DECAY OF TEMPERATURE FLUCTUATIONS IN MHD TURBULENT FLOW IN PRESENCE OF DUST PARTICLES BEFORE THE FINAL PERIOD

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ABSTRACT

This paper reports the decay of temperature fluctuation in homogeneous MHD turbulence in the presence of dust particles before the final period. We have considered the two and three point correlation equations and solved them after neglecting the fourth order correlations in comparison with the second and third order correlations. Finally, the energy decay law for temperature fluctuation of MHD turbulence in the presence of dust particles at times before the final period is obtained.

1. INTRODUCTION

Saffman[1] observed the effect of dust particles on the stability of the laminar flow of an incompressible fluid with constant mass concentration of dust particles and derived an equation which described the motion of a fluid containing small dust particles. It is of great interest of the behavior of dust particles in turbulent flow to many branches of science and technology, particularly if there is a substantial difference in density between the particles and the fluid. This behavior depends on the concentration and size of the particles with respect to the scale of turbulent flow. Deissler[2] developed a theory "Decay of homogeneous turbulence for times before the final period". He considered two and three point correlation equations neglecting fourth and higher order correlation terms. Using Deissler's theory, Kumar and Patel[3] studied the " First order reactants in homogeneous turbulent flow before the final period" for the case of multipoint and single time correlation. Corrsin[4,5] has already made an analytical attempt on the problem of turbulent temperature fluctuations using the approaches employed in the statistical theory of turbulence. Loeffler and Deissler[6] studied the decay of temperature fluctuation in homogeneous turbulence before the final period. In their approach they considered the two and three point correlation equations and solved these equations after neglecting the fourth and higher order correlation terms. Following Deissler's approach, Sarker and Rahman[7] also studied the decay of temperature fluctuations in MHD turbulence before the final period. In this problem, we studied the decay of temperature fluctuation in MHD turbulence before the final period in the presence of dust particles as an extension of the work of Sarker and Rahman.[7] Finally, the energy decay law for temperature field fluctuation of dusty fluid MHD turbulence before the final period is obtained.

2. Two point correlation and spectral equations

The induction equation of a magnetic field at the point p is

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=(\frac{\nu}{p_M})\frac{\partial^2 h_i}{\partial x_k \partial x_k} \quad (2.1)$$

and the energy equation at the point p' is

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$$\frac{\partial T_J^{/}}{\partial t}+u_k{}'\frac{\partial T_j{}'}{\partial x_k{}'}=(\frac{\nu}{p_r})\frac{\partial^2 T_j{}'}{\partial x_k{}'\partial x_k{}'} \quad (2.2)$$

where

u_k (x,t) = component of turbulent velocity,

$h_i(x,t)$ = component of magnetic field,

$\rho_M=\frac{\nu}{\lambda}=$ magnetic Prandtl number,

$\rho_r=\frac{\nu}{\gamma}$ = Prandtl number,

ν = kinematics viscosity,

$\lambda=(4\pi\mu\sigma)^{-1}$ = magnetic diffusivity,

$\gamma=\frac{k}{\rho c_p}$ = thermal diffusivity,

c_p = heat capacity at constant pressure,

T_j' = component of temperature fluctuation,

x_k = space co-ordinate.

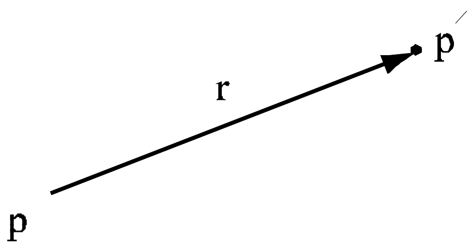

Fig.1. Vector configuration for two point correlation equations.

Multiplying equation (2.1) by T_j' and (2.2) by h_i, adding and taking ensemble average, we get

$$\frac{\partial}{\partial t}(\overline{h_i T_j'}) + \frac{\partial}{\partial x_k}(\overline{u_k h_i T_j'}) + \frac{\partial}{\partial x_k'}(\overline{u_k' h_i h_j'}) - \frac{\partial}{\partial x_k}(\overline{u_i h_k T_j'})$$
$$= \nu\left[\frac{1}{p_M}\frac{\partial^2}{\partial x_k \partial x_k}(\overline{h_i T_j'}) + \frac{1}{p_r}\frac{\partial^2}{\partial x_k' \partial x_k'}(\overline{h_i T_j'})\right] \quad (2.3)$$

The continuity equation is

$$\frac{\partial u_k}{\partial x_k} = \frac{\partial u_k'}{\partial x_k'} = 0 \,. \quad (2.4)$$

Substituting equation (2.4) into equation (2.3) yields

$$\frac{\partial}{\partial t}(\overline{h_i T_j'}) + \frac{\partial}{\partial x_k}(\overline{u_k h_i T_j'}) + \frac{\partial}{\partial x_k'}(\overline{u_k' h_i T_j'}) - \frac{\partial}{\partial_k}(\overline{u_i h_k T_j'})$$
$$= \nu\left[\frac{1}{P_M}\frac{\partial^2(\overline{h_i T_j'})}{\partial x_k \partial x_k} + \frac{1}{p_r}\frac{\partial^2(\overline{h_i T_j'})}{\partial x_k' \partial x_k'}\right] \quad (2.5)$$

Using the transformations

$$\frac{\partial}{\partial r_k} = -\frac{\partial}{\partial x_k} = \frac{\partial}{\partial x_k'}$$

and the relation (cf. Chandra Sekhar[8])

$\overline{u_k h_i T_j'} = -\overline{h_i u_k' T_j'}$ in equation (2.5), we get

$$\frac{\partial}{\partial t}(\overline{h_i T_j'}) + 2\frac{\partial}{\partial r_k}(\overline{u_k' h_i T_j'}) + \frac{\partial}{\partial r_k}(\overline{u_i h_k T_j'}) = \nu\left(\frac{1}{P_M} + \frac{1}{P_r}\right)\frac{\partial^2}{\partial r_k \partial r_k}(\overline{h_i T_j'}) \quad (2.6)$$

Now, we write this equation in spectral form by use of the three dimensional Fourier transforms

$$\overline{h_i T_j'}(r) = \int_{-\infty}^{\infty} \overline{l_i \tau_j'(k)} \exp(ik.r)dk \quad (2.7)$$

$$\overline{u_i h_k T_j'} = \int_{-\infty}^{\infty} \overline{\phi_i l_k \tau_j'(k)} \exp(ik.r)dk \quad (2.8)$$

Interchanging the subscripts i and j and then interchanging the points p and $p^{/}$, we have

$$\overline{u_k' h_i T_j'}(r) = \overline{u_k h_i h_j'}(-r) = \int_{-\infty}^{\infty} \overline{\phi_k l_i \tau_j'}(-k).\exp(ik.r)dk. \quad (2.9)$$

Putting (2.7), (2.8) and (2.9) into equation (2.6), we get

$$\frac{\partial}{\partial t}\overline{l_i \tau_j'(k)} + ik_k\left[2\overline{\phi_k l_i \tau_j(-k)} + \overline{\phi_i l_k \tau_j'(k)}\right] = -\nu\left[\frac{1}{P_M} + \frac{1}{P_r})K^2 \overline{l_i \tau_j(K)}\right] \quad (2.10)$$

The tensor equation (2.10) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial}{\partial t}(l_i \tau_i(k) + ik_k[2\overline{\phi_k l_i \tau_i(-k)} + \overline{\phi_i l_k \tau_i'}] = -\nu[(\frac{1}{p_M} + \frac{1}{p_r})k^2 \overline{l_i \tau_i'(k)}]. \quad (2.11)$$

3. Three point correlation and equations.

The momentum equation of MHD turbulence in the presence of dust particles at the point p, the induction equation at the point p$'$ and the energy equation at p$''$ as

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} - h_k \frac{\partial h_i}{\partial x_k} = -\frac{\partial W}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} + \frac{KN}{\rho}(v_i - u_i). \quad (3.1)$$

$$\frac{\partial h_i'}{\partial t} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \frac{\nu}{p_M} \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} \quad (3.2)$$

and

$$\frac{\partial T_j''}{\partial t} + u_k'' \frac{\partial T_j''}{\partial x''} = (\frac{\nu}{p_r}) \frac{\partial^2 T_j''}{\partial x_k'' \partial x_k''} \quad (3.3)$$

where,

$W = \frac{P}{\rho} + \frac{1}{2}\left|\bar{h}\right|^2$ = total MHD pressure,

P(x,t) = hydrodynamic pressure,

ρ = fluid density,

K = stock registance,

N = number density of dust particles,

v_i = component of the fluctuating velocity of dust particles.

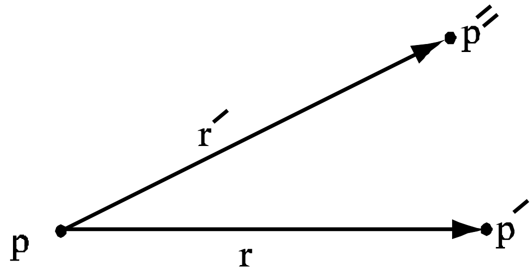

Fig.2. Vector configuration for three point correlation equations.

Multiplying the equation (3.1) by $h_i T_j''$, (3.2) by $u_i T_j''$, and (3.3) by $u_i h_i'$, adding three equations and taking space or time averages, we obtain

$$\frac{\partial}{\partial t}(\overline{u_i h_j' T_j''}) + \frac{\partial}{\partial x_k}(\overline{u_i u_k h_i' T_j''}) - \frac{\partial}{\partial x_k}(\overline{h_i h_k h_i' T_j''}) + \frac{\partial}{\partial x_k'}(\overline{u_i u_k' h_i' T_j''})$$
$$-\frac{\partial}{\partial x_k'}(\overline{u_i u_i' h_k' T_j''}) + \frac{\partial}{\partial x_k''}(\overline{u_i u_k'' h_i' T''_j}) = -\frac{\partial}{\partial x_i}(\overline{W h_i' T_j''}) + \nu \frac{\partial^2}{\partial x_k \partial x_k}(\overline{u_i h_i' T_j''})$$
$$+\nu\left[\frac{1}{P_M}\frac{\partial^2}{\partial x_k' \partial x_k'}(\overline{u_i h_i' T_j''}) + \frac{1}{P_r}\frac{\partial^2}{\partial x_k'' \partial x_k''}(\overline{u_i h_i' T_j''})\right] + f(\overline{v_i h_i' T_j''} - \overline{u_i h_i' T_j''}). \quad (3.4)$$

where $f = \frac{KN}{\rho}$ has the dimension of the frequency. Substituting the relations

$\frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'} \; and \; \frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'})$ into equation (3.4), we get

$$\frac{\partial}{\partial t}(\overline{u_i h_i' T_j''}) - \nu\Big[(1+\frac{1}{p_M})\frac{\partial^2}{\partial r_k \partial r_k}(\overline{u_i h_i' T_j''}) + (1+\frac{1}{p_r})\frac{\partial^2}{\partial r_k' \partial r_k'}(\overline{u_i h_i' T_j''})$$

$$+ 2\frac{\partial^2}{\partial r_k \partial r_k'}(\overline{u_i h_i' T_j''})\Big] = \frac{\partial}{\partial r_k}(\overline{u_i u_k h_i' T_j''}) + \frac{\partial}{\partial r_k'}(\overline{u_i u_k h_i' T_j''}) - \frac{\partial}{\partial r_k}(\overline{h_i h_k h_i' T_j''})$$

$$-\frac{\partial}{\partial r_k'}(\overline{h_i h_k h_i' T_j''}) - \frac{\partial}{\partial r_k'}(\overline{h_i h_k h_i' T_j''}) - \frac{\partial}{\partial r_k}(\overline{u_i u_k' h_i' T_j''}) + \frac{\partial}{\partial r_k}((\overline{u_i u_i' h_k T_j''})\text{s}$$

$$-\frac{\partial}{\partial r_k'}(\overline{u_i u_k'' h_i' T_j''}) + \frac{\partial}{\partial r_i}(\overline{W h_i' T_j''}) + \frac{\partial}{\partial r_i'}(\overline{W h_i' T_j''}) + f(\overline{v_i h_i' T_j''} - \overline{u_i h_i' T_j''}) \cdot \qquad (3.5).$$

Now, we write equation (3.5) in spectral form in order to reduce it to an ordinary differential equation and because of the physical significance of spectral quantities. For this, we use six dimensional Fourier transforms:

$$\overline{u_i h_i'(r) T_j''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \beta_i'(k)\theta_j''(k'')}.exp\left[i(k.r + k'..r')\right]dkdk', \qquad (3.6)$$

$$\overline{u_i u_i'(r) h_k'(r) T_j''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \phi_i'(k)\beta_k'(k)\theta_j''(k')}.exp\left[i(k.r + k'.r')\right]dkdk', \qquad (3.7)$$

$$\overline{u_i u_k'(r) h_i'(r) T_j''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \phi_k'(k)\beta_i'(k)\theta_j''(k')}.ex\left[i(k.r + k'.r')\right]dkdk' \qquad (3.8)$$

$$\overline{u_i u_k h_i'(r) T_j''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\phi_i \phi_k \beta_i'(k)\theta_j''(k')}.exp\left[i(k.r + k'.r')\right]dkdk' \qquad (3.9)$$

$$\overline{h_i h_k h_i'(r) T_j''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\beta_i \beta_k \beta_i' \theta_j''(k')}.exp\left[i(k.r + k'.r')\right]dkdk' \qquad (3.10)$$

$$\overline{W h_i'(r) T_j''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\gamma \beta_i'(k)\theta_j''(k')}.exp\left[i(k.r + k'.r')\right]dkdk', \qquad (3.11)$$

and

$$\overline{V_i h_i'(r) T_j''(r')} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\mu_i \beta_i'(k)\theta_j''(k')}.exp\left[i(k.r + k'.r')\right]dkdk' \qquad (3.12)$$

Interchanging of points p' and p'' along with the indices i and j , result in the relations

$$\overline{u_i u_k'' h_i' T_j''} = \overline{u_i u_k' h_i' T_j''}$$

By use of these facts and relations (3.6) - (3.12), one can write equation (3.5) in the form

$$\frac{\partial}{\partial t}\overline{\phi_i \beta_i' \theta_j''} + \nu\left[(1+\frac{1}{P_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2k_k k_k'\right]\overline{\phi_i \beta_i' \theta_j''}$$

$$= i(k_k + k_k')\overline{\phi_i\phi_k\beta_i'\theta_j''} - i(k_k + k_k')\overline{\beta_i\beta_k\beta_i'\theta_j''} - i(k_k + k_k')\overline{\phi_i\phi_k'\beta_i'\theta_j''}$$

$$+ i(k_k + k_k')\overline{\phi_i\phi_i'\beta_k'\theta_j''} + i(k_i + k_i')\overline{\gamma\beta_i'\theta_j''} + f(\overline{\mu_i\beta_i'\theta_j''} - \overline{\phi_i\beta_i'\theta_j''}) \tag{3.13}$$

The tensor equation (3.13) can be converted to scalar equation by contraction of the subscripts i and j

$$\frac{\partial}{\partial t}\overline{\phi_i\beta_i'\theta_j''} + \nu\left[(1+\frac{1}{P_M})k^2 + (1+\frac{1}{P_r})k'^2 + 2k_kk_k'\right]\overline{\phi_i\beta_i'\theta_j''} = i(k_k + k_k')\overline{\phi_i\phi_k\beta_i'\theta_i''}$$

$$- i(k_k + k_k')\overline{\beta_i\beta_k\beta_i'\theta_i''} - i(k_k + k_k')\overline{\phi_i\phi_k\beta_i'\theta_i''} + ik_k\overline{\phi_i\phi_i'\beta_k'\theta_i''}$$

$$+ i(k_k + k_k')\overline{\gamma\beta_i'\theta_j''} + f(\overline{\mu_i\beta_i'\theta_j''} - \overline{\phi_i\beta_i'\theta_j''}) \tag{3.14}$$

If we take the derivative with respect to x_i of the momentum equation (3.1) for the point p , we obtain

$$-\frac{\partial^2 W}{\partial x_i\partial x_i} = \frac{\partial^2}{\partial x_i\partial x_k}(u_iu_k - h_ih_k) - \frac{\partial}{\partial x_i}f(v_i - u_i) \tag{3.15}$$

Multiplying equation (3.15) by $h_i'T_j''$, taking averages and writing the equation in terms of the independent variables r and r'

$$-[\frac{\partial^2}{\partial r_i\partial r_i} + \frac{\partial^2}{\partial r_i'\partial r_i'} + 2\frac{\partial^2}{\partial r_i\partial r_i'}]\overline{wh_i'T_j''}$$

$$= \left[\frac{\partial^2}{\partial r_i\partial r_k} + \frac{\partial^2}{\partial r_i'\partial r_k} + \frac{\partial^2}{\partial r_i\partial r_k'} + \frac{\partial^2}{\partial r_i'\partial_k'}\right](\overline{u_iu_kh_i'T_j''} - \overline{h_ih_kh_i'T_j''})$$

$$+ f(\frac{\partial}{\partial r_i} + \frac{\partial}{\partial r_i'})(\overline{v_ih_i'T_j''} - \overline{u_ih_i'T_j''}). \tag{3.16}$$

Taking the Fourier transforms of equation (3.16), we get

$$-\overline{\gamma\beta_i'\theta_j''} = \left[(k_ik_k + k_i'k_k + k_ik_k' + k_i'k_k'(\overline{\phi_i\phi_k\beta_i'\theta_j''} - \overline{\beta_i\beta_k\beta_i'\theta_j''})\right.$$

$$\left.- if(k_i + k_i')(\overline{\mu\beta_i'\theta_j''} - \overline{\phi_i\beta_i'\theta_j''})\right]/(k^2 + k'^2 + 2k_ik_i'). \tag{3.17}$$

Equation (3.1) can be used to eliminate $\overline{\gamma\beta_i'\theta_j''}$ from equation (3.13).

4. Solution for times before the final period

To study the decay of MHD dusty fluid turbulence for times before the final period, the three point correlations are considered and the quadruple correlations are neglected. If this is happened then equation (3.17) shows that the term $\overline{\gamma\beta_i'\theta_j''}$ associated with the pressure correlations, should also be neglected. Thus we have from the equation (3.14)

$$\frac{\partial}{\partial t}\overline{\phi_i\beta_i'\theta_j''} + \nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{p_r})k'^2 + 2kk' - Rf\right]\overline{\phi_i\beta_i'\theta_i''} = 0. \tag{4.1}$$

where

$$R = \{\frac{(k_i + k_i^/)^2}{k^2 + k^{/2} + 2k_i k_i^/} - 1\}(s-1)$$

and $\mu\beta_i^{'}\theta_j^{''} = s\phi_i\beta_i^{'}\theta_j^{''}$,also R and s are arbitrary constants.

Integrating the equation (4.1) between t_o and t with inner multiplication by k_k gives

$$\overline{k_k\phi_i\beta_i^{'}\theta_i^{''}} = k_k\left[\overline{\phi_i\beta_i^{'}\theta_i^{''}}\right]_0 .exp\left[\{-\nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{pr})k^{'2} + 2kk^{'}cos\theta\right] + Rf\}(t-t_0)\right] \quad (4.2)$$

where θ is the angle between k and $k^{'}$. Now, by letting $r^{'}=0$ in equation (3.6) and comparing with the equation (2.8) and (2.9), we have

$$\overline{\phi_i l_k \tau_i^{'}(k)} = \int_{-\infty}^{\infty} \overline{\phi_i\beta_i^{'}\theta_i^{''}}\, dk^{'} \quad (4.3)$$

and

$$\overline{\phi_i l_i \tau_i^{'}(-k)} = \int_{-\infty}^{\infty} \overline{\phi_k\beta_i^{'}(-k)\theta_i^{''}(-k^{'})}dk^{'} \quad (4.4)$$

Substituting equations (4.2), (4.3) and (4.4) in equation (2.9), we get

$$\frac{\partial}{\partial t}\overline{l_i\tau_i^{'}(k)} + \nu(\frac{1}{p_M} + \frac{1}{p_r})k^2\overline{l_i\tau_i^{'}(k)} = \int_{-\infty}^{\infty}\left[\overline{\phi_i\beta_i^{'}\theta_i^{''}} + 2\overline{\phi_k\beta_i^{'}(-k)\theta_i^{''}(-k^{'})}\right]_0$$

$$.exp\left[\{-\nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{pr})k^{'2} + 2kk^{'}cos\theta\right] + Rf\}(t-t_0)\right]dk^{'} \quad (4.5)$$

Now, $dk^{'} = dk_1^{'}dk_2^{'}dk_3^{'}$ can be expressed intersms of $k^{'}$ and θ [3] as

$$dk^/ = -2\pi k^{/2} d(\cos\theta)dk^/ \quad (4.6)$$

Substituting equation (4.6) to equation (4.5) to give

$$\frac{\partial}{\partial t}\overline{l_i\tau_i^{'}(k)} + \nu(\frac{1}{p_M} + \frac{1}{p_r})k^2\overline{l_i\tau_i^{'}(k)} = -\int_0^{\infty} 2\pi i k_k\left[\overline{\phi_i\beta_i^{'}\theta_i^{''}} + 2\overline{\phi_k\beta_i^{'}(-k)\theta_i^{''}(-k^{'})}\right]_0 k^{'2}$$

$$.\left[\int_{-1}^{1} exp\left[\{-\nu\left[(1+\frac{1}{p_M})k^2 + (1+\frac{1}{pr})k^{'2} + 2kk^{'}cos\theta\right] + Rf\}(t-t_0)\right]d(cos\theta)\right]dk^{'} .. \quad (4.7)$$

In order to find the solution completely and following Loeffler and Deissler[6], we assume that

$$ik_k\left[\overline{\phi_i\beta_i^{'}\theta_i^{//}} + 2\overline{\phi_k\beta_i^{'}(-k)\theta_i^{''}(-k^{'})}\right]_0 = -\frac{\xi_o}{(2\pi)^2}(k^2k^{'4} - k^4k^{'2}) \quad (4.8)$$

where ξ_0 is a constant depending on the initial conditions.

Putting (4.8) in equation (4.7) and completing the integration with respect to $\cos\theta$, we get

$$\frac{\partial}{\partial t}(\overline{2\pi l_i\tau_i'(k)})+\nu(\frac{1}{p_M}+\frac{1}{p_r})k^2(\overline{2\pi l_i\tau_i'(k)})=-\frac{\xi_0}{2\nu(t-t_0)}\int_0^\infty(kk'^5-k^3k'^3)$$

$$.\left[exp\left[\{-\nu[(1+\frac{1}{p_M})k^2+(1+\frac{1}{pr})k'^2-2kk'\right]+Rf\}(t-t_0)\right]$$

$$.exp\left[\{-\nu[(1+\frac{1}{p_M})k^2+(1+\frac{1}{pr})k'^2+2kk']+Rf\}(t-t_0)]\right]dk' \qquad (4.9)$$

Multiplying both sides by k^2, we have

$$\frac{\partial Q}{\partial t}+\nu(\frac{1}{p_M}+\frac{1}{p_r})k^2Q=F \qquad , \qquad (4.10)$$

where

$$Q=2\pi k^2\overline{l_i\tau_i'(k)} \qquad (4.11)$$

is the magnetic energy spectrum function and F is the energy transfer term given by

$$F=-\frac{\xi_0}{\nu(t-t_0)}\int_0^\infty(k^3k'^5-k^5k'^3)\left[\ exp\left[\ \{-\nu[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2\right.\right.$$

$$-2kk']+Rf\}(t-t_o)]-\exp[\{-\nu[(1+\frac{1}{p_M})k^2+(1+\frac{1}{p_r})k'^2+2kk']+Rf\}(t-t_o)]]dk' \quad ..(4.12)$$

Integrating equation (4.12) with respect to k', we obtain

$$F=-\frac{\sqrt{\pi}\xi_0 p_r^{\frac{5}{2}}}{2\nu^{\frac{3}{2}}(t-t_o)^{\frac{3}{2}}(1+p_r)^{\frac{5}{2}}}exp\left[\{-\nu(1+\frac{1}{p_M}-\frac{p_r}{1+p_r})k^2+Rf\}(t-t_0)\right]$$

$$.\left[\frac{15p_rk^4}{4\nu^2(t-t_0)^2(1+p_r)}+\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\}\frac{k^6}{\nu(t-t_0)}+\{\ \frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{(1+p_r)}\ \}k^8\right] \qquad (4.13)$$

The series of equation (4.13) contains only even power of k.
It is interesting to note that

$$\int_0^\infty F.dk=0 \qquad (4.14)$$

This indicates that the condition of continuity and homogeneity are maintained.
The linear equation (4.10) can be solved to give

$$Q=exp\left[-\nu k^2(t-t_0)(\frac{1}{p_M}+\frac{1}{p_r})\right]\int F.exp[\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})(t-t_o)]dt$$

$$+C(k)exp\left[-\nu k^2(\frac{1}{p_M}+\frac{1}{p_r})(t-t_o)\right] \qquad (4.15)$$

where $C(k)=\frac{N_0k^2}{\pi}$(4.16) is a constant of integration and can be obtained as by corrsin.[4]
Substituting the values of F from equation (4.13) in equation (4.15) and integrating with respect to t, we get

$$Q(k,t)=\frac{N_0k^2}{\pi}exp\left[\{\ -\nu K^2(\frac{1}{p_M}+\frac{1}{p_r})+Rf\ \}(t-t_0)]+\frac{\sqrt{\pi}\xi_0 p_r^{\frac{5}{2}}}{2\nu^{\frac{3}{2}}(1+p_r)^{\frac{7}{2}}}exp[\{-\nu k^2\frac{(1+P_r+P_M)}{P_M(1+p_r)}+Rf\}(t-t_0)\right]$$

$$\left[\frac{3p_r k^4}{2\nu^2(t-t_0)^{\frac{5}{2}}}+\frac{P_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_0)^{\frac{3}{2}}}-\frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^2(t-t_0)^{\frac{1}{2}}}+\frac{8\sqrt{\nu}}{3}\frac{(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{\frac{5}{2}}p_r^{\frac{1}{2}}}N(\omega)\right] \quad (4.17)$$

where $N(\omega)=\exp(-\omega^2)\int_0^W \exp(x^2)dx$ and

$$\omega = k\left[\frac{\nu(t-t_0)}{p_M(1+p_M)}\right]^{\frac{1}{2}}.$$

The function $F(\omega)$ has been calculated numerically and tabulated in.[6] Let **r** =0 in equation (2.7) and use is made of the definition of Q as given by equation(4.11), the result is

$$\frac{\overline{T^2}}{2}=\frac{\overline{T_iT_j'}}{2}=\int_0^\infty Q(k)dk \quad (4.18)$$

Substituting equation (4.17) into (4.18) and after integration, one can obtain

$$\frac{\overline{T^2}}{2}=\frac{N_0P_M^{\frac{3}{2}}\nu^{\frac{-3}{2}}(t-t_0)^{-\frac{3}{2}}}{8\sqrt{2\pi}}+exp\{Rf(t-t_0)\}\xi_0 S\nu^{-6}(t-t_0)^{-5} \quad (4.19)$$

where

$$S=\frac{\pi P_M^{\,6}}{(1+P_M)(1+2P_M)^{\frac{3}{2}}}\left[\frac{9}{16}+\frac{5P_M(7P_M-6)}{16(1+2P_M)}-\frac{35P_M(3P_M^{\,2}-2P_M+3)}{8(1+2P_M)^2}+......\right].$$

Thus, the decay law for magnetic energy fluctuation before the final period in the presence of dust particle may be written as

$$\overline{T^2}=A(t-t_0)^{-\frac{3}{2}}+B(t-t_0)^{-5}exp\{Rf(t-t_0)\}, \quad (4.20)$$

where, $\overline{T^2}$ is the mean square of the magnetic field fluctuation, t is the time,

$A=\frac{N_0P_M^{\frac{3}{2}}\nu^{-\frac{3}{2}}}{4\sqrt{2\pi}}$, $B=2\xi_0 Q\nu^{-6}$ and t_0 are constants determined by the initial conditions.

For larger times, the last terms in the equation becomes negligible, leaving the –3/2 power decay law for the final period.

The results of the present study, obtained by neglecting the quadruple correlation's in the three points correlation equation, appear to represent the MHD turbulence for times before the final period. For clean fluid , i.e. in absence of dust particles we put f=0, the equation (4.20) becomes

$$\overline{T^2}=A(t-t_0)^{-\frac{3}{2}}+B(t-t_0)^{-5}$$

which was obtained earlier by Sarker and Rahman.[8] For larger times, the last term in the equation becomes negligible, giving the –3/2 power decay law for the final period.

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