J. Bangladesh Acad. Sci., Vol. 39, No. 2, 223-231, 2015

PRODUCTS AND MAPPINGS OF INTUITIONISTIC FUZZY T_0, T_1 AND T_2-SPACES

ESTIAQ AHMED*, M. S. HOSSAIN[1] AND D. M. ALI[1]

Department of Mathematics, Meherpur Government College, Meherpur-7100, Bangladesh

ABSTRACT

The basic concepts of the theory of intuitionistic fuzzy topological spaces have been defined by some workers. In this paper, the product and mappings of T_0, T_1 and T_2-spaces have been studied.

Key words: Intuitionistic fuzzy sets, Fuzzy topological spaces, Fuzzy mappings, Fuzzy product spaces

INTRODUCTION

After the introduction of fuzzy sets by Zadeh (1965), mathematicians introduced generalizations of the notion of fuzzy set. Among others, Atanassov (1984, 1986) introduced the notion of intuitionistic fuzzy set. Chang (1968) used fuzzy sets to introduce the concept of a fuzzy topology. Later this concept was extended to intuitionistic fuzzy spaces by Coker (1997). In this paper, we investigate the properties and features of T_0, T_1 and T_2-spaces.

Definition 1.1. Let X be a non-empty set and I be the unit interval [0, 1]. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$, where $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership and the degree of non-membership, respectively and $\mu_A(X) + \nu_A(x) \leq 1$.

Let I(X) denote the set of all intuitionistic fuzzy sets in X. Obviously every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form $(\mu_A, -\mu_A)$.

Throughout this paper, the authros used the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$ (Atanassov 1986).

Definition 1.2. Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP in short) of X is an IFS of X defined by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if } y = x \\ (0, 1) & \text{if } y \neq x \end{cases}$$

* Corresponding author: < estiaq1977@gmail.com >

[1] Department of Mathematics, Rajshahi University, Rajshahi-6205, Bangladesh.

In this case, x is called the support of $x_{(\alpha,\beta)}$ and α and β are called the value and non-value of $x_{(\alpha,\beta)}$, respectively. An IFP $x_{(\alpha,\beta)}$ is said to belong to an IFS $A = (\mu_A, \nu_A)$ of X, denoted by $x_{(\alpha,\beta)} \in A$, if $\alpha \leq \mu_A(x)$, $\beta \geq \nu_A(x)$ (Lee 2004)

Definition 1.3. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \leq \nu_B$.

(2) $A^c = (\nu_A, \mu_A)$.

(3) $A \cap B = (\mu_A \cap \mu_A; \nu_A \cup \nu_A)$.

(4) $A \cup B = (\mu_A \cup \mu_A; \nu_A \cap \nu_A)$.

(5) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$ (Atanassov 1986).

Definition 1.4 Let $\{A_i : i \in J\}$ be an arbitrary family of IFSs in X. Then

(a) $\cap A_i = (\cap \mu_{A1}, \cup \nu_{A1})$.

(b) $\cup A_i = (\cup \mu_{A1}, \cap \nu_{A1})$ (Coker 1997)

Definition 1.5. An intuitionistic fuzzy topology (IFT in short) on X is a family t of IFS's in X which satisfies the following axioms:

(1) $0_\sim = 1_\sim \in t$.

(2) if $A_1, A_2 \in t$, then $A_1 \cap A_2 \in t$.

(3) if $A_i \in t$ for each i, then $\cup A_i \in t$.

The pair (X, t) is called an intuitionistic fuzzy topological space (IFTS, in short), Let (X, t) be an IFTS. Then any member of t is called an intuitionistic fuzzy open set (IFOS, in short) in X. The complement of an IFOS in X is called an intuitionistic fuzzy closed set (IFCS, in short) in X.

Let I (X) denote the set of all intuitionistic fuzzy sets in X. Obviously every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form $(\mu_A, 1 - (\mu_A)$.

Throughout this paper, the authors used the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ (Coker 1997)

Definition 1.6. Let X and Y be two nonempty sets and $f : X \to Y$ be a function $B = \{(y, \mu_B(y), \nu_B(y)) \mid y \in Y\}$ is an IFS in Y, then the pre-image of B under f, denoted by f^{-1} (B) is the IFS in X defined by $f^{-1}(B) = \{(x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)) \mid x \in X\}$ and the image of A under f, denoted by $f(A) = \{(y, f(\mu_A), f(\nu_A) \mid y \in Y\}$, is an IFS of Y, for each $y \in Y$.

$$f(\mu_A)(y) = \begin{cases} x \in f^{-1} \overset{\sup}{\underset{0}{y}} \mu_A(x) \text{ if } f^{-1}(y) \neq \phi \\ \text{otherwise} \end{cases}$$

$$f(\nu_A)(y) = \begin{cases} x \in f^{-1} \overset{\sup}{\underset{1}{y}} \nu_A(x) \text{ if } f^{-1}(y) \neq \phi, \\ \text{otherwise} \end{cases}$$ (Atanassov 1986).

Definition 1.7. Let A = (X, μ_A, ν_A) and B = (Y, μ_B, ν_B) be IFS of X and Y respectively. Then the product of intuitionistic fuzzy sets A and B denoted by A × B is defined by A × B = {X × Y, μ $\mu_A \times \mu_B$, $\nu_A \times \nu_B$}, where $(\mu_A \times \mu_B)(x, y) = \min(\mu_A(x), \mu_B(y))$ and $(\nu_A \times \nu_B)(x, y) = \max(\nu_A(x), \nu_B(y))$ for all $(x, y) \in X \times Y$. Obviously, $0 \leq \mu_A \times \mu_B + \nu_A \times \nu_B \leq 1$.

This definition can be extended to an arbitrary family of IFS as follows:

If Ai = $((\mu_{A1}, \nu_{A1}), i \in J)$ is a family of IFS in X_i then their product is defined as the IFS in ΠX_i given by $\Pi A_i = (\Pi\mu_{A1}, \Pi\nu_{A1})$ where $\Pi\mu_{A1}(x) = \inf \mu_{A1}(x_i)$, for all $x = \Pi X_i \in X$ and $\Pi\nu_{A1}(x) = \sup \nu_{Ai}(x_i)$, for all $x = \Pi X_i \in X$ (Bayhan 1996).

Definition 1.8. Let (X_i, t_i), i = 1, 2 be two IFTSs, and then the product $t_1 \times t_2$ on $X_1 \times X_2$ is defined as the IFT generated by $\rho_i^{-1}(U_i) : U_i \in t_i$, i = 1, 2}, where $\rho_i : X_1 \times X_2 \to X_i$, i = 1, 2 are the projection maps and the IFTS $(X_1 \times X_2, t_1 \times t_2)$ is called product IFTS (Bayhan 1996).

2. Intuitionistic fuzzy T_0-spaces

Definition 2.1. An intuitionistic fuzzy topological space (X, t) is called

(1) IF-T_0(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) = 1$.

(2) IF-T_0(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) > 1$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) > 0$.

(3) IF-T_0(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$ or $\mu_A(y) > 0$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) = 1$.

(4) IF-T_0(iv) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) > 0$ or $\mu_A(y) > 0$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) = 0$ (Ahmed 2014).

Definition 2.2. Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α-IF-T_0(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ or $\mu_A(y) = 1$, $\nu_A(y) = 0$; $\mu_A(x) = 0$, $\nu_A(x) \geq \alpha$.

(b) α-IF-T_0(ii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) \geq \alpha, \nu_A(x) = 0; \mu_A(y) = 0, \nu_A(y) \geq \alpha$ or $\mu_A(y) \geq \alpha, \nu_A(y) = 0; \mu_A(x) = 0, \nu_A(x) \geq \alpha$.

(c) α-IF-T_0(iii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A) \in t$ such that $\mu_A(x) > 0, \nu_A(x) = 0; \mu_A(y) = 0, \nu_A(y) \geq \alpha$ or $\mu_A(y) > 0, \nu_A(y) = 0; \mu_A(x) = 0, \nu_A(x) \geq \alpha$ (Ahmed 2014).

Theorem 2.3. Let (X, t) and (Y, s) be two intuitionistic fuzzy topological space and $f : X \rightarrow Y$ be one-one, onto, continuous open mapping, then

(1) (X, t) is IF-T_0(i) $\Leftrightarrow$ (Y, s) is IF-T_0(i).

(2) (X, t) is IF-T_0(ii) $\Leftrightarrow$ (Y, s) is IF-T_0(ii).

(3) (X, t) is IF-T_0(iii) $\Leftrightarrow$ (Y, s) is IF-T_0(iii).

(4) (X, t) is IF-T_0(iv) $\Leftrightarrow$ (Y, s) is IF-T_0(iv).

(5) (X, t) is α-IF-T_0(i) $\Leftrightarrow$ (Y, s) is α-IF-T_0(i).

(6) (X, t) is α-IF-T_0(ii) $\Leftrightarrow$ (Y, s) is α-IF-T_0(ii).

(7) (X, t) is α-IF-T_0(iii) $\Leftrightarrow$ (Y, s) is α-IF-T_0(iii).

Proof (1). Suppose the IFTS (X, t) is IF-T_0(i). The authors shall show that the IFTS (Y, s) is IF-T_0(i). Let $y_1, y_2 \in Y, y_1 \neq y_2$. Since *f* is onto, then there exists $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_x) = y_2$. Again, since $y_1 \neq y_2$, then $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto. Hence $y_1 \neq y_2$. Therefore, since (X, t) is IF-T_0(i), then there exists $A = (\mu_A, \nu_A) \in t$ such that $\{\mu_A(x_1) = 1, \nu_A(x_1) = 0; \mu_A(x_2) = 0, \nu_A(x_2) = 1\}$ or $\{\mu_A(x_2) = 1, \nu_A(x_2) = 0; \mu_A(x_1) = 0, \nu_A(x_1) = 1\}$. Suppose $\{\mu_A(x_1) = 1, \nu_A(x_1) = 0; \mu_A(x_2) = 0, \nu_A(x_2) = 0$. Now $\{(f(\mu_A)) (y_1) = \mu_A(f^{-1}(y_1)) = \mu_A(x_1) = 1, \{(f(\nu_A)) (y_1) = \nu_A(f^{-1}(y_1)) = \nu_A(x_1) = 0\};\{\{(f(\mu_A)) (y_2) = \mu_A(f^{-1}(y_2)) = \mu_A(x_2) = 0, \{(f(\nu_A)) (y_2) = \nu_A(f^{-1}(y_2)) = \nu_A(x_2) = 1\}$. Since f is IF-continuous, then $(f(\mu_A), (f(\nu_A) \in s$ with $(f(\mu_A)) (y_1) = 1, (f(\mu_A)) (y_1) = 0; (f(\mu_A)) (y_1) = 0, (f(\mu_A)) (y_1) = 1$. Therefore, the IFTS (Y, s) is IF-T_0(i).

Conversely, suppose the IFTS (Y, s) is IF-T_0(i). The authors shall show that the IFTS (X, t) is IF-T_0(i). Let $x_1, x_2 \in X$ with $y_1 \neq y_2$. Since f is one-one, then there exists $y_1, y_2 \in Y$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ and $f(x_1) \neq f(x_2)$. That is, $y_1 \neq y_2$. Again, since (Y, s) is IF-T_0(i), then there exists $B = (\mu_B, \nu_B) \in s$ such that $\{\mu_B(y_1) = 1, \nu_B(y_1) = 0; \mu_B(y_2) = 0, \nu_B(y_2) = 1\}$ or $\{\mu_B(y_2) = 1, \nu_B(y_2) = 0; \mu_B(y_1) = 0, \nu_B(y_1) = 1\}$

Suppose $\{\mu_B(y_1) = 1, \nu_B(y_1) = 0; \mu_B(y_2) = 0, \nu_B(y_2) = 1$. Now, $\{(f^{-1}(\mu_B) (x_1) = \mu_B(f(x_1)) = \mu_B(x_1) = 1, \{(f^{-1}(\nu_B) (x_1) = \nu_B(f(x_1)) = \mu_B(y_1) = 0\}; \{(f^{-1}(\mu_B) (x_2) = \mu_B(f(x_2)) = \mu_B(x_2) = 0, \{(f^{-1}(\mu_B) (x_2) = \mu_B(f(x_2)) = \mu_B(x_2) = 1\}$. Since f is IF-continuous, then $(f^{-1}(\mu_B), (f^{-1}(\nu_B) \in t$ with $f^{-1}(\mu_B) (x_1) = 1, f^{-1}(\nu_B) (x_2) = 0, f^{-1}(\nu_B) (x_2) = 1$. Therefore, the IFTS (X, t) is IF-T_0(i).

(2), (3), (4), (5), (6), (7) can be proved in the similar way.

Theorem 2.4. Let $\{(X_m, t_m) : m \in J\}$ be a family of intuitionistic fuzzy topological spaces and let (X, t) be their product IFTS. Then the product IFTS $(\Pi X_m, \Pi t_m)$ is IF-T_0(i) if each (X_m, t_m) is IF-T_0(i).

Proof: Suppose (X_m, t_m) is IF-T_0(i) for all $m \in J$. The authors shall show that the product space (X, t) is IF-T_0(i). Choose $x, y \in X$, $x \neq y$. Let $x = \Pi X_m$, $y = \Pi y_m$. Then there exists j = J such that $x_j \neq y_j$. Now, since (X_j, t_j) is IF-T_0(i), then there exists Aj $(\mu_{Aj}, \nu_{aj}) \in t$ such that $\{\mu_{Aj}(x_j) = 1, \nu_{aj}(x_j) = 0; \mu_{Aj}(y_j) = 0, \nu_{aj}(x_j) = 1\}$ or $\{\mu_{Aj}(y_j) = 1, \nu_{aj}(y_j) = 0; \mu_{Aj}(y_j) = 0, \nu_{aj}(y_j) = 1\}$. Suppose $\mu_{Aj}(x_j) = 1, \nu_{aj}(x_j) = 0; \mu_{Aj}(y_j) = 0, \nu_{aj}(x_j) = 1$.

Now consider the basic IFOSs $\Pi A_k \in \Pi t_k$ where $A_k = (1^{\sim}, 0^{\sim})$ for $k \in J$, $k \neq j$ and $A_k = A_j$ when k = j. Then $\Pi A_k(x) = \left(\overset{\inf}{k \in J}\, \mu_{A_k}(x_k)\right), \overset{\sup}{k \in J}\, \nu_{A_k}(x_k)) = (1, 0)$; $\Pi A_k(y) = \left(\overset{\inf}{k \in J}\, \mu_{A_k}(y_k)\right), \overset{\sup}{k \in J}\, \nu_{A_k}(y_k)) = (0, 1)$. Hence (X, t) is IF-$T_0$(i).

Theorem 2.5. Let $\{(X_m, T_m) : m \in J\}$ be a family of IFTS and (X, t) be their product IFTS. Then the product IFTS $(\Pi X_m, \Pi t_m)$ is IF-T_0(n) if each (X_m, T_m) is IF-T_0(n), n = ii, iii, iv.

Proof: The above theorem can be proved in the similar way.

Theorem 2.6. Let $\{(X_m, T_m) : m \in J\}$ be a family of IFTS and let (X, t) be their product. Then the product IFTS $(\Pi X_m, \Pi t_m)$ is α-IF-T_0(n) if each IFTS (X_m, T_m) is α-IF-T_0(n), n = i, ii, iii.

Proof: The above theorem can be proved in the similar way.

3. Intuitionistic fuzzy T_1-spaces

Definition 3.1. An intuitionistic fuzzy topological space (X, t) is called

(1) IF-T_1(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1, \nu_A(x) = 0; \mu_A(y) = 1, \nu_A(y) = 1$ and $\mu_B(y) = 1, \nu_B(y) = 0; \mu_B(x) = 0, \nu_B(x) = 1$.

(2) IF-T_1(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1, \nu_A(x) = 0; \mu_A(y) = 0, \nu_A(y) > 0$ and $\mu_B(y) = 1, \nu_B(y) = 0; \mu_B(x) = 0, \nu_B(x) > 0$.

(3) IF-T_1(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0, \nu_A(x) = 0; \mu_A(y) = 0, \nu_A(y) = 1$ and $\mu_B(y) > 0, \nu_B(y) = 0; \mu_B(x) = 0, \nu_B(x) = 1$.

(4) IF-T_1(iv) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0, \nu_A(x) = 0; \mu_A(y) = 0, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) = 0; \mu_B(x) = 0, \nu_B(x) > 0$ (Ahmed 2014).

Definition 3.2. Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α-IF-T_1(i) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$.

(b) α-IF-T_1(ii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) \geq \alpha$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) \geq \alpha$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$.

(c) α-IF-T_1(iii) if for all $x, y \in X$, $x \neq y$ there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) = 0$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$ (Ahmed 2014).

Theorem 3.3. Let (X, t) and (Y, s) be two intuitionistic fuzzy topological space and $f : X \to Y$ be one-one, onto, continuous open mapping, then

(1) (X, t) is IF-T_1(i) $\Leftrightarrow$ (Y, s) is IF-T_1(i).

(2) (X, t) is IF-T_1(ii) $\Leftrightarrow$ (Y, s) is IF-T_1(ii).

(3) (X, t) is IF-T_1(iii) $\Leftrightarrow$ (Y, s) is IF-T_1(iii).

(4) (X, t) is IF-T_1(iv) $\Leftrightarrow$ (Y, s) is IF-T_1(iv).

(5) (X, t) is α-IF-T_1(i) $\Leftrightarrow$ (Y, s) is α-IF-T_1(i).

(6) (X, t) is α-IF-T_1(ii) $\Leftrightarrow$ (Y, s) is α-IF-T_1(ii).

(7) (X, t) is α-IF-T_1(iii) $\Leftrightarrow$ (Y, s) is α-IF-T_1(iii).

Theorem 3.4. Let $\{(X_m, T_m) : m \in J\}$ be a family of IFTS and let (X, t) be their product IFTS. Then the product IFTS $(\Pi X_m, \Pi t_m)$ is IF-T_1(i) if each (X_m, T_m) is IF-T_1(i).

Proof: Suppose (X_m, T_m) is IF-T_1(i) for all $m \in J$. The authors shall show that the product space (X, t) is IF-T_1(i). Choose, $y \in X$, $x \neq y$. Let $x = \Pi x_m$, $y = \Pi y_m$), then there exists $j \in J$ such that $x_j \neq y_j$. Now, since (X_j, t_j) is IF-T_1(i), then there exists $A_j = (\mu_{A_j}, \nu_{A_j})$, $B_j = (\mu_{B_j}, \nu_{B_j}) \in t_j$ such that $(\mu_{A_j}(x_j) = 1$, $\nu_{A_j}(x_j) = 0$; $\mu_{A_j}(y_j) = 0$, $\nu_{A_j}(y_j) \geq 1$ and $\mu_{B_j}(y_j) = 0$, $\nu_{B_j}(y_j) = 0$; $\mu_{B_j}(x_j) = 0$, $\nu_{B_j}(x_j) \geq \alpha)$. Now consider the basic IFOSs ΠA_k and ΠB_k where $A_k = (1^\sim, 0^\sim)$, $B_k = (1^\sim, 0^\sim)$ for $k \in J$, $k \neq j$ and $A_k = A_j$, $B_k = B_j$ when $k = j$. Then

$$\{\prod A_k(x) = (\inf_{k \in J} \mu_{A_k}(x_k), \sup_{k \in J} \nu_{A_k}(x_k)) = (1, 0);\ \prod B_k(x) = (\inf_{k \in J} \mu_{B_k}(y_k),$$

$$\sup_{k \in J} \nu_{B_k}(y_k)) = (0, 1)\} \text{ and } \{\prod B_k(y) = (\inf_{k \in J} \mu_{B_k}(x_k), \sup_{k \in J} \nu_{B_k}(x_k)) = (1, 0);$$

$$\{\prod B_k(x) = (\inf_{k \in J} \mu_{B_k}(x_k), \sup_{k \in J} \nu_{B_k}(y_k)) = (0, 1)\}.$$ Hence (X, t) is IF-T_1(i).

Theorem 3.5. Let $\{(X_m, t_m) : m \in J\}$ be a family of IFTS and (X, t) be their product IFTS. Then the product IFTS $(\Pi X_m, Xt_m)$ is IF-T_1(n) if each (X_m, t_m) is IF-T_1(n), n = ii, iii, iv.

Proof: The above theorem can be proved in the similar way.

Theorem 3.6. Let $\{(X_m, t_m) : m \in J\}$ be a family of IFTSs and let (X, t) be their product IFTS. Then the product IFTS $(\Pi X_m, Xt_m)$ is α-IF-T_1(n) if each (X_m, t_m) is α-IF-T_1(n), n = i, ii, iii.

Proof: The above theorem can be proved in the similar way.

4. Intuitionistic fuzzy T_2-spaces

Definition 4.1. An intuitionistic fuzzy topological space (X, t) is called

(1) IF-T_2(i) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $(\mu_A(x) = 1, \nu_A(x) = 0; \mu_B(y) = 0, \nu_B(y) = 0$ and $A \cup B = 0_{\sim}$.

(2) IF-T_2(ii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $(\mu_A(x) = 1, \nu_A(x) = 0; \mu_B(y) > 0, \nu_B(y) = 0$ and $A \cup B = (0^{\sim}, y^{\sim})$ where $\gamma \in (0, 1]$.

(3) IF-T_2(iii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $(\mu_A(x) > 0, \nu_A(x) = 0; \mu_B(y) = 1, \nu_B(y) = 0$ and $A \cup B = (0^{\sim}, y^{\sim})$ where $\gamma \in (0, 1]$.

(4) IF-T_2(iv) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $(\mu_A(x) > 0, \nu_A(x) = 0; \mu_B(y) > 0, \nu_B(y) = 0$ and $A \cup B = (0^{\sim}, y^{\sim})$ where $\gamma \in (0, 1]$ (Ahmed 2014).

Definition 4.2. Let $\alpha \in (0, 1)$. An intuitionistic fuzzy topological space (X, t) is called

(a) α-IF-T_2(i) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $(\mu_A(x) = 1, \nu_A(x) = 0; \mu_B(y) \geq \alpha, \nu_B(y) = 0$ and $A \cup B = 0_{\sim}$.

(b) α-IF-T_2(ii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $(\mu_A(x) \geq \alpha, \nu_A(x) = 0; \mu_B(y) \geq \alpha, \nu_B(y) = 0$ and $A \cup B = (0^{\sim}, y^{\sim})$ where $\gamma \in (0, 1]$.

(c) α-IF-T_2(iii) if for all $x, y \in X, x \neq y$ there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $(\mu_A(x) > 0, \nu_A(x) = 0; \mu_B(y) \geq \alpha, \nu_B(y) = 0$ and $A \cup B = (0^{\sim}, y^{\sim})$ where $\gamma \in (0, 1]$ (Ahmed 2014).

Theorem 4.3. Let (X, t) and (Y, s) be two intuitionistic fuzzy topological space and $f : X \to Y$ be one-one, onto, continuous open mapping, then

(1) (X, t) is IF-T_2(i) $\Leftrightarrow$ (Y, s) is IF-T_2(i).

(2) (X, t) is IF-T_2(ii) $\Leftrightarrow$ (Y, s) is IF-T_2(ii).

(3) (X, t) is IF-T_2(iii) $\Leftrightarrow$ (Y, s) is IF-T_2(iii).

(4) (X, t) is IF-T_2(iv) $\Leftrightarrow$ (Y, s) is IF-T_2(iv).

(5) (X, t) is α-IF-T_2(i) $\Leftrightarrow$ (Y, s) is α-IF-T_2(i).

(6) (X, t) is α-IF-T_2(ii) $\Leftrightarrow$ (Y, s) is α-IF-T_2(ii).

(7) (X, t) is α-IF-T_2(iii) $\Leftrightarrow$ (Y, s) is α-IF-T_2(iii).

Proof: The above theorem can be proved as the theorem 2.3.

Theorem 4.4. Let $\{(X_m, t_m) : m \in J\}$ be a family of IFTS and let (X, t) be their product IFTS. Then the IFTS $(\Pi X_m, \Pi t_m)$ is IF-T_2(i) if each (X_m, t_m) is IF-T_2(i).

Proof: Suppose (X_m, t_m) is IF-T_2(i) for all $m \in J$. The authors shall show that the product space (X, t) is IF-T_2(i). Choose, $y \in X$, $x \neq y$. Let $x = \Pi x_m$, $y = \Pi y_m$, then there exists $j \in J$ such that $x_j \neq y_j$. Now, since (X_j, t_j) is IF-T_2(i), then there exists $A_j = (\mu_{A_j}, \nu_{A_j})$, $B_j = (\mu_{B_j}, \nu_{B_j}) \in t_j$ such that $(\mu_{A_j}(x_j) = 1$, $\nu_{A_j}(x_j) = 0$; $\mu_{A_j}(y_j) = 1$, $\nu_{A_j}(y_j) = 0$ and $Aj \cap Bj = 0_\sim$. Now consider the basic IFOSs ΠA_k and ΠB_k where $A_k = (1^\sim, 0^\sim)$, $B_k = (1^\sim, 0^\sim)$ for $k \in J$, $k \neq j$ and $A_k = A_j$, $B_k = B_j$ when $k = j$. Then $\{\prod A_k(x) = (\overset{\inf}{k \in J}\, \mu_{A_k}(x_k), \overset{\sup}{k \in J}\, \nu_{A_k}(x_k)) = (1, 0); \prod B_k(x) = (\overset{\inf}{k \in J}\, \mu_{B_k}(y_k), \overset{\sup}{k \in J}\, \nu_{B_k}(y_k)) = (1, 0)\}$ and $\{\Pi A_k \cap \{\Pi B_k = 0_\sim$ because for $j \in J$, $A_j \cap B_j = 0_\sim$. Hence (X, t) is IF-T_1(i).

Theorem 4.5. Let $\{(X_m, t_m) : m \in J\}$ be a family of IFTSs and (X, t) be their product IFTS. Then the product IFTS $(\Pi X_m, \Pi t_m)$ is IF-T_2(n) if each (X_m, t_m) is IF-T_2(n), n = ii, iii, iv.

Proof: The above theorem can be proved in the similar way.

Theorem 4.6. Let $\{(X_m, t_m) : m \in J\}$ be a family of intuitionistic fuzzy topological space and let (X, t) be their product IFTS. Then the product IFTS IFTS $(\Pi X_m, \Pi t_m)$ is α-IF-T_2(n) if each (X_m, t_m) is α–IF–T_2(n), n = i, ii, iii.

Proof: The above theorem can be proved in the similar way.

REFERENCES

Ahmed, E., M. S. Hossain and D. M. Ali. 2014. On intuitionistic fuzzy T_0-spaces. *J. Bangladesh Acad. Sci.* **38**(2): 197-207.

Ahmed, E., M. S. Hossain and D. M. Ali. 2014. On intuitionistic fuzzy T_1-spaces. *Journal of Physical Sciences* **19**: 59-66.

Ahmed, E., M. S. Hossain and D. M. Ali. 2014. On intuitionistic fuzzy T_2-spaces. *IOSR Journal of Mathematics* **10**(6): 26-30.

Atanassov, K. T. 1986. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **20**: 87-96.

Bayhan, S. and D. Coker. 2003. On T_1 and T_2 separation axioms in intuitionistic fuzzy topological space. *J. Fuzzy Mathematics* **11**(3): 581-592.

Bayhan, S. and D. Coker. 1996. On fuzzy separation axioms in intuitionistic fuzzy topological space. *BUSEFAL* **67**: 77-87.

Chang, C. L. Fuzzy topological space. 1968. *J. Math. Anal. Appl*. **24**: 182-190

Coker, D. 1997. An introduction to intuitionistic fuzzy topological space. *Fuzzy Sets and Systems* **88**: 81-89.

Lee, S. j. and E. P. Lee. 2004. Intuitionistic Fuzzy proximity Spaces. *IJMMS* **49**: 2617-2628.

Singh, A. K. and R. Srivastava. 2012. Separation axioms in intuitionistic fuzzy topological spaces. *Advanced in Fuzzy Systems*. Article id604396.

Zadeh, L. A. 1965. Fuzzy Sets. *Information and Control* **8**: 338-353.

(*Received revised manuscript on 7 October, 2015*)