J. Bangladesh Acad. Sci., Vol. 39, No. 2, 249-255, 2015

DERIVATIONS ON LIE IDEALS OF ±:-PRIME d-RINGS

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ABSTRACT

The authors extend and generalize some results of previous workers to ±:-prime Γ-ring. For a ±:-square closed Lie ideal *U* of a 2-torsion free ±:-prime Γ-ring *M*, let $d: M \to M$ be an additive mapping satisfying *d*(*u*¥:*u*)=*d*(*u*)¥: *u* + *u*¥:*d*(*u*) for all $u \in U$ and ¥: ∈ Γ. The present authors proved that *d*(*u*¥:*v*) = *d*(*u*)¥:*v* + *u*¥:*d*(*v*) for all $u, v \in U$ and ¥:∈ Γ, and consequently, every Jordan derivation of a 2-torsion free ±:-prime Γ-ring *M* is a derivation of *M*.

Key words: Lie ideal, ±:-square closed Lie ideal, ±:-prime Γ-ring, Derivation

INTRODUCTION

Oukhtite and Salhi (2008) worked on left derivations of ±:-prime rings and proved that, if *U* is a nonzero σ:-square closed Lie ideal of a ring *R* then $U \subseteq Z(R)$, centre of *R* or $d(U) = 0$. They described additive mappings $d : R \to R$ such that $d(u^2) = 2u\, d(u) \forall u \in U$, where *U* is a nonzero ±:-square closed Lie ideal of a 2-torsion free σ:-prime ring *R* and proved that $d(uv) = ud(v) + vd(u)\ \forall u, v \in U$. Oukhtite *et al.* (2007) also studied Jordan generalized derivations of ±:-prime rings and proved that every Jordan generalized derivations on *U* of *R* is a generalized derivations on *U* of *R*, where *U* is a σ:-square closed Lie ideal of a 2-torsion free ±:-prime ring *R*. Some significant results developed on Lie ideals and generalized derivations in ±:-prime rings by Khan and Khan (2012). Some characterizations of centralizing automorphisms on a ±:-square closed Lie ideals of ±:-prime Γ-rings have been developed by Dey *et al.* (2015). They studied Jordan left derivations on a σ:-square closed Lie ideals and proved that such type of Jordan derivations is a derivation on a ±:-square closed Lie ideals of a ±:-prime Γ-ring. Paul and Chakraborty (2015) studied ±:-prime Γ-rings and proved that if a derivation *d* acting as homomorphism and an anti-homomorphism in a σ:-Lie ideal *U* of a σ:-prime Γ-ring *M*, then $d = 0$ or $U \subseteq Z(M)$. An example of an involution and an example of a ±:-prime Γ-ring which is not a prime Γ-ring appeared in Dey and Paul (2015). On the other hand, various remarkable characterizations of ±:-prime rings on σ:-square closed Lie ideals have been studied by many authors *viz*. Bergun (1981), Herstein (1969), Khan *et al.* (2010), Oukhtite and Salhi (2006), Paul and Rahman (2015).

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The authors proved that if $d : M \to M$ is an additive mapping satisfying *d*(*u*¥⁼*u*) = *d*(*u*)¥⁼*u* + *u*¥⁼*d*(*u*) for all $u \in U$ and ¥⁼∈ Γ then *d*(*u*¥⁼*v*) = *d*(*u*)¥⁼*v* + *u*¥⁼*d*(*v*) for all *u*, *v* ∈ *U* and ¥⁼ ∈ Γ, where *U* is a ±⁼-square closed Lie ideal of a 2-torsion free ±⁼-prime Γ-ring *M*, and hence every Jordan derivations on a ±⁼-prime Γ-ring *M* is a derivation on *M*.

Throughout this paper, the authors consider *M* an associative Γ-ring with centre *Z*(*M*). Define [*x*, *y*]¥⁼ = *x*¥⁼*y* – *y*¥⁼*x* which is known as the commutator of *x* and *y* with respect to ¥⁼. The authors assume the condition (*) *x*~*y* *z* = *x* *y*~*z* ∀ *x*, *y*, *z* ∈ *M* and ~ ∈ Γ. Using this condition the basic commutator identities become [*x* *y*, *xz*]¥⁼ = [*x*, *z*]¥⁼ *y* + *x* [*y*, *z*]¥⁼ and [*x*, *y* *z*]¥⁼ = [*x*, *y*]¥⁼ *z* + *y* [*x*, *z*]¥⁼ for all *x*, *y*, *z* ∈ *M* and , ¥⁼ ∈ Γ. An additive subgroup *U* of a Γ-ring *M* is called a Lie ideal if $[U, M]_\Gamma \subseteq U$. An additive mapping $d : M \to M$ is called a derivation if *d*(*a*¥⁼*b*) = *d*(*a*)¥⁼*b* + *a*¥⁼*d*(*b*)∀¥⁼, *b* ∈ *M*, ¥⁼ ∈ Γ and *d* is a Jordan derivation if *d*(*a*¥⁼*a*) = *d*(*a*)¥⁼*a* + *a*¥⁼*d*(a)∀ *a* ∈ *M*, ¥⁼ ∈ Γ. Clearly every derivation is a Jordan derivation but the converse is not true in general. A Γ-ring *M* is prime if *a*Γ*M*Γ*b* = *0* implies that *a* = 0 or *b* = 0 for every *a*, *b* ∈ *M*. An additive mapping $f : M \to M$ is called a generalized derivation with the associated derivation $d : M \to M$ if *f*(*a*¥⁼*b*) = *f*(*a*)¥⁼*a* + *a*¥⁼*d*(*a*)∀ *a* ∈ *M*, ¥⁼ ∈ Γ, and it is called a Jordan generalized derivation with the associated derivation *d* of *M* if *f*(*a*¥⁼*a*) = *f*(*a*) ¥⁼*b* + *a*¥⁼*d*(*b*) for all *a*, *b* ∈ *M*, ¥⁼ ∈ Γ.

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Let *M* be a Γ-ring. A mapping ±⁼: $M \to M$ is called an involution if σ⁼(*a* + *b*) = ±⁼(*a*) + σ⁼(*b*), σ⁼²(*a*) = *a* and σ (*a*¥⁼*b*) = σ (*b*)¥⁼σ⁼(*a*) for all *a*, *b* ∈ *M* and ¥⁼ ∈ Γ. A Lie ideal *U* of a Γ-ring *M* is called a σ⁼-Lie ideal if σ⁼(*U*) = *U* and it is called a σ⁼-square closed Lie ideal if it is a ±⁼-Lie ideal and for all *u* ∈ *U* ¥⁼ ∈ Γ, *u*¥⁼*u* ∈ *U*. A Γ-ring *M* with involution σ⁼is said to be a σ⁼-prime Γ-ring if *a*Γ*M*Γ*b* = *a*Γ*M*Γ±⁼ = {0} implies that *a* = 0 or *b* = 0. It is worthwhile to note that every prime Γ-ring having an involution ±⁼ is ±⁼-prime but the converse is not true in general. We define the set $S_{a\pm}$(*M*) = {*x* = *M* : σ (*x*) = ± *x*} which is known as the set of symmetric and skew symmetric elements of *M*. Let *U* be a Lie ideal of a Γ-ring *M*. The present authors define centralizer of *U* with respect to *M* by $C_M(U)$ = {*m* ∈ *M* : *m*¥⁼*u* = *u*¥⁼*m* ∀ *u* ∈ *U*, ¥⁼ ∈ Γ}.

Lemma 2.1. [(Rahman and Paul 2013), Lemma 2.5] Let *M* be a Γ-ring and *U* be a Lie ideal of *M* such that *u*~*u* ∈ *U* for all *u* ∈ *U* and ~ ∈ Γ. If *d* is a Jordan derivation on *U* of *M*, then for all *a*, *b*, *c* ∈ *U* and ~, ∈ Γ, the following statements hold:

(i) *d*(*a*~*b* + *b*~*a*) = *d*(*a*)~*b* + *d*(*b*)~*a* + *a*~*d*(*b*) + *b*~*d*(*a*)

(ii) *d*(*a*~*b* *a* + *a* *b*~*a*) = *d*(*a*)~*b* *a* + *d*(*a*) *b*~*a* + *a*~*d*(*b*) *a* + a *d*(*b*)*a*~ + *a*~*b* *d*(*a*) + *a* *b* *a*(*a*)

In particular, if M is 2-torsion free and satisfies the condition (*), then

(iii) $d(a\alpha b\beta a) = d(a)\alpha b\beta a + a\alpha d(b)\beta a + a\alpha b\beta d(a)$

(iv) $d(a\alpha b\beta c + c\alpha b\beta a) = d(a)\alpha b\beta c + d(c)\alpha b\beta a + a\alpha d(b)\beta c + c\alpha d(b)\beta a + a\alpha b\beta d(c)$

$+ c\alpha b\beta b(a)$

Lemma 2.2. [(Rahman and Paul 2013), Lemma 2.8] Let M be a 2-torsion free Γ-ring satisfying the condition (*) and U be a Lie ideal of M. If d is a Jordan derivation on U of M then for all $u, v, w \in U$ and $\alpha, \beta \in \Gamma$, $\delta_\alpha(u, v)\beta w\beta[u, v]_\alpha + [u, v]_\alpha\beta w\beta\delta_\alpha(u, v) = 0$.

Lemma 2.3. [(Rahman and Paul 2013), Lemma 2.11] Let M be a 2-torsion free prime Γ-ring and U be an admissible Lie ideal of M. If $a, b \in M$ or $a \in M$, $b \in U$ such that $a\alpha x\beta b + b\alpha x\beta a = 0$ for all $x \in U$ and $\alpha, \beta \in \Gamma$ then $a\alpha x\beta b + b\alpha x\beta a = 0$.

Lemma 2.4. Let M be a 2-torsion free σ-prime Γ-ring and U be a σ-Lie ideal of M. Let $u \in U$ such that $[u,[u,x]_\alpha]_\alpha = 0$ for all $x \in M$ and $\alpha \in \Gamma$, then $[u, x]_\alpha = 0$.

Proof. Since $[u,[u,x]_\alpha]_\alpha = 0$ for all $x \in M$ and $\alpha \in \Gamma$. Let $y \in M$ and $\beta \in \Gamma$ be arbitrary elements.

Replacing x by $x\beta y$, we obtain

$$\begin{aligned} 0 &= [u,[u,x\beta y]_\alpha]_\alpha \\ &= [u, x\beta[u,y]_\alpha + [u,x]_\alpha\beta y]_\alpha \\ &= [u, x\beta[u,y]_\alpha]_\alpha + [u,[u,x]_\alpha\beta y]_\alpha \\ &= x\beta[u,[u,y]_\alpha]_\alpha + [u,x]_\alpha\beta[u,y]_\alpha + [u,[u,x]_\alpha]_\alpha\beta y + [u,x]_\alpha\beta[u,y]_\alpha \\ &= 2[u,x]_\alpha\beta[u,y]_\alpha . \end{aligned}$$

Since M is 2-torsion free, so $[u,x]_\alpha\beta[u,y]_\alpha = 0$. For every $z \in M$ we have $z\beta x \in M$. Putting $z\beta x$ for y, we have $[u,x]_\alpha\beta[u,z\beta x]_\alpha = 0$. Therefore,

$$\begin{aligned} 0 &= [u,x]_\alpha\beta(z\beta[u,x]_\alpha + [u,z]_\alpha\beta x) \\ &= [u,x]_\alpha\beta z\beta[u,x]_\alpha + [u,x]_\alpha\beta[u,z]_\alpha\beta x \\ &= [u,x]_\alpha\beta z\beta[u,x]_\alpha . \end{aligned}$$

Therefore, $[u,x]_\alpha\gamma M\gamma[u,x]_\alpha = 0$. Since $\sigma(U)=U$, we have $\sigma(u)=u$, for all $u \in U$.

Let $x \in S_{\alpha\sigma}(M)$. Then $\sigma(x) = \pm x$. If $\sigma(u) = u$ and $\sigma(x) = -x$, then

$$\begin{aligned} \sigma([u,x]_\alpha) &= \sigma(u\alpha x - x\alpha u) = \sigma(u\alpha x) - \sigma(x\alpha u) \\ &= \sigma(x)\alpha\sigma(u) - \sigma(u)\alpha\sigma(x) = -x\alpha u + u\alpha x = [u,x]_\alpha \end{aligned}$$

Hence $[u,x]_\alpha\beta M\beta[u,x]_\alpha = [u,x]_\alpha\beta M\beta\sigma([u,x]_\alpha) = 0$. By the σ-primeness of M, $[u,x]_\alpha = 0$.

Lemma 2.5. Let M be a 2-torsion free σ⁚-prime Γ-ring and U be a nonzero σ -Lie ideal and a ±⁚-sub Γ-ring of M. Then either $U \subseteq Z(M)$ or U contains a nonzero σ -ideal of M.

Proof. First let it be assumed that, U as a ±⁚ - Γ-ring which is not commutative. Then for some $u, v \in U$, $[u, v]_{\sim} \neq 0$ and $[u, v]_{\sim} \in U$. Therefore, the ideal S of M generated by $[u, v]_{\sim}$ is nonzero, $S \subseteq U$ and ±⁚(S) = S. On the other hand, let it be assumed that U is commutative. Then for every $u \in U$ $[u,[u,x]_{\sim}]_{\sim} = 0$ for all $x \in M$ and $\sim \in \Gamma$. Hence by Lemma 2.4, $[u,x]_{\sim} = 0$ for all $x \in M$ and $\sim \in \Gamma$. This shows that $U \subseteq Z(M)$.

Lemma 2.6 If $U \not\subseteq Z(M)$ is a ±⁚-Lie ideal of a ±⁚-prime Γ-ring M, then $C_M(U) \subseteq Z(M)$.

Proof. $C_M(U)$ is both a ±⁚-sub Γ-ring and a ±⁚-Lie ideal of M and $C_M(U)$ contains no nonzero ±⁚-ideal of M. In view of Lemma 2.5, $C_M(U) \subseteq Z(M)$. Therefore, $C_M(U) = Z(M)$.

Lemma 2.7. Let U be a ±⁚-Lie ideal of a ±⁚-prime Γ-ring M and $a \in M$. If $[\sim, [U, U]_\Gamma]_\Gamma = 0$ then $[U, U]_\Gamma = 0$, that is, $C_M([U, U]) = C_M(U)$.

Proof. If $[U, U]_\Gamma \not\subseteq Z(M)$, then by Lemma 2.6, $\sim \in Z(M)$, so a centralizes U.

On the contrary, let $[U, U]_\Gamma \subseteq Z(M)$, then $[u, [u, x]_{¥⁚}]_{¥⁚} = 0$ $\forall u \in U, x \in M$ and ¥⁚ $\in \Gamma$.

In view of Lemma 2.4, $[u, x]_{¥⁚} = 0$. This yields that $U \subseteq Z(M)$. For both the cases $\sim \in C_M(U)$.

This gives that $C_M([U, U] = C_M(U)$.

Lemma 2.8. Let $U \not\subseteq Z(M)$ be a ±⁚-square closed Lie ideal of a 2-torsion free ±⁚-prime Γ-ring M and $d : M \to M$ be an additive mapping satisfying $d(u¥⁚u)=d(u)¥⁚u + u¥⁚d(u)$ for all $u \in U$ and ¥⁚ $\in \Gamma$. If $¨_{\sim}(u, v) = d(u¥⁚v) - d(u)¥⁚v - u¥⁚d(v)$ for all $u, v \in U$ and $\in \Gamma$, then $¨_{\sim}(u, v)¥⁚w¥⁚⁚[u, v]_{¥⁚⁚} = 0$ for all $w \in U$.

Proof. Since $U \not\subseteq Z(M)$ is a ±⁚-square closed Lie ideal of a 2-torsion free ±⁚-prime Γ-ring M and $d : M \to M$ is an additive mapping satisfying $d(u¥⁚u)= d(u)¥⁚u + u¥⁚d(u)$ for all $u \in U$ and ¥⁚⁚ $\in \Gamma$. So by Lemma 2.2, $¨_{\sim}(u, v)¥⁚w¥⁚⁚[u, v]_{¥⁚⁚} + [u, v]¥⁚w¥⁚¨_{\sim}(u, v) = 0$ for all $u, v, w \in U$ and $\sim, ¥ \in \Gamma$. (1)

Applying Lemma 2.3, for every $w \in U$, (1) implies that $¨_{\sim}(u, v)¥⁚w¥⁚[u, v]_{¥⁚} = 0$.

Lemma 2.9. Let U be a ±⁚-square closed Lie ideal of a 2-torsion free ±⁚-prime Γ-ring M and $a, b \in M$ such that $\sim$ $U¥⁚b = \sim$ $U¥⁚±⁚(b) = 0$, then $a = 0$ or $b = 0$.

Proof. Suppose U contains an element u_0 in $S_{a±⁚}(M)$ such that M $u_0 \in U$. Let $\sim \neq 0$, there are two several cases. First consider $u_0 \in Z(M)$. If $m \in M$ and $a\sim m$ $u_0¥⁚b$

= *a*~*m* u_0¥± (*b*) = 0 then *a*~*M* u_0¥*b* = *a*~*M* u_0¥± (*b*) = *a*~*M* (u_0¥*b*) = 0

⇒ u_0¥*b* = 0.

Since u_0∈ *Z*(*M*), then u_0 *M*¥*b* = ±(u_0) *M*¥*b* = 0 ⇒ *b* = 0.

Next, consider u_0 ∈ *Z*(*M*). Suppose *a*~ [*t*, u_0]¥ = 0 ∀ *t* ∈ *M*, then *a*~ [*t* *m*, u_0]¥ = *a*~*t* [*m*, u_0]¥ = 0.

So *a*~*M* [*m*, u_0]¥ = 0 = *a*~*M* ([*m*, u_0]¥) ⇒ [*m*, u_0]¥ = 0 ∀*m* ∈ *M* which contradicts the assumption u_0 ∈ *Z*(*M*).

Thus there exists *t* ∈ *M* such that *a*~ [*t*, u_0]¥ ≠ 0.

From *a*~ [*t*, u_0]¥*m* *b* = *a*~ [*t*, u_0]¥*m* ± *(b)* = 0 it follows that *a*~ [*t*, u_0]¥*M* *b* = *a*~ [*t*, u_0]¥*M* ±*(b)* = 0 and by the ±-primeness of *M*, *b* = 0.

Similarly, if *b* ≠ 0 then *a* = 0.

Theorem 2.10. Let *U* be a ±-square closed Lie ideal of a 2-torsion free ±-prime Γ-ring *M* and *d* : *M* → *M* be an additive mapping satisfying *d*(*u*¥*u*)=*d*(*u*)¥*u* + *u*¥*d*(*u*) for all *u* ∈ *U* and ¥ ∈ Γ then *d*(*u*¥*v*) = *d*(*u*)¥*v* + *u*¥*d*(*v*) for all *u*, *v* ∈ *U* and ¥ ∈ Γ.

Proof. If *U* is a non-commutative Lie ideal of *M*, then *U* ≠ *Z* (*M*).

By Lemma 2.8, ¨~ (*a*, *b*) *w*¥ [*a, b*]~ = 0 for all *a, b, w* ∈ *U* and ~, , ¥ ∈ Γ.

Let *a*, *b* ∈ *U* ∩ $S_{a\sim}$ (*M*). Since ± (*U*) = *U*, so ± [*a*, *b*]~ = [*a*, *b*]~ , as [*a*, *b*]~ ∈ *U*. If ±(*b*) = – *b* and ± (*a*) = – *a*, then

± ([*a*, *b*]~) = ± (*a*~*b* – *b*~*a*) = ± (*b*)~± (*a*) – ± (*a*)~± (*b*) = –*b*~*a* + *a*~*b* = [*a*, *b*]~.

Also, if ± (*b*) = *b* and ± (*a*) = – *a*, then ± ([*a*, *b*]~) = [*a*, *b*]~. Therefore,

¨~ (*a*, *b*) *w*¥[*a*, *b*]~ = ¨~(*a*, *b*) *w*¥±[*a*, *b*]~) = 0.

Applying Lemma 2.9 in the above relation,

¨~ (*a*, *b*) = 0 or [*a*, *b*]~ =*0*, for all *a*, *b* ∈ *U* ∩ $S_{a\sim}$ (*M*).

Let I_a = {*b* ∈ *U* : ¨~ (*a*, *b*) = 0} and J_a = {*b* ∈ *U* : [*a*, *b*]~ = 0}. Then I_a and J_a are additive subgroups of *U* such that I_a ∪ J_a = *U*. Then by Brauer's trick I_a = *U* or J_a = *U*.

Using the similar argument, *U* = {*a* ∈ *U* : *U* = I_a} or *U* = {*a* ∈ *U* : *U* = J_a}.

If *U* = {*a* ∈ *U* : *U* = J_a} then $[a, b]_\alpha = 0$ which yields that *U* ⊆ *Z* (*M*), by Lemma 2.5. Which is a contradiction to the fact that *U* ⊈ *Z*(*M*). So *U* = {*a* ∈ *U* : *U* = I_a} and hence ¨~ (*a*, *b*) = 0, for all *a*, *b* ∈ *U* ∩ $S_{a\sim}$ (*M*). This implies that

d(*a*~*b*) = *d*(*a*)~*b* + *a*~*b*(*b*), ∀ *a*, *b* ∈ *U* ∩ $S_{a\sim}$ (*M*). (2)

Now let $u, v \in U$. Define $u_1 = u + \text{±}(u)$, $u_2 = u - \text{±}(u)$, $v_1 = v + \text{±}(v)$, v_2

$$= v - \text{±}(v).$$

Then $u_1, u_2, v_1, v_2 \cap S_{a\sim}(M)$ and $2u = u_1 + u_2$, $2v = v_1 + v_2$.

Therefore, in view of (2)

$d(2u\text{¥}_2v) = d(u_1\text{¥}v_1 + u_1\text{¥}v_2 + u2\text{¥}v_1 + u_2\text{¥}v_2)$

$= d(u_1)\text{¥}v_1 + u_1\text{¥}(v_1) + d(u_1)\text{¥}v_2 + u_1\text{¥}d(v_2) + d(u_2)\text{¥}v_1 + u_2\text{¥}d(v_1) + d(u_1)\text{¥}v_2 + u_2\text{¥}d(v_2)$

$= (d(u_1) + d(u_2))\text{¥}v_1 + (u_1 + u_2)\text{¥}d(v_1) + (d(u_1) + d(u_2))\text{¥}v_2 + (u_1 + u_2)\text{¥}d(v_2)$

$= d(u_1 + u_2)\text{¥}v_1 + 2u\text{¥}d(v_1) + d(u_1 + u_2)\text{¥}v_2 + 2u\text{¥}d(v_2)$

$= d(2u)\text{¥}v_1 + 2u\text{¥}d(v_1) + d(2u)\text{¥}v_2 + 2u\text{¥}d(v_2)$

$= 2d(u)\text{¥}(v_1 + v_2) + 2u\text{¥}d(v_1 + v_2)$

$= 2d(u)\text{¥}2v + 2u\text{¥}d(2v)$

$= 4d(u)\text{¥}v + 4u\text{¥}d(v)$.

Thus $4d(u\text{¥}v) = 4(d(u)\text{¥}v + u\text{¥}d(v))$.

Since M is 2-torsion free, so $d(u\text{¥}v) = d(u)\text{¥}v + u\text{¥}d(v)$.

If U is a commutative ±-Lie ideal of M, then by Lemma 2.5, $U \subseteq Z(M)$.

Therefore, using 2-torsion freeness of M and in view of the Lemma 2.1(i)

$d(u\text{¥}v) = d(u)\text{¥}v + u\text{¥}d(v)\ \forall\ u, v \in U$ and ¥ $\in \Gamma$.

The following corollary is an immediate consequence of the above theorem.

Corollary 2.11. If M is a 2-torsion free ±-prime Γ-ring, then every Jordan derivation of M is a derivation of M.

REFERENCES

Bergun, J., I. N. Herstein and J. W. Kerr. 1981. Lie ideals and derivations of prime rings. *J. Algebra* **71:** 259-267.

Herstein, I. N. 1969. Topics in Ring Theory. *The University of Chicago Press, Chicago* 111 – London.

Dey, K. K., A. C. Paul and V. Davvaj. 2015. On centralizing automorphisms and Jordan left derivations on ±-prime Γ-rings. *Hacettepe J. Math. Stat.* **44**(1): 51-57.

Khan, M. R., D. Arora and M. A. Khan. 2010. Notes on derivations and lie ideals in sigma-prime rings. *Advances in Algebra* **3**(1): 19-23.

Khan, M. S. and M. A. Khan. 2012. Lie ideal and generalized derivations in sigma-prime rings *J. Algebra* **6**(29): 1419 - 1429.

Oukhtite, L. and S. Salhi. 2006. On generalized derivations of sigma-prime rings. *African Diaspora J. Math.* **5**(1): 19-23.

Oukhtite, L. and S. Salhi. 2006. On commutativity of sigma-prime rings. *Glasnik Mathematicki* **41**(1): 57-64.

Oukhtite, L. and S. Salhi. 2006. Derivations and commutativity of sigma-prime rings. *Int. J. Contemp.* **1:** 439-448.

Oukhtite, L. and S. Salhi. 2007. On derivations in sigma-prime rings. *Int. J. Algebra* **1:** 241-246.

Oukhtite, L. and S. Salhi. 2007. Sigma-Lie ideals with derivations as homomorphisms and anti-homomorphisms. *Int. J. Algebra* **1:** 235-239.

Oukhtite, L., S. Salhi and L. Taofiq. 2007. Jordan generalized derivations on sigma-prime rings. *Int. J. Algebra* **1(**5): 231-234.

Oukhtite, L. and S. Salhi. 2008. Centralizing automorphisms and Jordan left derivations on sigma-prime rings. *Advances in Algebra* **1**(1): 19-26.

Paul, A.C and M. M. Rahman. 2015. Jordan Derivations on Lie Ideals of ±÷-Prime Rings. *GANIT, Journal of Bangladesh Mathematical Society* (Accepted for publication in its upcoming issue).

Paul, A.C. and S. Chakraborty. 2015. Derivations acting as homomorphisms and as anti-homomorphisms in ±÷-Lie ideals of ±÷-prime Γ-rings. *Math.Stat.* **3**(1): 10-15.

Rahman, M. M and A. C. Paul. 2013. Jordan derivations on lie ideals of prime Γ-Rings. *Mathematical Theory and Modeling* **3**(3): 128-135.

(*Received revised manuscript on 9 November, 2015*)