J. Bangladesh Acad. Sci., Vol. 39, No. 2, 203-211, 2015

ON T_2 SPACE IN L-TOPOLOGICAL SPACE

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ABSTRACT

The purpose of this paper was to construct seven concepts of T_2-space in L-topological spaces. After giving the fundamental definitions, the authors established some relations among them. Further, the authors proved that all these definitions satisfy 'good extension' property. Finally, it is shown that these definitions are hereditary, productive and projective.

Key words: Fuzzy topology, L-topology

INTRODUCTION

American mathematician Zadeh (1965) for the first time in 1965 introduced the concept of fuzzy sets. Chang (1968) and Lowen (1976) developed the theory of fuzzy topological space by using fuzzy sets. Separation axioms in fuzzy topological spaces have been developed and studied by many researchers from different view points (Jin-xuan, and Ren Bai-lin 1998, Kandil and El-Shafee 1991). FP-T_2 separation axioms have been introduced earlier by Kandil and El-Shafee (1991) and Nouh (1996). In this paper the workers defined possible seven definitions of T_2 space in L-topological space. All these definitions satisfy 'good extension' property and the authors established some implications among them. Finally it was shown that all these definitions are hereditary, productive and projective and preserved under one-one, onto and continuous maps.

Definition: Let X be a non-empty set and I = [0, 1]. A fuzzy set in X is a function $u : X \rightarrow I$ which assign to each element $x \in X$, a degree of membership, $u(x) \in I$ (Zadeh 1965). Example: Let $X = \{a, b, c\}$ and $I = [0, 1]$. If $u(a) = 0.2$, $u(b) = 0.4$, $u(c) = 0.5$ then $\{(a, 0.2), (b, 0.4), (c, 0.5)\}$ is a fuzzy set in X.

Definition: Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. An L-fuzzy set in X is a function $a : X \rightarrow L$ which assign to each element $x \in X$, a degree of membership, $\sim(x) \in L$.

Definition: Let ~ be an L-fuzzy set in X. Then $1 - \sim = \sim'$ is called the complement of ~ in X (Zadeh 1965).

Definition: If $r \in L$ and ~ is an L-fuzzy sets in X defined by $\sim(x) = r$? $x \in X$ then we refer to ~ as a constant L-fuzzy sets and denoted it by r itself.

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In particular, the workers had the constant L-fuzzy sets 0 and 1.

Definition: An L-fuzzy point p in X is a special L-fuzzy sets with membership function $p(\mathrm{x}) = r$ *if* $x = x_0$, $p(x) = 0$ *if* $x \neq x_0$ where $r \in L$ (Zadeh 1965).

Definition: An L-fuzzy point p is said to belong to an L-fuzzy set ~ in $X(p \in$ ~$)$ if and only if $p\ (x) <$ ~ (x) and $p\ (y) \ \leq$ ~ (y). That is $x_r \in$ ~ implies $r <$ ~ (x).

Definition: Let I = [0, 1], X be a non-empty set and I^X be the collection of all mappings from X into I, i.e. the class of all fuzzy sets in X. A fuzzy topology on X is defined as a family t of members of I^X, satisfying the following conditions:

(i) $1, 0 \in t$ (ii) if $u_i \in t$ for each $i \in \Delta$ then $\bigcup_{i\in\Delta} u_i \in t$ (iii) if $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.

The pair (X, t) is called a fuzzy topological space (fts, in short) and the members of t are called t-open (or simply open) fuzzy sets. A fuzzy set v is called a t-closed (or simply closed) fuzzy set if $1 - v \in t$ (Chang 1968).

Definition: Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. Suppose that ² be the sub collection of all mappings from X to L i.e. ² $\subseteq L^X$. Then ² is called L-topology on X if it satisfies the following conditions:

(i) $0^*, 1^* \in$ ²

(ii) If $u_1, u_2 \in$ ² then $u_1 \cap u_2 \ \in$ ²

(iii) If $u_i \in$ ² for each $i \in \Delta$ then $\cup_{i\in\Delta} u_i \in$ ².

Then the pair (X, τ) is called a L-topological space (lts, for short) and the members of ² are called open L-fuzzy sets. An L-fuzzy sets v is called a closed L-fuzzy set if $1 - v \in$ ² (Jin-xuan, and Ren Bai-lin 1998) .

Definition: Let ª be an L-fuzzy set in 1st $(X,$ ²$)$. Then the closure of ª is denoted by $\overline{ª}$ and defined as $\overline{ª} = \cap\{$« : ª $\subseteq$ « $\in$ ²$^c\}$.

The interior of ª written ª^0 is defined by ª$^0 = \cup\{$« : « $\subseteq$ ª, « $\in \tau\}$.

Definition: If $(X,$ ²$)$ is an lts and $A \subseteq X$ then ²$_A = \{u \mid A : u \in$ ²$\}$ is called the sub space L-topology on A and $(A,$ ²$_A)$ is referred to as an L-sub space of (X, τ).

Definition: Let x_r be an L-fuzzy point in an lts $(X,$ ²$)$. An L-fuzzy set α in X is called an L-fuzzy neighborhood (in short, nhd) of x_r if and only if there exists an open L-fuzzy set in Y such that $x_r \in \ \subseteq$ ~.

Definition: An L-fuzzy singleton in X is an L-fuzzy set in X which is zero everywhere except at one point say x, where it takes a value say r with $0 < r \leq 1$ and $r \in L$. We denote it by x_r and $x_r \in$ ~ iff $r \leq$ ~ (x).

Definition: An L-fuzzy singleton x_r is said to be quasi-coincident (q-coincident, in short) with an L-fuzzy set ~ in *X*, denoted by $x_r q$~ iff $r +$~ $(x) > 1$. Similarly, an L-fuzzy set ~ in *X* is said to be q-coincident with an L-fuzzy set   in *X*, denoted by ~*q*   if and only if ~ (x) + $(x) > 1$ for some $x \in X$. Therefore iff ~ (x) + $(x) \leq 1$ for all $x \in X$, where denote an L-fuzzy set ~ in *X* is said to be not q-coincident with an L-fuzzy set   in *X*.

Definition: Let $f : X \rightarrow Y$ be a function and *u* be fuzzy set in *X*. Then the image $f(u)$ is a fuzzy set in *Y* which membership function is defined by

$(f(u))(y) = \{\sup(u(x)) \mid f(x) = y\}$ if $f^{-1}(y) \neq$ ¤, $x \in X$

$(f(u))(y) = 0$ if $f^{-1}(y) =$ ¤, $x \in X$ (Chang 1968).

Definition: Let *f* be a real-valued function on an L-topological space. If $\{x : f(x) >$ ~} is open for every real ~, then f is called lower-semi continuous function (lsc, in short)

Definition: Let (*X*, ²) and (*Y*, *s*) be two L-topological space and *f* be a mapping from (*X*, ²) into (*Y*, *s*) i.e. *f* : (*X*, ²) $\rightarrow$ (*Y*, *s*). Then *f* is called –

(i) Continuous iff for each open L-fuzzy set $u \in s \Rightarrow f^{-1}(u) \in \tau$.

(ii) Open iff *f* («)$\in s$ for each open L-fuzzy set « $\in \tau$.

(iii) Closed iff $f(\lambda)$ is s-closed for each ª $\in \tau^c$ where τ^c is closed L-fuzzy set in *X*.

(iv) Homeomorphism iff *f* is bijective and both *f* and f^{-1} are continuous.

Definition: Let *X* be a nonempty set and *T* be a topology on . Let ² = $\mu(T)$ be the set of all lower semi continuous (lsc) functions from (*X*, *T*) to *L* (with usual topology). Thus $\mu(T) = \{u \in L^X : u^{-1}(\alpha, 1] \in T\}$ for each $\alpha \in L$. It can be shown that $\mu(T)$ is a L-topology on *X*. Let "P" be the property of a topological space (*X*, *T*) and LP be its L-topological analogue. Then LP is called a "good extension" of P "if the statement (*X*, *T*) has P iff (*X*, $\mu(T)$) has LP" holds good for every topological space (*X*, *T*).

Definition: Let {(X_i, ²$_i$) : $i \in \Delta$} be a family of L-topological space. Then the space (ΠX_i, $\Pi\tau_i$) is called the product lts of the family {(X_i, ²$_i$) : $i \in \Delta$} where Π²$_i$ denote the usual product L-topologies of the families {²$_i$: $i \in \Delta$} of L-topologies on *X*.

T_2 SPACE IN L-TOPOLOGY

We now give the following definitions of T_2 L-topological spaces.

Definition: An lts (*X*, ²) is called-

(a) $L - T_2$(i) if $\forall x, y \in X, x \neq y, \exists u, v \in$ ² such that $u(x) = 1, u(y) = 0, v(x) = 0, v(y) = 1$ and $u \cap v = 0$.

(b) $L - T_2(ii)$ if for any pair of distinct L-fuzzy points $x_r, y_s \in X$, $\exists u, v \in$ ² such that $x_r \in u, y_s \notin u$ and $x_r \notin v, y_s \in v$ and $u \cap v = 0$.

(c) $L - T_2(iii)$ if for all pairs of distinct L-fuzzy singletons $x_r, y_s \in S(X)$ such that $x_r \bar{q} y_s, \exists u, v \in \tau$ such that $x_r \subseteq u, y_s \bar{q} u$ and $y_s \subseteq v, x_r \bar{q} v$ and $u \cap v = 0$.

(d) $L - T_2(iv)$ if for any pair of distinct L-fuzzy points $x_r, y_s \in S(X)$, $\exists u, v \in$ ² such that $x_r \in u, u\bar{q}y_s$ and $y_s \in v, v\bar{q}x_r$ and $u \cap v = 0$.

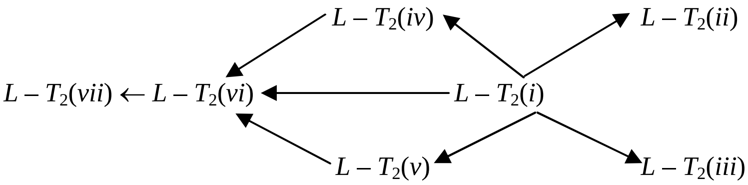
(e) $L - T_2(v)$ if for any pair of distinct L-fuzzy points $x_r, y_s \in S(X)$, $\exists u, v \in$ ² such that $x_r \in u \subseteq co y_s$, $y_s \in v \subseteq co x_r$ and $u \subseteq co v$.

(f) $L - T_2(vi)$ if $\forall x, y \in X, x \neq y, \exists u, v \in$ ² such that $u(x) > 0$, $u(y) = 0$ and $v(x) > 0$, $v(y) = 0$.

(g) $L - T_2(vii)$ if $\forall x, y \in X, x \neq y, \exists u, v \in$ ² such that $u(x) > u(y)$ and $v(y) > v(x)$.

Here, the present authors establish a complete comparison of the definitions $L - T_2(ii)$, $L - T_2(iii)$, $L - T_2(iv)$, $L - T_2(v)$, $L - T_2(vi)$ and $L - T_2(vii)$ with $L - T_2(i)$.

Theorem: Let (X, τ) be an lts. Then the authors have the following implications:



The reverse implications are not true in general, except $L - T_2(vi)$ and $L - T_2(vii)$.

Proof: $L - T_2(i) \Rightarrow L - T_2(ii)$, $L - T_2(i) \Rightarrow L - T_2(iii)$ can be proved easily. Now $L - T_2(i) \Rightarrow L - T_2(iv)$ and $L - T_2(\text{i}) \Rightarrow L - T_2(v)$, since $L - T_2(ii) \Leftrightarrow L - T_2(iv)$ and $L - T_2(iv) \Leftrightarrow L - T_2(v)$. $L - T_2(i) \Rightarrow L - T_2(vi)$; It is obvious. $L - T_2(i) \Rightarrow L - T_2(vii)$, since $- T_2(vi) \Rightarrow L - T_2(vii)$.

The reverse implications are not true in general, except $L - T_2(vi)$ and $L - T_2(vii)$, as can be seen through the following counter-examples:

Example 1: Let $X = \{x, y)$, ² be the L-topology on X generated by{~ : ~ $\in L\} \cup \{u, v\}$ where $u(x) = 0.5$, $u(y) = 0$ $v(x) = 0$, $v(y) = 0.6$, $L = \{0, 0.05, 0.1, 0.15, ..., 0.95, 1\}$ and $r = 0.4$, $s = 0.3$.

Example 2: Let $X = \{x, y)$, ² be the L-topology on X generated by {~ : ~ $\in L\} \cup \{u, v\}$ where $u(x) = 0.5$, $u(y) = 0$ $v(x) = 0$, $v(y) = 0.4$, $L = \{0, 0.05, 0.1, 0.15, ..., 0.95,1\}$ and $r = 0.5$, $s = 0.4$.

Proof: $L-T_2(ii) \not\Rightarrow L-T_2(i)$: From example-1, we see that the lts (X, ²) is clearly $L - T_2(ii)$ but it is not $L - T_2(i)$. Since there is no L-fuzzy set in ² which grade of membership is 1.

$L-T_2(iii) \not\Rightarrow L-T_2(i)$: From example-2, we see the lts (X, τ) is clearly $L - T_2(iii)$ but it is not $L - T_2(ii)$. Since $L-T_2(iii) \not\Rightarrow L-T_2(ii)$ and $L-T_2(ii) \not\Rightarrow L-T_2(i)$ so $L-T_2(iii) \not\Rightarrow L-T_2(i)$.

$L-T_1(i) \not\Rightarrow L-T_1(iv)$: As for the distinct L-fuzzy singletons x_1, y_1 in ² there does not exist $u, v \in$ ² such that $x_1 \subseteq u, y_1 \overline{q} u$ and $y_1 \subseteq v, x_1 \overline{q} v$.

$L-T_2(iv) \not\Rightarrow L-T_2(i)$: This follows from the fact that

$L-T_2(ii) \Leftrightarrow L-T_2(iv)$ and it has already been shown that $L-T_2(ii) \not\Rightarrow L-T_2(i)$ so $L-T_2(iv) \not\Rightarrow L-T_2(i)$.

$L-T_2(v) \not\Rightarrow L-T_2(i)$: Since $L-T_2(iv) \Leftrightarrow L-T_2(v)$ and $L-T_2(iv) \not\Rightarrow L-T_2(i)$ so $L-T_2(v) \not\Rightarrow L-T_2(i)$. But $L-T_2(vii) \Rightarrow L-T_2(vi) \Rightarrow T_2(i)$ is obvious.

The authors showed that all definitions $L - T_2(i)$, $L - T_2(ii)$, $L - T_2(iii)$, $L - T_2(iv)$, $L - T_2(v)$, $L - T_2(vi)$ and $L - T_2(vii)$ are 'good extensions' of T_2, as shown below:

Theorem: Let (X, T) be a topological space. Then (X, T) is T_2 iff (X, μ (T)) is $L - T_2(j)$, where $j = i, ii, iii, iv, v, vi, vii$.

Proof: Let (X, T) be T_2. Choose $x, y \in X$ with $x \neq y$. Then $\exists$ $U, V \in T$ such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$ and $U \cap V = \varnothing$. Now consider the lower semi continuous function 1_U, 1_V. Then 1_U, $1_V \in \mu$ (T) such that $1_U(x) = 1$, $1_U(y) = 0$ and $1_V(x) = 1$, $1_v(y) = 1$ and so that$1_U \cap 1_V = 0$. Thus iff (X, μ (T)) is $L - T_2(i)$.

Conversely, let (X, μ (T)) be $L - T_2(i)$. To show that (X, T) is T_2. Choose $x, y \in X$ with $x \neq y$. Then $\exists u, v \in \mu$ (T) such that $u(x) = 1$, $u(y) = 0$, $v(x) = 0$, $v(y) = 1$ and $u \cap v = 0$. Let $u(y) < u(x)$ and $v(x) < v(y)$. Choose r such that $u(y) < r < u(x)$ and $v(x) < r < v(y)$ and consider u^{-1} $(r, 1]$ and v^{-1} $(r, 1]$. Then u^{-1} $(r, 1]$, v^{-1} $(r, 1] \in T$ and is $x \notin u^{-1}$ $(r, 1]$, $y \notin u^{-1}$ $(r, 1]$, $x \in v^{-1}$ $(r, 1]$, $y \notin v^{-1}$ $(r, 1]$ and v^{-1} $(r, 1] \cap v^{-1}$ $(r, 1] = \varnothing$ as $u \cap v = 0$. Hence (X, T) is T_2.

Similarly the authors can easily show that $L - T_2(ii)$, $L - T_2(iii)$, $L - T_2(iv)$, $L - T_2(v)$, $L - T_2(vi)$, $L - T_2(vii)$, are also hold 'good extension' property.

Theorem: Let (X, τ) be an lts, $A \subseteq X$ and ²$_A$ = $\{u \mid A : u \in$ ²$\}$, then

(X, ²) is $L - T_2(i) \Rightarrow$ (A, ²$_A$) is $L - T_2(i)$

(X, ²) is $L - T_2(ii) \Rightarrow$ (A, ²$_A$) is $L - T_2(ii)$

(X, ²) is $L - T_2(iii) \Rightarrow$ (A, ²$_A$) is $L - T_2(iii)$

(X, ²) is $L - T_2(iv) \Rightarrow$ (A, ²$_A$) is $L - T_2(iv)$

(X, τ) is $L - T_2(v) \Rightarrow (A, {}^2{}_A)$ is $L - T_2(v)$

(X, τ) is $L - T_2(vi) \Rightarrow (A, {}^2{}_A)$ is $L - T_2(vi)$

(X, τ) is $L - T_2(vii) \Rightarrow (A, {}^2{}_A)$ is $L - T_2(vii)$.

Proof: The authors proved only (a). Suppose (X, τ) is L-topological space and $L - T_2(i)$.

We shall prove $(A, {}^2{}_A)$ is $L - T_2(i)$. Let $x, y \in A$ with $x \neq y$, then $x, y \in X$ with $x \neq y$ as $A \subseteq X$. Since $(X, {}^2\,)$ is $L - T_2(i)$, $\exists u, v \in {}^2$ such that $u(x) = 1$, $u(y) = 0$, $v(x) = 0$, $v(y) = 1$ and $u \cap v = 0$. For $A \subseteq X$ we find $u \mid A, v \mid A \in {}^2{}_A$ and $u \mid A(x) = 1$, $u \mid A(y) = 0$ and $v \mid A(x) = 0$, $v \mid A(y) = 1$ and $u \mid A \cap v \mid A = (u \cap v) \mid A = 0$ as $x, y \in A$. Hence it is clear that the subspace $(A, {}^2{}_A)$ is $L - T_2(i)$.

Similarly, (b), (c), (d), (e), (f), (g) can be proved.

Theorem: Given $\{(X_i, {}^2{}_i) : i \in \Delta\}$ be a family of L-topological space. Then the product of L-topological space $(X_i, \Pi{}^2{}_i)$ is $L - T_2\,(j)$ iff each coordinate space $(X_i, {}^2{}_i)$ is $L - T_2(j)$, where $j = i, ii, iii, iv, v, vi, vii$.

Proof: Let each coordinate space $\{(X_i, {}^2{}_i) : i \in \Delta\}$ be $L - T_2(i)$. Then the writers showed that the product space is $L - T_2(i)$. Suppose $x, y \in X$ with $x \neq y$, again suppose $x = \Pi x_i$, $y = \Pi y_i$ then $x_j \neq y_j$ for some $j \in \Delta$. Now consider $x_j, y_j \in X_j$. Since $(X_i, {}^2{}_i)$ is $L - T_2(i)$, $\exists u_j, v_j \in {}^2{}_j$ such that $u_j(x) = 1$, $u_j\,(y_j) = 0$, $v_j(x) = 1$, $v_j\,(y_j) = 1$ and $u_j \cap v_j = 0$. Now take $u = \Pi u'_j$, $v = \Pi v'_j$ where $u'_j = u_j$, $v'_j = v_j$ and $u_i = v_i = 1$ for $i \neq j$. Then $u, v \in \Pi{}^2{}_i$ such that $u(x) = 1$, $u(y) = 0$, $v(x) = 0$, $v(y) = 1$ and $u \cap v = 0$. Hence the product L-topological space $(\Pi X_i, \Pi{}^2{}_i)$ then is $L - T_2(i)$.

Conversely, let the product L-topological space $(\Pi X_i, \Pi{}^2{}_i)$ is $L - T_2(i)$. Take any coordinate space $(X_j, {}^2{}_j)$, choose $x_j, y_j \in X_j$, $x_j \neq y_j$. Now construct $x, y \in X$ such that $x = \Pi x'_i$, $y = \Pi y'_i$ where x'_i, y'_i for $i \neq j$ and $x'_j = y'_j$, $y'_j = y_j$. Then $x \neq y$ and using the product space $L - T_2(i)$ $\exists$ $u, v \in \Pi{}^2{}_i$ such that $u(x) = 1$, $u(y) = 0$, $v(x) = 0$, $v(y) = 1$ and $u \cap v = 0$. Now choose any L-fuzzy point x_r in u. Then $\exists$ a basic open L-fuzzy set $\prod u_j^r \in \prod r_j$ such that $x_r \in \prod u_j^r \subseteq u$ which implies that $r < \prod u_j^r(x)$ or that $r < in\,f_j u_j^r(x'_j)$ and hence $r < \prod u_j^r(x'_j) \forall j \in \Lambda$ ... (i) and $u(y) = 0 \Rightarrow \Pi u_j(y) = 0$ ... (ii). Similarly, corresponding to a fuzzy point $y_s \in v$ there exists a basic fuzzy open set $\prod v_j^s \in \prod r_j$ such that $y_s \in \prod v_j^s \subseteq v$ which implies that $s < v_j^s(j) \forall j \in \Lambda$ ... (iii) and $\prod v_j^s(y) = 0$ ... (iv). Further, $\prod u_j^r(y) = 0 \Rightarrow u_j^r(y_i) = 0$, since for $j \neq i$, $x'_j = y'_j$ and hence from (i), $u_j^r(y_j) = u_j^r(x_j) > r$. Similarly, $\prod v_j^s(x) = 0 \Rightarrow v_i^s(x_i) = 0$ using (iii). Thus we have $u_i^r(x_i) > r$, $u_i^r(y_i) = 0$ and $v_i^s(y_i) > s, v_i^s(x_i) = 0$. Now consider $\sup_r u_i^r = u_i$, $\sup_s v_i^s = v_i \in \tau_i$ then $u_i(x_i) = 1$, $u_i(y_i) = 0$, $v_i(x_i) = 0$, $v_i(y_i) = 1$ and $u_i \cap v_i = 0$ showing that (X_i, τ_i) is $L - T_2(i)$.

Moreover one can easily verify that

(X, τ), $i \in \Delta$ is $L - T_2(ii) \Leftrightarrow (\Pi X_i, \Pi\ {}^{2}{}_{i})$ is $L - T_2(ii)$.

(X, τ), $i \in \Delta$ is $L - T_2(iii) \Leftrightarrow (\Pi X_i, \Pi\ {}^{2}{}_{i})$ is $L - T_2(iii)$.

(X, τ), $i \in \Delta$ is $L - T_2(iv) \Leftrightarrow (\Pi X_i, \Pi\ {}^{2}{}_{i})$ is $L - T_2(iv)$.

(X, τ), $i \in \Delta$ is $L - T_2(v) \Leftrightarrow (\Pi X_i, \Pi\ {}^{2}{}_{i})$ is $L - T_2(v)$.

(X, τ), $i \in \Delta$ is $L - T_2(vi) \Leftrightarrow (\Pi X_i, \Pi\ {}^{2}{}_{i})$ is $L - T_2(vi)$.

(X, τ), $i \in \Delta$ is $L - T_2(vii) \Leftrightarrow (\Pi X_i, \Pi\ {}^{2}{}_{i})$ is $L - T_2(vii)$.

Hence it is seen that

$L - T_2(i)$, $L - T_2(ii)$, $L - T_2(iii)$, $L - T_2(iv)$, $L - T_2(v)$, $L - T_2(vi)$, $L - T_2(vii)$ properties are productive and projective.

MAPPING OF L-TOPOLOGICAL SPACES

The workers showed that $L - T_2(j)$ property is preserved under one-one, onto and continuous maps for $j = i, ii, iii, iv, v, vi, viii$.

Theorem: Let $(X, {}^{2})$, and (Y, s) be two L-topological space and f : $(X, {}^{2}) \rightarrow (Y, s)$ be one-one, onto and L-open map, then-

(a) (X_i, τ_i) is $L - T_2(i) \Rightarrow (Y, s)$ is $L - T_2(i)$.

(b) (X_i, τ_i) is $L - T_2(ii) \Rightarrow (Y, s)$ is $L - T_2(ii)$.

(c) (X_i, τ_i) is $L - T_2(iii) \Rightarrow (Y, s)$ is $L - T_2(iii)$.

(d) (X_i, τ_i) is $L - T_2(iv) \Rightarrow (Y, s)$ is $L - T_2(iv)$.

(e) (X_i, τ_i) is $L - T_2(v) \Rightarrow (Y, s)$ is $L - T_2(v)$.

(f) (X_i, τ_i) is $L - T_2(vi) \Rightarrow (Y, s)$ is $L - T_2(vi)$.

(g) (X_i, τ_i) is $L - T_2(vii) \Rightarrow (Y, s)$ is $L - T_2(vii)$.

Proof: Suppose (X, τ) is $L - T_2(i)$. Then the writers shall prove that (Y, s) is $L - T_2(i)$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, $\exists x_1, x_2 \in X$ such that $f(x_1) = y_1$, $f(x_2) = y_2$ and $x_1 \neq x_2$ as f is one-one. Again since (X, τ) is $L - T_2(i)$ $\exists u, v \in {}^{2}$ such that $u(x_1) = 1$, $u(x_2) = 0$, $v(x_1) = 1$, $v(x_2) = 1$ and $u \cap v = 0$. Now

$f(u)(y_1) = \{\sup u(x_1) : f(x_1) = y_1\} = 1$

$f(u)(y_2) = \{\sup u(x_2) : f(x_2) = y_2\} = 0$

$f(v)(y_1) = \{\sup v(x_1) : f(x_1) = y_1\} = 0$

$f(v)(y_2) = \{\sup v(x_2) : f(x_2) = y_1\} = 1$

and

$f(u \cap v)(y_1) = \{\sup (u \cap v)(x_1) : f(x_1) = y_1\}$

$f(u \cap v)(y_2) = \{\sup (u \cap v)(x_2) : f(x_2) = y_2\}$

Hence $f(u \cap v) = 0 \Rightarrow f(u) \cap f(v) = 0$

Since f is L-open, $f(u), f(v) \in s$. Now it is clear that $\exists\, f(u), f(v) \in s$ such that $f(u)(y_1) = 1$, $f(u)(y_2) = 0$, $f(v)(y_1) = 0$ $f(v)(y_2) = 1$, and $f(u) \cap f(v) = 0$. Hence it is clear that the L-topological space (Y, s) is $L - T_2(i)$.

Similarly (b), (c), (d), (e), (f), (g) can be easily proved.

Theorem: Let $(X, {}^{2})$ and (Y, s) be two L-topological space and $f : (X, {}^{2}) \rightarrow (Y, s)$ be L-continuous and one-one map, then-

(a) (Y, s) is $L - T_2(i) \Rightarrow (X, {}^{2})$ is $L - T_2(i)$.

(b) (Y, s) is $L - T_2(ii) \Rightarrow (X, {}^{2})$ is $L - T_2(ii)$.

(c) (Y, s) is $L - T_2(iii) \Rightarrow (X, {}^{2})$ is $L - T_2(iii)$.

(d) (Y, s) is $L - T_2(iv) \Rightarrow (X, {}^{2})$ is $L - T_2(iv)$.

(e) (Y, s) is $L - T_2(v) \Rightarrow (X, {}^{2})$ is $L - T_2(v)$.

(f) (Y, s) is $L - T_2(vi) \Rightarrow (X, {}^{2})$ is $L - T_2(vi)$.

(g) (Y, s) is $L - T_2(vii) \Rightarrow (X, {}^{2})$ is $L - T_2(vii)$.

Proof: Suppose (Y, s) is $L - T_2(i)$. The authors shall prove that $(X, {}^{2})$ is $L - T_2(i)$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ as f is one-one. Since (Y, s) is $L - T_2(i)$ $\exists u, v \in s$ such that $u(f(x_1)) = 1$, $u(f(x_2)) = 0$, $v(f(x_1)) = 0$ $v(f(x_2)) = 1$ and $u \cap v = 0$. This implies that $f^{-1}(u)(x_1) = 1$, $f^{-1}(u)(x_2) = 0$, $f^{-1}(v)(x_1) = 0$, $f^{-1}(v)(x_2) = 1$ and $f^{-1}(v)(x_1) = 0 \Rightarrow f^{-1}(u) \cap f^{-1}(v) = 0$. Hence $f^{-1}(u)$, $f^{-1}(v) \in {}^{2}$ as f is L-continuous and $u, v \in s$. Now it is clear that $f^{-1}(u)$, $f^{-1}(v) \in {}^{2}$ such that $f^{-1}(u)(x_1) = 1$, $f^{-1}(u)(x_2) = 0$, $f^{-1}(v)(x_1) = 0$, $f^{-1}(v)(x_2) = 1$, and $f^{-1}(u) \cap f^{-1}(v) = 0$. Hence the L-topological space $(X, {}^{2})$ is $L - T_2(i)$.

Similarly (b), (c), (d), (e), (f), (g) can be proved.

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(*Received revised manuscript on 20 April, 2015*)