SEPARATION AXIOMS IN MIXED FUZZY TOPOLOGICAL SPACES

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ABSTRACT

We deal with fuzzy topology. In this paper, we introduce the concept of mixed fuzzy topology which is constructed from two fuzzy topologies on the same fuzzy set X and study several features of this mixed fuzzy topology.

Keywords: Fuzzy Topological Spaces, Mixed fuzzy topology

1. INTRODUCTION

The origin of mixed topology lies on the work of Alexiewicz and Z.Semadeni [(1)], when they introduced two norm spaces around the middle of the last century. Separation axioms in fuzzy setting have been studied by several researchers from the early eighties. WUYTS AND LOWEN [(15)] have been studied separation properties of fuzzy topological spaces, fuzzy neighbourhood and fuzzy uniform space. AHMED [(9)] has introduced some fuzzy separation axioms to study their hereditary and productive properties. We consider the study of mixed fuzzy topology as a new field of research, which has already been introduced by Das and Baishya in their paper [(8)]. In this paper, we explore the impact of structural properties of the original fuzzy topologies on the mixed fuzzy topology and study separation axioms in mixed fuzzy topological spaces in more detail.

2. PRELIMINARIES

We briefly touch upon the terminological concepts, definitions and some results, which are needed in the sequel. The following are essential in our study and can be found in the paper referred to.

2.1 Definition [(6)]. Let $I = [0,1]$ and X be a non-empty set. We denote the set of all fuzzy sets in X by I^X . A fuzzy topology on a set X is a family t of fuzzy sets in X satisfying the following conditions:

(i) $1, 0 \in t$

(ii) if $a, b \in t$, then $a \cap b \in t$ and

(iii) if $\{a_i : i \in I\}$ is a family of fuzzy sets in T , then $\bigcup_{i \in I} a_i \in t$.

Then the pair (X, t) is called a fuzzy topological space (in short fts) and the members of t are called the t-open fuzzy sets and their complements are called the t-closed fuzzy sets.

2.2 Definition [11]**.** A fuzzy point in X is a fuzzy set in X which is zero everywhere except at one point, say x, where it takes value, say r with $r \in (0,1)$ i.e. $0 < r < 1$. We denote it by x_r and we call the point x its support and r its value.

2.3 Definition [11]**.** A fuzzy point x_r is said to belong to a fuzzy set α in X, denoted $x_r \in \alpha$ if and only if $r < \alpha(x)$. Evidently, a fuzzy set α in X is the union of all its fuzzy points.

2.4 Definition [11]**.** Let x_r be a fuzzy point in an fts (X,τ). A fuzzy set α is a neighborhood (in short nhd) of x_r if and only if there exist an open fuzzy set β such that $x_r \in \beta \subseteq \alpha$. A nhd α is an open nhd if and only if α is open; a nhd α is closed nhd if and only if α is closed.

2.5 Definition. A fuzzy singleton in X is a fuzzy set in X which is zero everywhere except at one point, say x, where it takes value, say r with $r \in (0,1]$ i.e. $0 < r \leq 1$. We denote it by x_r. Also, $x_r \in \alpha$ if and only if $r \leq \alpha(x)$.

2.6 Definition. A fuzzy singleton x_r in X is said to be quasi-coincident (in short q-coincident) with a fuzzy set α in X, denoted by $x_r q \alpha$ if and only if $r + \alpha(x) > 1$.

2.7 Definition. A fuzzy set α in X is called q-coincident with a fuzzy set β in X, denoted $\alpha q \beta$ if and only if $\alpha(x) + \beta(x) > 1$, for some $x \in X$. It is clear that if $x_r \not{q} \alpha$, then $r + \alpha(x) \leq 1$, for every $x \notin X$ and if $\alpha q \beta$, then $\alpha(x) + \beta(x) \leq 1$, for every $x \in X$.

2.8 Definition. A fuzzy set α in an fts (X,τ) is called a q-nhd of a fuzzy singleton x_r in X if and only if there exist $\beta \in \tau$ such that $x_r q \beta$ and $\beta \subseteq \alpha$.

2.9 Definition. Let (X,τ) be an fts and x_r be a fuzzy point in X. Then the family N_{x_r} consisting of all the q-nhds of x_r is called the system of q-nhds of x_r.

2.10 Proposition [11]**.** Let (X,τ) be an fts. Then for each x_r in X, N_{x_r} satisfies the followings:

(i) x_r is a quasi-coincident with α, for every, $\alpha \in N_{x_r}$.

(ii) if $\alpha, \beta \in N_{x_r}$, then $\alpha \cap \beta \in N_{x_r}$.

(iii) if $\alpha \in N_{x_r}$ and $\alpha \subset \beta$, then , $\beta \in N_{x_r}$.

Conversely, for each fuzzy point x_r in X, if N_{x_r} is the family of fuzzy sets in X satisfying the conditions (i), (ii) and (iii), then the family t of all fuzzy sets α such that $\alpha \in N_{x_r}$ whenever $x_r q \alpha$ is a fuzzy topology for X.

2.11 Definition. Let α be a fuzzy set in an fts (X,t). Then the closure of α is denoted by $\overline{\alpha}$ and defined as the intersection of all closed supersets of α i.e. $\overline{\alpha} = \cap\{\beta : \beta \supseteq \alpha, \beta \in t^c\}$.

2.12 Definition. Let λ be a fuzzy set in an fts (X,t). Then the interior of λ is denoted by λ^0 and defined as the union of all open subsets of λ i.e. $\lambda^0 = \cup\{\mu : \mu \subseteq \lambda, \mu \in t\}$.

2.13 Definition. A fuzzy topological space (X,t) is called a fuzzy T_0-space if and only if for any pair of fuzzy singletons x_r, y_s $(x \neq y)$ in X, there exists $u \in t$ such that $x_r \in u$ or $y_s \in u$.

2.14 Definition. A fuzzy topological space (X,t) is called a fuzzy T_1-space if and only if for any pair of fuzzy singletons x_r, y_s $(x \neq y)$ in X, there exists $u, v \in t$ such that $x_r \in u \subseteq$ com y_s and $y_s \in v \subseteq$ com x_r.

2.15 Definition [12]. A fuzzy topological space (X,t) is called a fuzzy hausdorff or T_2-space if and only if for any pair of fuzzy singletons x_r, y_s $(x \neq y)$ in X, there exists $u, v \in t$ such that $x_r \in u$, $y_s \in v$ and $u \cap v = 0$.

2.16 Definition. A fuzzy topological space (X,t) is called a fuzzy regular if and only if for all $x \in X$ and closed fuzzy set u with $x_r \notin u$, there exist $v, w \in t$ such that $x_r \in v, u \subseteq w$ and $v \subseteq 1 - w$.

2.17 Proposition [3]. A fuzzy topological space (X,t) is fuzzy regular if and only if for all $x \in X$, $r \in (0,1)$ and $\alpha \in t$ with $r < \alpha(x)$, there exists $\beta \in t$ such that $r < \beta(x)$ and $\bar{\beta} \subseteq \alpha$.

2.18 Definition. A fuzzy topological space (X,t) is called fuzzy normal if and only if for each closed fuzzy set m and open fuzzy set u with $m \subseteq u$, there exist $v \in t$ such that $m \subseteq v^0 \subseteq \bar{v} \subseteq u$.

2.19 Definition [14]. A family τ of fuzzy sets is a cover of fuzzy set α if and only if $\alpha \subseteq \cup\{\alpha_i : \alpha_i \in \tau\}$. It is called an open cover if each member α_i is an open fuzzy set. A subcover of τ is a subfamily of τ which is also a cover of α .

2.20 Definition [6]. A fuzzy topological space (X,τ) is compact if and only if every open cover has a finite subcover.

2.21 Definition [11]. A fuzzy set α in a fuzzy topological space (X,τ) is said to be disconnected if and only if there exist two non-empty fuzzy sets α_1 and α_2 such that α_1 and α_2 are Q-separated and $\alpha = \alpha_1 \cup \alpha_2$. A fuzzy set is called connected if and only if it is not disconnected.

3. SEPARATION AXIOMS IN MIXED FUZZY TOPOLOGICAL SPACES

Now we come to our main discussion.

3.1 Definition. Let (X,t_1) and (X,t_2) be two fuzzy topological spaces. We define $t_1(t_2) = \{\alpha \in I^X : \text{for every } x_r q\alpha$, there exists a t_2-quasi-neighborhood β of x_r such that t_1-closure, $\bar{\beta} \subseteq \alpha\}$. Then $t_1(t_2)$ is fuzzy topology on X . This fuzzy topology is called a mixed fuzzy topology and the pair $(X,t_1(t_2))$ is called a mixed fuzzy topological space.

3.2 Theorem [8]. Let (X,t_1) and (X,t_2) be two fuzzy topological spaces and let $t_1(t_2) = \{\alpha \in I^X : \text{for every } x_r q\alpha$, there exists a t_2-quasi-neighborhood β of x_r such that t_1-closure, $\bar{\beta} \subseteq \alpha\}$. Then $t_1(t_2)$ is fuzzy topology on X .

3.3 Lemma [8]. Let t_1 and t_2 be two fuzzy topologies on a set X . If every t_1-quasi-nhd of x_r is t_2-quasi-nhd of x_r for all fuzzy singletons x_r, then t_1 is coarser then t_2, in symbol $t_1 \subseteq t_2$.

3.4 Theorem [8]. Let t_1 and t_2 be two fuzzy topologies on a set X . Then the mixed fuzzy topology $t_1(t_2)$ is coarser than t_2, in symbol $t_1(t_2) \subseteq t_2$.

3.5 Theorem. Let (X,t_1) and (X,t_2) be two fuzzy topological spaces. If (X,t_1) is fuzzy T_0-space and $t_1 \subseteq t_2$, then $(X,t_1(t_2))$ is a fuzzy T_0-space.

Proof. Let $x_r, y_s \in t_1, x \neq y$. Since (X,t_1) is a fuzzy T_0-space, then there exists $u_1 \in t_1$ such that $x_r \in u_1$ or $y_s \in u_1$. Let u be the t_1-quasi-nhd of

x_r or y_s. Then for $u_1 \in t_1$ we have $x_r q u_1$ or $y_s q u_1$ and $u_1 \subseteq u$. So, $r + u_1(x) > 1$ or $s + u_1(y) > 1$ and $0 < r \leq 1, 0 < s \leq 1$. This implies that, $r + u(x) > 1$ or $s + u(y) > 1 \Rightarrow x_r q u$ or $y_s q u$. Since $t_1 \subseteq t_2$ and $u_1 \in t_1$, then u_1 is a t_2-quasi-nhd of x_r or y_s and $\bar{u}_1 \subseteq u$. Thus we have $u \in t_1(t_2)$. Also, $x_r \in u_1 \subseteq u$ or $y_s \in u_1 \subseteq u \Rightarrow x_r \in u$ or $y_s \in u$. This shows that $(X, t_1(t_2))$ is a fuzzy T_0-space. This completes the proof of the theorem.

3.6 Theorem. Let (X, t_1) and (X, t_2) be fuzzy topological spaces. If (X, t_1) is fuzzy T_1-space and $t_1 \subseteq t_2$, then $(X, t_1(t_2))$ is a fuzzy T_1-space.

Proof. Let $x_r, y_s \in t_1, x \neq y$. Since (X, t_1) is a fuzzy T_1-space, then there exist $u_1, v_1 \in t_1$ such that $x_r \in u_1 \subseteq$ com y_s and $y_s \in v_1 \subseteq$ com x_r. Let u and v be the t_1-quasi-nhds of x_r and y_s respectively. Then for $u_1, v_1 \in t_1$ we have $x_r q u_1$ and $y_s q u_1$ and $u_1 \subseteq u, v_1 \subseteq v$. So, $r + u_1(x) > 1$ and $s + v_1(y) > 1$ and $0 < r \leq 1, 0 < s \leq 1$. This implies that $r + u(x) > 1$ and $s + v(y) > 1 \Rightarrow x_r q u$ and $y_s q v$. Since $t_1 \subseteq t_2$ and $u_1, v_1 \in t_1$, then u_1 and v_1 are t_2-quasi-nhds of x_r and y_s respectively and $\bar{u}_1 \subseteq u$, $\bar{v}_1 \subseteq v$. Thus we have $u, v \in t_1(t_2)$. Also, $x_r \in u_1 \subseteq u$ and $y_s \in v_1 \subseteq v \Rightarrow x_r \in u \subseteq$ com y_s and $y_s \in v \subseteq$ com x_r. This shows that $(X, t_1(t_2))$ is a fuzzy T_1-space. This completes the proof of the theorem.

3.7 Theorem. A fts $(X, t_1(t_2))$ is a fuzzy T_1-space if and only if every fuzzy singleton set in X is closed.

Proof. Suppose $(X, t_1(t_2))$ is a fuzzy T_1-space. We show that $\{x\}^c$ is open. Let x_r and y_s be two fuzzy singletons, $x \neq y$. Let $x_r \in \{x\}^c$. Then there exists a fuzzy open set $u \in t_1(t_2)$ such that $x_r \in u$ but $y_s \notin u$. Thus we have $x_r \in u \subseteq \{x\}^c$ and hence $\{x\}^c = \cup\{u : x_r \in \{x\}^c\}$. Accordingly $\{x\}^c$, being a union of fuzzy open sets, is fuzzy open and $\{x\}$ is closed.

Conversely, let $\{x\}$ be closed for every $x \in X$. Let x_r and y_s be two fuzzy singletons, $x \neq y$. Now, $x \neq y \Rightarrow y_s \in \{x\}^c$, hence $\{x\}^c$ is an open set containing y_s but not containing x_r. Similarly, $\{y\}^c$ is an open set containing x_r but not containing y_s. Thus $\{x\}^c, \{y\}^c \in t_1(t_2)$ and $(X, t_1(t_2))$ is a fuzzy T_1-space.

3.8 Example. Let $t_1(t_2) = \{0, \{x\}, 1\}$ and $X = \{x, y\}$. Then 1 is the only fuzzy open set containing y but it also contains x. Hence $(X, t_1(t_2))$ is not a fuzzy T_1-space. In this case the fuzzy singleton set $\{x\}$ is not closed because $\{x\}^c = \{y\}$ is not open.

3.9 Theorem. Let (X, t_1) and (X, t_2) be fuzzy topological spaces. If (X, t_1) is fuzzy T_2-space and $t_1 \subseteq t_2$, then $(X, t_1(t_2))$ is a fuzzy T_2-space.

Proof. Let $x_r, y_s \in t_1, x \neq y$. Since (X, t_1) is a fuzzy T_2-space, then there exist $u_1, v_1 \in t_1$ such that $x_r \in u_1$, $y_s \in v_1$ and $u_1 \wedge v_1 = 0$. Let u and v be t_1-quasi-nhds of x_r and y_s respectively. Then for $u_1, v_1 \in t_1$ we have $x_r q u_1$, $y_s q u_1$ and $u_1 \subseteq u, v_1 \subseteq v$. So $r + u_1(x) > 1$, $s + v_1(y) > 1$ and $0 < r \leq 1, 0 < s \leq 1$. This implies that $r + u(x) > 1$, $s + v(y) > 1 \Rightarrow x_r q u, y_s q v$. Since $t_1 \subseteq t_2$ and $u_1, v_1 \in t_1$, then u_1 and v_1 are the t_2-quasi-nhds of x_r and y_s respectively and $\bar{u}_1 \subseteq u, \bar{v}_1 \subseteq v$. Thus we have $u, v \in t_1(t_2)$.
Also, $x_r \in u_1 \subseteq u$, $y_s \in v_1 \subseteq v$ and $u_1 \wedge v_1 = 0 \Rightarrow x_r \in u, y_s \in v$ and $u \wedge v = 0$. This shows that $(X, t_1(t_2))$ is a fuzzy T_2-space. This completes the proof of the theorem.

3.10 Theorem. Let (X, t_1) and (X, t_2) be fuzzy topological spaces. If (X, t_1) is fuzzy regular space and $t_1 \subseteq t_2$, then $(X, t_1(t_2))$ is a fuzzy regular space.

Proof. Let $u \in t_1(t_2)$. Then for every fuzzy singleton x_r, we have $x_r q u$ and there exist $u_1 \in t_1$ is a t_2-quasi-nhd of x_r such that t_1-closure, $\bar{u}_1 \subseteq u$.
Also, $x_r q u \Rightarrow r + u(x) > 1$. Put $1 - r = s$, then $0 < s \leq 1$. So, $s < u(x)$.
Since (X, t_1) is fuzzy regular space, then by proposition **(2.18)** for $x \in X$, $0 < s \leq 1$ and $u_1 \in t_1$ with $s < u_1(x)$ there exists $v_1 \in t_1$ such that $s < v_1(x)$ and $\bar{v}_1 \subseteq u_1$. Now, $s < u_1(x) \Rightarrow r + u_1(x) > 1$ and since $t_1 \subseteq t_2$ then v_1 is a t_2-quasi-nhd of x_r. This implies that $u_1 \in t_1(t_2)$.
Therefore we have for $x \in X$, $0 < s \leq 1$ and $u \in t_1(t_2)$ with $s < u(x)$ there exists $u_1 \in t_1(t_2)$ such that $s < u_1(x)$ and $\bar{u}_1 \subseteq u \Rightarrow$ By the Proposition

(2.18) $(X, t_1(t_2))$ is a fuzzy regular space. This completes the proof of the theorem.

3.11 Example. Let $X = \{x, y\}$, $t_1 = t_2 = \{0, x_{.4}, 1\}$, where $x_{.4}$ is a fuzzy singleton and $t_1(t_2) = \{0,1\}$. Here $t_1 \neq t_2$ and $1 \notin t_1(t_2) \Rightarrow 0.4 + 1(x) > 1 \Rightarrow 0.4 < 1(x)$. But there exists $0 \in t_1(t_2) \Rightarrow 0.4 + 0(x) > 1 \Rightarrow 0.4 < 0(x)$ and $\overline{0} \subseteq 1$. So $(X, t_1(t_2))$ is not a fuzzy regular space.

3.12 Theorem. Let (X, t_1) and (X, t_2) be fuzzy topological spaces. If (X, t_2) is a fuzzy normal space, then the mixed fuzzy topological space $(X, t_1(t_2))$ is a fuzzy normal space.

Proof. Let $w \in (t_1(t_2))^c$ and $u \in t_1(t_2)$ with $w \subseteq u$. Let $m \in t_2^c$ and $u_1 \in t_2$ with $m \subseteq u_1$. Then there exists $v \in t_2$ such that $m \subseteq v^0 \subseteq \overline{v} \subseteq u_1$. By **theorem (3.4)** we have $u \in t_2$, $v \in t_1(t_2)$ and therefore $m \subseteq v^0 \subseteq \overline{v} \subseteq u$. Now, $w \in (t_1(t_2))^c \Rightarrow 1 - w \in t_1(t_2) \Rightarrow 1 - w \in t_2 \Rightarrow w \in t_2^c$. Thus $w \subseteq m$ and hence $w \subseteq v^0 \subseteq \overline{v} \subseteq u$. This implies that the mixed fuzzy topological space $(X, t_1(t_2))$ is a fuzzy normal space. This completes the proof of the theorem.

3.13 Theorem. A mixed fuzzy topological space $(X, t_1(t_2))$ is fuzzy normal if and only if for any two closed fuzzy sets m and n in X with $m \subseteq 1 - n$, there exist $u, v \in t_1(t_2)$ such that $m \subseteq u$, $n \subseteq v$ and $\overline{u} \subseteq 1 - \overline{v}$.

Proof. Suppose that $(X, t_1(t_2))$ is fuzzy normal. Then for any fuzzy closed set $m \in (t_1(t_2))^c$ and fuzzy open set $u \in t_1(t_2)$ with $m \subseteq u$, there exists $v \in t_1(t_2)$ such that $m \subseteq v^0 \subseteq \overline{v} \subseteq u$. Let $n \in (t_1(t_2))^c$ be such that $n = 1 - u$. Then $m \subseteq 1 - n$ and $\overline{v} \subseteq 1 - n$. Now, $\overline{v} \subseteq 1 - n \Rightarrow n \subseteq 1 - \overline{v}$ and since $u \in t_1(t_2)$, then $\overline{u} \in (t_1(t_2))^c$. So we have $\overline{u} \subseteq n \Rightarrow \overline{u} \subseteq 1 - \overline{v}$. Also, $m \subseteq 1 - n \Rightarrow n \subseteq 1 - m \subseteq v \Rightarrow n \subseteq v$.
Conversely, let $m, n \in (t_1(t_2))^c$ with $m \subseteq 1 - n$. Then there exists $u, v \in t_1(t_2)$ such that $m \subseteq u$, $n \subseteq v$ and $\overline{u} \subseteq 1 - \overline{v}$. Let $u = 1 - n$. Then $m \subseteq v^0 \subseteq \overline{v} \subseteq 1 - \overline{u} \subseteq 1 - n = u \Rightarrow m \subseteq v^0 \subseteq \overline{v} \subseteq u$. This shows that $(X, t_1(t_2))$ is fuzzy normal. This completes the proof of the theorem.

4. COMPACTNESS AND CONNECTEDNESS

In this section we establish some simple results about compactness and connectedness.

4.1 Theorem. If wα is τ_2-compact, then α is $\tau_1(\tau_2)$- compact.

Proof. Suppose α is τ_2-compact. We have by **Theorem (3.4)** that if τ_1 and τ_2 are two fuzzy topologies on a set X, then the mixed fuzzy topology $\tau_1(\tau_2)$ is coarser than τ_2, in symbol $\tau_1(\tau_2) \subseteq \tau_2$.Form the definition of compactness, we can easily show that α is $\tau_1(\tau_2)$- compact. This completes the proof of the theorem.

4.2 Theorem. If α is τ_2-connected, then α is $\tau_1(\tau_2)$- connected.

Proof: If possible suppose that α is $\tau_1(\tau_2)$-disconnected. Then there exist relatively $\tau_1(\tau_2)$ closed fuzzy sets α_1 and α_2 such that $\alpha_1 \cap \alpha \neq 0$, $\alpha_2 \cap \alpha \neq 0$, $\alpha_1 \cap \alpha_2 = 0$ and $\alpha \subset \alpha_1 \cup \alpha_2$.Since $\tau_1(\tau_2) \subset \tau_2$, then $\tau_1(\tau_2)$ relatively closed fuzzy sets α_1 and α_2 are α_2-relatively closed fuzzy sets. Therefore α is τ_2-disconnected, which contradicts the fact that α is τ_2-connected. Hence the proof of the theorem is complete.

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Journal of Bangladesh Academy of Sciences, Vol. 32, No. 2, 211-220, 2008