) FPT_0(*iv*) space iff for every pair of fuzzy singletons x_p, $y_r \in S(X)$ such that $x \neq y$, there is a t_1-open fuzzy set or a t_2-open fuzzy set which is Q-nbd of one of the fuzzy singletons and not quasi-coincident with other (Nouh 1996).

(e) FPT_0(*v*) space iff for every pair of fuzzy singletons x_p, $y_r \in S(X)$ with $x \neq y$, there exists a fuzzy set $\mu \in t_1 \cup t_2$ such that ($x_p \in \mu$. $\mu \subseteq (y_r)^c$) or ($x_r \in \mu$. $\mu \subseteq (y_r)^c$) (Abu *et al.* 1994).

(f) FPT_0(*vi*) iff for any two distinct fuzzy points p, q in X there exists a fuzzy set $\mu \in t_1 \cup t_2$ such that ($p \in \mu$, $q \subseteq \mu^c$) or ($q \in \mu$, $q \subseteq \mu^c$) (Abu *et al.* 1994).

Theorem: Let (X, t_1, t_2) be an fbts. Then the workers have the following implications:

(a) $\Leftrightarrow$ (b) $\Leftrightarrow$ (c) $\Leftrightarrow$ (d) $\Leftrightarrow$ (e) and (d) $\Rightarrow$ (f), but (f) $\not\Rightarrow$ (d).

Proof: (a) $\Rightarrow$ (b): Let (X, t_1, t_2) be FPT_0(*i*)-space. Then by definition, for the singletons x_p, $y_r \in S(X)$ with $x \neq y$, choose $r^* \in (0, 1)$ such that $r^* > 1 - r$. Since (X, t_1, t_2) is FPT_0(*i*), then there exists a fuzzy set $\mu \in t_1 \cup t_2$ such that $x_{r^*} \in \mu$ and $y_1 \bar{q} \mu$.

Since $\mu(x) \geq r^*$ and $r^* > 1 - r$, then $\mu(x) > 1 - r$, that is, $r + \mu(x) > 1$ and so $x_r q \mu$.

Also, $y_1 \bar{q} \mu \Rightarrow 1 + \mu(y) \leq 1 \Rightarrow \mu(y) \leq 1 - 1 = 0$, that is, $\mu(y) = 0$. Now for any fuzzy singletons x_r, $y_s \in S(X)$, it is seen that, $x_r q \mu$ and $y_s \cap \mu = 0$. Hence (X, t_1, t_2) is FPT_0(*ii*)-space.

(b) $\Rightarrow$ (c): Let x_r, y_s be two distinct fuzzy points in X. Choose $r^* \in (0, 1)$ such that $r^* < 1 - r$. Since (X, t_1, t_2) is FPT_0(*ii*) and x_{r^*}, y_s are distinct fuzzy singletons, then

$$\left(\exists \mu \in N_Q(x_{r^*}, t_1), \mu \cup y_s = 0\right) \text{ or } \left(\exists \mu \in N_Q(x_{r^*}, t_2), (\mu \cup y_s = 0)\right).$$

Now, let $\mu \in N_Q(x_{r^*}, t_1)$ ($\mu \cap y_s = 0$). That is, $r^* + \mu(x) > 1$ and ($\mu \cap y_s = 0$). Since $r^* + \mu(x) > 1$ and $r^* < 1 - r$, then $\mu(x) > r$ and so $x_r \in \mu$. Hence (X, t_1, t_2) is FPT_0(*iii*)-space.

(c) $\Rightarrow$ (d): Let x_r, $y_s \in S(X)$ with $x \neq y$. Choose r^*, $s^* \in (0, 1)$ such that $r^* > 1 - r$ and $s^* > 1 - s$. Since (X, t_1, t_2) is FPT_0(*iii*) and x_{r^*}, y_{s^*} are distinct fuzzy points, then there exists a fuzzy set $\mu \in t_1 \cup t_2$ such that $x_{r^*} \in \mu$ and $y_{s^*} \cap \mu = 0$. Since $\mu(x) > r^*$ and $r^* > 1 - r$, then we have $\mu(x) > 1 - r$, that is, $r + \mu(x) > 1$ and so $x_r q \mu$.

Again, $y_{s^*} \cap \mu = 0$ implies $y_s \cap \mu = 0$, since $\mu(y) = 0$. So, $y_s \bar{q} \mu$. Hence (X, t_1, t_2) is FPT_0(*iv*)-space.

(d) $\Rightarrow$ (e): Let x_r, y_s be two distinct fuzzy singletons in X. Choose $r^* \in (0, 1)$ such that $r^* < 1 - r$. Since (X, t_1, t_2) is FPT_0(*iv*), then there exists a $\mu \in t_1 \cup t_2$ such that $r^* q \mu$

and $y_s \bar{q} \mu$.

Now, since $r^* + \mu(x) > 1$ and $r^* < 1 - r$, then $\mu(x) > r$. So, $x_r \in \mu$. Also, $y_s \bar{q} \mu \Rightarrow s + \mu(y) \leq 1 \Rightarrow s \leq 1 - \mu(y)$. So, $\mu \subseteq (y_s)^c$. Hence (X, t_1, t_2) is $\text{FP}T_0(v)$-space.

(e) $\Rightarrow$ (a): Let $x_r, y_s \in S(X)$ with $x \neq y$. Since (X, t_1, t_2) is $\text{FP}T_0(v)$, then there exists a fuzzy set $\mu \in t_1 \cup t_2$ such that $x_r \in \mu$ and $\mu \subseteq (y_s)^C$.

Also, $\mu \subseteq (y_s)^C \Rightarrow \mu(\text{y}) \leq 1 - \text{s} \Rightarrow \text{s} + \mu(\text{y}) \leq 1$ and so, $y_s \bar{q} \mu$. Hence (X, t_1, t_2) is $\text{FP}T_0(i)$-space.

(d) $\Rightarrow$ (f): Let x_r, y_s be two distinct fuzzy points in X. Choose $r^* \in (0, 1)$ such that $r^* < 1 - r$. Since (X, t_1, t_2) is $\text{FP}T_0(iv)$ and x_{r^*}, y_s are distinct fuzzy singletons, then there exists a fuzzy set $\mu \in t_1 \cup t_2$ such that $x_{r^*} q \mu$ and $y_s \bar{q} \mu$.

Since $r^* + \mu(x) > 1$ and $r^* < 1 - r$, then one has $\mu(x) > r$ and so $x_r \in \mu$. Also, $y_s \bar{q} \mu \Rightarrow s + \mu(y) \leq 1 \Rightarrow s \leq 1 - \mu(y)$. So, $y_s \subseteq \mu^c$. Hence (X, t_1, t_2) is $\text{FP}T_0(vi)$-space.

(f) $\not\Rightarrow$ (d): Example: Let $X = 1$, $t_1 = t_2 = \{0, \lambda : \lambda(x) > 0, \forall x \in X\}$. Let x_r and y_s be distinct fuzzy singletons in X and $\gamma = \min\{1 - r, 1 - s\}$

Now, if $\gamma \neq 0$, the authors define μ as follows: $\mu(x) = 1$ and $\mu(y) = \dfrac{\gamma}{8}$, if $y \neq x$.

Again, if $\gamma = 0$, one can define μ as follows: $\eta(x) = 1$ and $\eta(y) = 0.1$, if $y \neq x$.

Then $\mu \in t_1 = t_2$. For any pair of distinct fuzzy points x_r, y_s is X, it is seen that $x_r \in \mu$ and $y_s \subseteq \mu^c$. Therefore, (X, t_1, t_2) is $\text{FP}T_0(vi)$-space. But if one takes $x_1, y, \in S(X)$, then it can be seen that $x_1 q \eta$ and $y_1 q \eta$. Hence (X, t_1, t_2) is $\text{FP}T_0(\text{i}v)$-space.

Theorem: Let (X, s, t) be a fuzzy topological space, $A \subset X$, and $S_A = \left\{\dfrac{u}{A} : u \in s\right\}$, $t_A = \left\{\dfrac{v}{A} : v \in t\right\}$. Then

(a) (X, s, t) is $\text{FP}T_0(i) \Rightarrow (A, S_A, t_A)$ is $\text{FP}T_0(i)$.

(b) (X, s, t) is $\text{FP}T_0(ii) \Rightarrow (A, S_A, t_A)$ is $\text{FP}T_0(ii)$.

(c) (X, s, t) is $\text{FP}T_0(iii) \Rightarrow (A, S_A, t_A)$ is $\text{FP}T_0(iii)$.

(d) (X, s, t) is $\text{FP}T_0(iv) \Rightarrow (A, S_A, t_A)$ is $\text{FP}T_0(iv)$.

(e) (X, s, t) is $\text{FP}T_0(v) \Rightarrow (A, S_A, t_A)$ is $\text{FP}T_0(v)$.

(f) (X, s, t) is $\text{FP}T_0(vi) \Rightarrow (A, S_A, t_A)$ is $\text{FP}T_0(vi)$.

Proof: (a) Suppose (X, s, t) is $\text{FP}T_0(i)$. One has to show that (A, S_A, t_A) is $\text{FP}T_0(i)$. Let $a_r, b_p \in S(A)$ with $a \neq b$. Since (X, s, t) is $\text{FP}T_0(i)$, there exists a $u \in s \cup t$ such that $a_r \in u$ and $b_p \bar{q} u$, that is, $u(a) \geq r$ and $u(b) + p \leq 1$.

Now $\dfrac{u}{A} \in S_A \bigcup t_A$ and $\left(\dfrac{u}{A}\right)(a) = u(a)$. Then $(u/A)\,(a) \geq r$. So, $a_r \in \dfrac{u}{A}$.

Also $\left(\dfrac{u}{A}\right)(b) + p \leq 1$, since $\left(\dfrac{u}{A}\right)(b) = u(b)$. Hence (A, S_A, t_A) is FP$T_0(i)$.

(f) Suppose (X, s, t) is FP$T_0(vi)$. The authors have to show that (A, S_A, t_A) is FP$T_0(vi)$. Let a_r, b_p be two distinct fuzzy points in A. Then a_r, b_p are two distinct fuzzy points in X. Since (X, s, t) is FP$T_0(vi)$, there exists a $u \in s \bigcup t$ such that $a_r \in u$ and $b_p \subseteq u^c$, that is, $u(a) > r$ and $1 - u(b) \geq p$.

Now $\dfrac{u}{A} \in S_A \bigcup t_A$ and $\left(\dfrac{u}{A}\right)(a) = u(a)$. Then $(u/A)\,(a) > r$. So, $a_r \in \dfrac{u}{A}$.

Also $1 - \left(\dfrac{u}{A}\right)(b) \geq p$. since $\left(\dfrac{u}{A}\right)(b) = u(b)$ and so $b_p \subseteq \left(\dfrac{u}{A}\right)^C$. Hence (A, S_A, t_A) is FP$T_0(vi)$.

The proofs of (b), (c), (d) and (e) are similar.

Theorem: Let (X, T_1, T_2) be a bitopological space. Then

(a) (X, T_1, T_2) is P$T_0 \Leftrightarrow (X, \omega(T_1), \omega(T_2)$ is FP$T_0(i)$.

(b) (X, T_1, T_2) is P$T_0 \Leftrightarrow (X, \omega(T_1), \omega(T_2)$ is FP$T_0(ii)$.

(c) (X, T_1, T_2) is P$T_0 \Leftrightarrow (X, \omega(T_1), \omega(T_2)$ is FP$T_0(iii)$.

(d) (X, T_1, T_2) is P$T_0 \Leftrightarrow (X, \omega(T_1), \omega(T_2)$ is FP$T_0(iv)$.

(e) (X, T_1, T_2) is P$T_0 \Leftrightarrow (X, \omega(T_1), \omega(T_2)$ is FP$T_0(v)$.

(f) (X, T_1, T_2) is P$T_0 \Leftrightarrow (X, \omega(T_1), \omega(T_2)$ is FP$T_0(vi)$.

Proof: (a) Suppose that (X, T_1, T_2) is PT_0. Then the authors are to show that $(X, \omega(T_1), \omega(T_2)$ is FP$T_0(i)$. Let $x_p, y_r \in S(X)$ with $x \neq y$. Since (X, T_1, T_2) is PT_0 then

$(\exists \bigcup \in N(x, T_1), (y \notin \bigcup)$ or $(\exists V \in N(x, T_2), (y \notin V)$ Then $1_U \in N(x_p, \omega(T_1)), y_r \bar{q} 1u)$, or $1_V \in N(x_p, \omega(T_2)), y_r \bar{q} 1v)$. Thus $(X, \omega(T_1), \omega(T_2)$ is FP$T_0(i)$.

Conversely, suppose that $(X, \omega(T_1), \omega(T_2)$ is FP$T_0(i)$, then one has to show that (X, T_1, T_2) is PT_0. Let $x, y \in X$ such that $x \neq y$. Since $(X, \omega(T_1), \omega(T_2)$ is FP$T_0(i)$, then $(\exists \mu \in N(x1, \omega(T_1)), y_1 \bar{q} \mu)$ or $(\exists \eta \in N(x1, \omega(T_2)), y_1 \bar{q} \eta)$.

Now, let $\mu \in N(x_1, \omega(T1)), y_1 \bar{q} \mu$. i.e. $\mu(x) = 1$ and $\mu(y) = 0$. Hence $x \in \mu^{-1}(0,1) \in T_1$ and $y \notin \mu^{-1}(0,1) \in T_1$. Hence (X, T_1, T_2) is PT_0.

(f) Suppose that (X, T_1, T_2) is PT_0. One has to show that $(X, \omega(T_1), \omega(T_2))$ is

FP$T_0(vi)$.

Let x_p, y_r be two distinct fuzzy points in X. Since (X, T_1, T_2) is PT_0,

then $(\exists U \in N(x, T_1), (y, U)$ or $(\exists V \in N(x, T_2), (y \neq V)$.

Then $1_U \in N(x_p, \omega(T_1)), (1_U \cap y_r = 0))$ or $1_v \in N(x_p, \omega(T_2)), (1_v \cap y_r = 0))$ which implies $1_U \in N(x_p, \omega(T_1)), y_r \subseteq (1_U)^c)$ or $1_v \in N(x_p, \omega(T_2)), y_r \subseteq (1_v)^c)$. Thus $(X, \omega(T_1), \omega(T_2))$ is FP$T_0(vi)$.

Conversely, suppose that $(X, \omega(T_1), \omega(T_2))$ is FP$T_0(vi)$ is FP$T_0(vi)$. One has to show that (X, T_1, T_2) is PT_0. Let x, $y \in X$ such that x $\neq y$. Take r such that $0.5 < r < 1$. Since $(X, \in (T_1), \in (T_2))$ is FP$T_0(vi)$ is FP$T_0(vi)$, then $(\exists \mu \in N(x_r, \in (T_1), y_r \subseteq \mu^c)$ or $(\exists \eta \in N(x_r, \in (T_2), y_r \subseteq \eta^c)$.

Now, let $\mu \omega N(x_r, \omega(T_1)), y_r \subseteq \mu^c$. i.e. $\mu(x) > r$ and $\mu(y) \leq 1 - r$.

Hence $x \in \mu^{-1}(0.5, 1) \in T_1)$ and $y \notin \mu^{-1}(0.5, 1) \in T_1$. Hence (X, T_1, T_2) is PT_0.

Proofs of (b), (c) and (e) are similar and for the proof of (d) (Nouh 1996).

Theorem: Product of any two FP$T_0(j)$-spaces is FP$T_0(j)$-space where $j = i, ii, iii, iv, v, vi$.

Proof: Suppose (X_1, s_1, t_1) and (X_2, s_2, t_2) are FP$T_0(i)$, then one has to show that $(X_1 \times X_2, s_1 \times s_2, t_1 \times t_2)$ is FP$T_0(i)$. Let $(x, y)p$, $(x_1, y_1)q \in X_1 \times X_2$ with $(x, y) \neq (x_1, y_1)$. It can be assumed without loss of generality that $x \neq x_1$.

Since x_p, $(x_1)_q \in S(X_1)$ with $x \neq x_1$ and (X_1, s_1, t_1) is FP$T_0(i)$, then $(\exists \mu \in N(x_p, s_1), ((x_1)_q \bar{q} \mu))$ or $(\exists \eta \in N(x_p, t_1), ((x_1)_q \bar{q} \eta))$.

Now, let $\mu \in N(x_p, s_1), ((x_1)_q \bar{q} \eta)$. That is, $\mu(x) \geq p$ and $\mu(x_1) + q \leq 1$.

Since $\mu \times X_2 \in s_1 \times s_2$ and $(\mu \times X_2)(x, y) = \min\{\mu(x), X_2(y)\} = \mu(x) \geq p$, then one has $(x, y)_p \in \mu \times X_2$.

Also, $(\mu \times X_2)(x_1, y_1) + q = \mu(x_1) + q \leq 1$. Hence $(X_1 \times X_2, s_1 \times s_2, t_1 \times t_2)$ is FP$T_0(i)$.

Suppose (X_1, s_1, t_1) and (X_2, s_2, t_2) are FP$T_0(vi)$, then one has to show that $(X_1 \times X_2, s_1 \times s_2, t_1 \times t_2)$ is FP$T_0(vi)$. Let $(x, y)_p$, $(x_1, y_1)_q$ be two distinct fuzzy points in $X_1 \times X_2$. One assumes without loss of generality that $x \neq x_1$. Since x_p, $(x_1)_q$ are two distinct fuzzy points in X_1 and (X_1, s_1, t_1) is FP$T_0(vi)$, then $(\exists \mu N(x_p, s_1), ((x_1)_q \subseteq \mu^c))$ or $(\exists \eta N(x_p, t_1), ((x_1)_q \subseteq \eta^c))$.

Now, let $\mu \in N(x_p, s_1)$, $(x_1)q \subseteq \mu^c$. That is, $\mu(x) > p$ and $q \leq 1 - \mu(x_1)$.

Since $\mu \times X_2 \in s_1 \times s_2$ and $(\mu \times X_2)(x, y) = \min\{\mu(x), X_2(y)\} = \mu(x) > p$, the authors have $(x, y), \in \mu \times X_2$.

Also, for all $(x, y) \in X_1 \times X_2$, we have $1 - (\mu \times X_2)(x, y) = 1 - \mu(x) \geq q$.

So, $(x_1, y_1) \subseteq (\mu \times X_2)^c$. Hence $(X_1 \times X_2, s_1 \times s_2, t_1 \times t_2)$ is FP$T_0(vi)$.

Proofs of the other claims are similar.

Theorem: A bijective mapping from an fts (X, t) to an fts (Y, s) preserves the value of a fuzzy singleton (fuzzy point).

Proof: Let c_r be a fuzzy singleton in X. So, there exist a point $a \in Y$ such that $f(c) = a$. Now $f(c_r)(a) = f(c_r)(f(c)) = \sup c_r(c) = c_r(c) = r$, since f is bijective. Hence a_r has same value as c_r.

Note: Preimage of any fuzzy singleton (fuzzy point) under bijective mapping preserves its value.

Theorem: Let (X, s, t) and (Y, s_1, t_1) and be two fuzzy bitopological spaces and let f : $X \to Y$ be bijective and FP-open. Then

(a) (X, s, t) is FP$T_0(i) \Rightarrow (Y, s_1, t_1)$ is FP$T_0(i)$.

(b) (X, s, t) is FP$T_0(ii) \Rightarrow (Y, s_1, t_1)$ is FP$T_0(ii)$.

(c) (X, s, t) is FP$T_0(iii) \Rightarrow (Y, s_1, t_1)$ is FP$T_0(iii)$.

(d) (X, s, t) is FP$T_0(iv) \Rightarrow (Y, s_1, t_1)$ is FP$T_0(iv)$.

(e) (X, s, t) is FP$T_0(v) \Rightarrow (Y, s_1, t_1)$ is FP$T_0(v)$.

(f) (X, s, t) is FP$T_0(vi) \Rightarrow (Y, s_1, t_1)$ is FP$T_0(vi)$.

Proof: (a) Suppose (X, s, t) is FP$T_0(i)$. The authors shall now show that (Y, s_1, t_1) is FP$T_0(i)$. Let $a_r, b_q \in S(Y)$ with $a \neq b$. Since f is bijective, then there exists $c_r, d_q \in S(Y)$ such that $f(c) = a$, $f(d) = b$ and $c \neq d$. Again since (X, s, t) is FP$T_0(i)$, then

$(\exists \mu\, N(c_r, s), (d_q \overline{q} \mu)$ or $(\exists \eta\, N(c_r, t), (d_q \overline{q} \eta)$.

Now, let $\mu \in N(c_r, s), (d_q \overline{q} \mu)$. That is, $\mu(c) \geq r$ amd $\mu(d) + q \leq 1$.

Now $f(\mu)(a) = f(\mu)(f(c) = \sup \mu(c) = \mu(c) \geq r$. So, $a_r \in f(\mu)$.

Also $f(\mu)(b) + q = f(\mu)(f(d)) + q = \mu(d) + q \leq 1$. So, $b_q \overline{q} f(\mu)$.

Since f is FP-open, then $(\mu) \in s_1$. Hence (Y, s_1, t_1) is FP$T_0(i)$.

(f) Suppose (Y, s, t) is FP$T_0(vi)$. We shall show that (Y, s_1, t_1) is FP$T_0(vi)$. Let a_r, b_q be two distinct fuzzy points in Y. Since f is bijective, then there exists $c_r, d_q \in S(X)$ such that $f(c) = a$, $f(d) = b$ and $c \neq d$. Again since (Y, s, t) is FP$T_0(vi)$, then

$(\exists\, \mu \in N(c_r, s), (d_q \subseteq \mu^c))$ or $(\exists\, \eta \in N(c_r, t), (d_q \subseteq \eta^c))$.

Now, let $\mu \in N(c_r, s)$, $(d_q \subseteq \mu^c)$. That is, $\mu(c) > r$ and $q \leq 1 - \mu(d)$.

Now $f(\mu)\,(a) = f(\mu)\,(f(c)) = \sup\mu(c) = \mu(c) > r$. So, $a_r, \in f(\mu)$..

Also, $1 - f(\mu)\,(b) = 1 - f(\mu)\,(f(d)) = 1 - \mu(d) \geq q$. So, $b_q \subseteq (f(\mu))^C$. Since c is FP-open, then $(\mu) \in s_1$. Hence (Y, s_1, t_1) is FP$T_0(vi)$.

Proofs of (b), (c), (d) and (e) are similar.

Theorem: Let (Y, s, t) and (Y, s_1, t_1) be two fuzzy bitopological spaces and $f : X \rightarrow Y$ be FP-continuous and bijective. Then

(a) (Y, s_1, t_1) is FP$T_0(i) \Rightarrow (Y, s, t)$ is FP$T_0(i)$.

(b) (Y, s_1, t_1) is FP$T_0(ii) \Rightarrow (Y, s, t)$ is FP$T_0(ii)$.

(c) (Y, s_1, t_1) is FP$T_0(iii) \Rightarrow (Y, s, t)$ is FP$T_0(iii)$.

(d) (Y, s_1, t_1) is FP$T_0(iv) \Rightarrow (Y, s, t)$ is FP$T_0(iv)$.

(e) (Y, s_1, t_1) is FP$T_0(v) \Rightarrow (Y, s, t)$ is FP$T_0(v)$.

(f) (Y, s_1, t_1) is FP$T_0(vi) \Rightarrow (Y, s, t)$ is FP$T_0(vi)$.

Proof: The authors shall prove (a) and (f) only.

(a) Suppose (Y, s, t) is FP$T_0(i)$. One has to show that (Y, s, t) is FP$T_0(i)$. Let $c_r, d_q \in S(X)$ such that $c \neq d$. Then there exist $a_r, b_q \in S(Y)$ such that $f(c) = a$, $f(d) = b$ and $a \neq b$, since f is one-one. Again since (Y, s_1, t_1) is FP$T_0(i)$, then $(\exists\, \mu\, N(a_r, s_1), (b_q \bar{q} \mu))$ or $(\exists \eta\, N(a_r, t_1), (b_q \bar{q} \eta))$.

Now, let $\mu \in N(a_r, s_1)$, $(b_q \bar{q} \mu)$. That is, $\mu(a) \geq r$ and $\mu(b) + q \leq 1$.

Since $f^{-1}(\mu)\,(c) = \mu(f(c)) = \mu(a) \geq r$, then $c_r \in f^{-1}(\mu)$. So, $a_r q f^{-1}(\mu)$.

Also, $f^{-1}(\mu)\,(d) + q = \mu(f(d)) + q = \mu(b) + q \leq 1$. So, $b_q \bar{q} \mu$..

Since f is FP-continuous, then $f^{-1}(\mu) \in s$. Hence (Y, s, t) is FP$T_0(i)$.

(f) Suppose (Y, s_1, t_1) is FP$T_0(vi)$. Then one has to show that (Y, s, t) is FP$T_0(vi)$. Let c_r, d_q be two distinct fuzzy points in X. Then there exists distinct fuzzy points a_r, b_q in Y such that $f(c) + a$, $f(d) = b$ and $a \neq b$, since f is one-one. Again since (Y, s_1, t_1) is FP$T_0(vi)$, then $(\exists\, \mu\, N(a_r, s_1), (b_q \in \mu^c))$ or $(\exists \eta\, N(a_r, t_1), (b_q \in \eta^c))$.

Now, let $\mu \in N(a_r, s_1)$, $(b_q \in \mu^c)$, that is, $\mu(a) > r$ and $1 - \mu(b) \geq q$.

Since $f^{-1}(\mu)$ (c) $= \mu(f(c)) = \mu(a) > r$, then $c_r \in f^{-1}(\mu)$.

Also, $1 - f^{-1}(\mu)\,(d) = 1 - \mu(f(d)) = 1 - \mu(b) \geq q$. So, $d_q \in (f^{-1}(\mu))^c$.

Since f is FP-continuous, then $f^{-1}(\mu) \in s$. Hence (Y, s, t) is FP$T_0(vi)$.

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