STUDY OF ATTRACTIVE NONLINEARITY BY 1-D NONLINEAR SCHRÖDINGER EQUATION IN THE BOSE –EINSTEIN CONDENSATE

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ABSTRACT

The work represents and investigates the stationary solutions of the one-dimensional Non-linear Schrödinger Equation (NLSE), for attractive non-linearity, in the Bose-Einstein condensates (BEC) under the box boundary condition and calculates the characteristics of internal modes of bright solitons (eigen modes of small perturbation of the condensate).

INTRODUCTION

Bose-Einstein condensate (BEC) was created for lithium [1-3] in case of attractive nonlinearity as predicted by the nonlinear Schrödinger equation (NLSE) for three dimensions [4, 5, and 6]. The condensate collapsed when the number of particles became large. However, in one or quasi-one-dimension, no collapse is predicted [7] and in one dimension the NLSE has a wide application in fiber optics [8] as well as in other fields [9-12].

In this paper we present stationary solutions to the one-dimensional NLSE for attractive nonlinearity under box boundary conditions. The Bose-Einstein Condensate (BEC) is said to be in the quasi-one-dimensional regime when its transverse dimensions are of the order of its healing length, and its longitudinal dimension is much longer than its transverse ones. In this case the 1-D limit of the 3-D NLSE is appropriate, rather than a true 1-D mean-field theory [13], as would be the case when the transverse dimension is the order of the atomic interaction length or the atomic size itself. Under these criteria the condensate is well out of the Thomas-Fermi limit; i.e., the kinetic energy in the transverse dimension is very high. It is this high kinetic-energy which prevents the condensate from collapsing. We have numerically illustrated the stability of the condensate in quasi-1-D case.

BOSE-EINSTEIN CONDENSATION (BEC)

Bose-Einstein condensation is based on the indistinguishability and wave nature of particles, both of which are at the heart of quantum mechanics. In a simplified picture, atoms in a gas may be regarded as quantum-mechanical wavepackets which have an extent of the order of a thermal de Broglie wavelength $\lambda_{dB} = (2\pi\hbar^2 / Mk_B T)^{1/2}$ where T is the temperature and M the mass of the atom. λ_{dB} can be regarded as the position uncertainty associated with the thermal momentum distribution. The lower is the temperature, the longer is λ_{dB}. When atoms are cooled to the point where λ_{dB} is comparable to the interatomic separation, the atomic wavepackets "overlap" and the indistinguishability of particles becomes important. At this temperature, bosons undergo a phase transition and form a Bose-Einstein condensate, a coherent cloud of atoms all occupying the same quantum mechanical state. The transition temperature and the peak atomic density n are related as $n\lambda_{dB}^3 \cong 2.612$.

In a Bose condensed gas, the separation between atoms a (charectarized by the s-wave scattering length) is equal to or smaller than the thermal de Broglie wavelength. The largest length scale is the confinement, either characterized by the size of the box potential or by the oscillator length $a_{HO}=\sqrt{\hbar/M\omega}$ which is the size of the ground state wavefunction in a harmonic oscillator potential with frequency ω .

Atom-atom interactions are described by a mean field energy $U_{\mathrm{int}} = 4\pi\hbar^2 na/M$. In most experiments, $k_B T > U_{\mathrm{int}}$, but the opposite case has also been realized [14,15]. In comparison, superfluid helium is a strongly interacting quantum liquid - the size of the atom, the healing length, the thermal de Broglie wavelength and the separation between atoms are all comparable, creating a complex rich situation.

FORM OF NONLINEAR SCHRÖDINGER EQUATION (NLSE) IN BOSE-EINSTEIN CONDENSATIONS (BEC)

The interaction between N atoms of mass M, confined in an external potential V(r), is taken into account by including a term $g|\psi|^2$ in the Schrödinger equation, proportional to the square of the wave function [16]

$$\left[-\frac{\hbar^2}{2M}\nabla^2 + g|\psi(\vec{r},t)|^2 + V(\vec{r})\right]\psi(\vec{r},t) = i\hbar\partial_t\,\psi(\vec{r},t) \tag{1}$$

where $|\psi(\vec{r},t)|^2$ is the single particle density such that $\rho(\vec{r},t) = N|\psi(\vec{r},t)|^2$, the coupling constant $g \equiv 4\pi\hbar^2 aN/M$, [16,17] and a is the s-wave scattering length for binary collisions between atoms. The case of attractive interactions considered here corresponds to a < 0.

$V(\vec{r})$ is defined to be a three-dimensional rectangular box of length L and small transverse area A_t. In the transverse directions the wave function is required to vanish on the surface of the container; in the longitudinal direction we require either box or periodic boundary conditions.

The characteristic length scale over which the condensate density attains its average value away from a sharp defect or from a perfectly confining wall is the healing ξ

$$\xi \equiv \left(8\pi\bar{\rho}|a|\right)^{-1/2}$$

where $\bar{\rho} \equiv N/(LA_t)$ is the mean particle density, L is the longitudinal length of the confining potential, and L_y and L_z are the transverse lengths , $A_t \equiv L_Y L_Z$ is the transverse area.

The BEC is in the quasi-1D regime when L_y and L_Z satisfy the following criteria:

$$L_y, L_z \sim \xi \text{ and } L_y, L_z \ll L .$$

The former ensures that the condensate remains in the ground state in the two transverse dimensions, while the latter ensures that longitudinal excitations are much lower in energy than possible transverse excitations. Under these conditions one may make an adiabatic separation of longitudinal and transverse variables,

$$\psi(\vec{r},t) = (L, A_t)^{-1/2} f(x) h(y,z) e^{-i\mu t/\hbar}$$

where $f(x)$ and $h(y,z)$ are dimensionless functions and the time dependence of a stationary state has been assumed, μ being the chemical potential. This reduces the three-dimensional NLSE (1) to,

$$\left[-\frac{\hbar^2}{2M}\nabla^2 + \frac{g\left|f(x)h(y,z)\right|^2}{LA_t} e^{-i\mu t/\hbar} e^{i\mu t/\hbar} + V(\vec{r})\right] (L, A_t)^{-1/2} f(x) \cdot$$

$$h(y,z)e^{-i\mu t/\hbar} = (L, A_t)^{-1/2} f(x) h(y,z) i\hbar \frac{\partial}{\partial t}(e^{-i\mu t/\hbar})$$

or, $$\left[-\frac{\hbar^2}{2M}\nabla^2 + \frac{g\left|f(x)h(y,z)\right|^2}{LA_t} + V(\vec{r})\right] (L, A_t)^{-1/2} f(x)\cdot h(y,z) \qquad (2)$$

$$= (L, A_t)^{-1/2} f(x) h(y,z)\, \mu$$

Equation (2) may be projected on to the ground state of *h(y,z)* and integrated over the transverse dimensions y and z

$$\int_0^{L_y} dy \int_0^{L_Z} dz\, h^*_{gs}(y,z)\left[-\mu - \frac{\hbar^2}{2M}\nabla^2 + \frac{g\left|f(x)h(y,z)\right|^2}{LA_t} + V(x,y,z)\right] f(x)h(y,z) = 0 \qquad (3)$$

where $h_{gs}(y,z)$ is the ground state quantum mechanics particle in a box solution.

Requiring $h_{gs}(y,z) = h_0 \sin\frac{\pi y}{L_y}\sin\frac{\pi z}{L_z}$ is normalized to 1,where $h_0 = 2$, equation (3) becomes,

$$\int_0^{L_y} dy \int_0^{L_Z} dz\, h^*_{gs}(y,z)\left[-\mu - \frac{\hbar^2}{2M}(\partial_y^2 + \partial_z^2 + \partial_x^2) + \frac{g\left|f(x)h_{gs}(y,z)\right|^2}{L\,A_t} + V(x,y,z)\right] f(x)h_{gs}(y,z) = 0$$

or, $$\left[-\left(\mu - \frac{\hbar^2}{2M}\frac{\pi^2}{L_y^2} - \frac{\hbar^2}{2M}\frac{\pi^2}{L_z^2}\right) - \frac{\hbar^2}{2M}\partial_x^2 + \frac{\hbar^2}{2M\xi^2}\,\frac{9}{4}\,\frac{1}{L_y L_z}\left|f(x)\right|^2 + V(x,y,z)\right] f(x) = 0 \qquad (4)$$

Multiplying equation (4) by $\frac{2M\xi^2}{\hbar^2}$ we get,

$$\left[-\left(\tilde{\mu} - \frac{\pi^2\,\xi^2}{L_y^2} - \frac{\pi^2\,\xi^2}{L_z^2}\right) - \xi^2\partial_x^2 - \frac{9}{4}\,\frac{1}{L_y L_z}\left|f(x)\right|^2 + \tilde{V}(x,y,z)\right] f(x) = 0 \qquad (5)$$

where *f* is a dimensionless wave function describing excitations along L; $|f|^2/L$ is the longitudinal part of the single particle density. $\tilde{\mu} \equiv \frac{2M\xi^2}{\hbar^2}\mu$ is a dimensionless chemical potential which is now the eigen value of the problem and $\widetilde{V}(\widetilde{x}) \equiv \frac{2M\xi^2}{\hbar^2} V(\widetilde{x})$ is the confining potential.

Now combining the longitudinal length of the confining potential and healing length into a single dimensionless scaling parameter $\lambda = \xi / L$, equation (5) becomes,

$$\left[-\left(\widetilde{\mu} - \frac{\pi^2 \xi^2}{L_y^2} - \frac{\pi^2 \xi^2}{L_z^2} \right) - \lambda^2 \partial^2 \widetilde{x} - \frac{9}{4} |f(\widetilde{x})|^2 + \widetilde{V}(\widetilde{x}) \right] f(\widetilde{x}) = 0 \tag{6}$$

Using the approximations for L_y and L_z , and dividing by integrating factor of 9/4,

$$\left[-\widetilde{\mu}_{eff} - \lambda^2{}_{eff} \partial^2 \widetilde{x} + \widetilde{V}_{eff}(\widetilde{x}) - |f(\widetilde{x})|^2 \right] f(\widetilde{x}) = 0 \tag{7}$$

where, $\widetilde{\mu}_{eff} = \frac{4}{9}\widetilde{\mu} - \frac{8\pi^2}{9}$, $\lambda^2{}_{eff} = \frac{4\lambda^2}{9}$ and $\widetilde{V}_{eff}(\widetilde{x}) = \frac{4}{9}\widetilde{V}(\widetilde{x})$

We can simply drop the ‘eff ‘ subscripts

$$\left[-\lambda^2 \partial^2 \widetilde{x} - |f(\widetilde{x})|^2 + \widetilde{V}(\widetilde{x}) \right] f(\widetilde{x}) = \widetilde{\mu} f(\widetilde{x}) \tag{8}$$

For comparison with experimental result [18] the conversion factors from the dimensionless $\widetilde{\mu}$ to μ, in μK are given below. The general conversion μ =(8.34 $\times$ 10^{-15}) $(\bar{\rho}a/M)\widetilde{\mu}$, where M is in atomic mass units, $\bar{\rho}$ is in cm^{-3}, and a is in nm. Using common experimental values [18] of $\bar{\rho} \sim 10^{14}$, for ^{23}Na, a~2 .75, and for ^{87}Rb, a~5.77, the conversion factors are 0.0723 and 0.0401 respectively. Since the dimensionless chemical potentials found will be of the order of 1 -10 this gives a sense of the energy scale of the solutions on the order of 0.1 to 1 μK. In this paper two test scales of $\lambda = 1/10, \lambda = 1/25$ will be used for illustrative purposes.

As $|f(\widetilde{x})|^2$ is a single particle density, it is normalized to 1 rather than N.

$$\int_0^1 d\widetilde{x} |f(\widetilde{x})|^2 = 1 \tag{9}$$

The number of atoms N which is proportional to the coefficient to the nonlinear term in Eq (1), is then contained in the ratio of the healing length to the box length, $\lambda \propto N^{-1/2}$ the NLSE (8), subject to normalization (9) and box boundary conditions, is the equation which will be solved.

BOX BOUNDARY CONDITION ON THE BEC AND GENERAL FROM OF THE STATIONARY SOLUTION

We now consider the solution of Eq.(8) in regions of constant potential, which may be taken to beV(x) =0 without loss of generality. We note first that if $f(x)$ vanishes anywhere in an interval as for example at the edges of the box, then $f(x)$ may be taken to be purely real throughout that interval. This is easily established by considering a Taylor series expansion of $f(x)$ in the neighborhood of the point at which it vanishes. Thus we may remove the absolute value symbol in Eq. (8) and so recover an ordinary nonlinear equation for a real function.

$$-\lambda^2 f'' + f^3 - \tilde{\mu} f = 0 \tag{10}$$

The most general solution of the equation given above is a Jacobian elliptic function [15]. For the attractive case; both the *cn* and *dn* functions offer solutions. However, the *dn* function has no real zeros, so the *cn* function provides all solutions which satisfy box boundary conditions [15].

Considering the solution in constant potential, we may take $\tilde{V}(\tilde{x}) = 0$. So we get from equation (8)

$$\left[-\lambda^2 \frac{\partial^2}{\partial \tilde{x}^2} - |f(\tilde{x})|^2 \right] f(\tilde{x}) = \tilde{\mu} f(\tilde{x}) \tag{11}$$

The boundary conditions are,

$$f(0) = f(1) = 0 \tag{12}$$

Now most general form of the solution,

$$f(\tilde{x}) = A\, cn(k\tilde{x} + \delta | m) \tag{13}$$

Where $0 \le m \le 1$ is the parameter of Jacobian elliptic functions, $k \text{ and } \delta$ will be determined by the boundary conditions,while *A* and *m* will be determined by substitution of Eq. (13) into the NLSE and by normalization.

The function $cn(x|m)$ is periodic in x, with period equal to 4*K(m*), where *K(m)* is an elliptic integral of the first kind. Since the quarter period of *cn* is the complete elliptic integral of the first kind, $K(m)$, and since $cn(0|m) = 1$, we find that $k = 2jK(m)$ and $\delta = -K(m)$,where $j \in \{1,2,3,.......\}$. j-1 gives the number of nodes in the *cn* function

Now, $f(\tilde{x}) = A\, cn(2jK(m)\tilde{x} + \delta | m)$

Putting the value of $f(\tilde{x})$ in equation (11) we get

$$-\lambda^2 \frac{\partial^2}{\partial \tilde{x}^2} A\, cn(2jK(m)\tilde{x} + \delta | m) + |f(\tilde{x})|^2 f(\tilde{x}) = \tilde{\mu} f(\tilde{x}) \tag{14}$$

$$or, -\lambda^2 [2jK(m)]^2 (2m-1) + \lambda^2 [2jK(m)]^2\, 2m\, cn^2(2jK(m)\tilde{x} + \delta | m) = A^2\, cn^2(2jK(m)\tilde{x} + \delta | m) + \tilde{\mu} \tag{15}$$

Now equating the coefficient of equal powers of $cn\left(2jK(m)\tilde{x}+\delta\,|m\right)$ we find

$$A^2=\lambda^2\,2m[2j\,K(m)]^2 \tag{16}$$

$$and,\tilde{\mu}=-\lambda^2(2m-1)[2j\,K(m)]^2 \tag{17}$$

Substituting Eq. (16) into Eq. (9) and noting that the integral over sn^2 can be defined in multiples of the quarter period *K(m),* we obtain the normalization condition,

$$2(2j)^2\lambda^2K(m)[E(m)-(1-m)K(m)]=1$$

$$or,\ \left\{8\lambda^2K(m)\,[E(m)\,-(1-m)K(m)]\,\right\}^{-1/2}=j \tag{18}$$

If we carefully observe the Eq. (18), we see that the left side of the equation is a continuous and decreasing function of "*m*" but the right side gives only certain allowed discrete values (j = 1,2,3,4,5 ……….) where *E(m)* is the complete elliptic integral of the second kind.

Now, As $f(\tilde{x})=A\,cn\left(k\tilde{x}+\delta|m\right)and\ k=2jK(m),\delta=-K(m)$

So equation (13) becomes,

$$\therefore f(\tilde{x})=\sqrt{2m}\,\left[2jK(m)\right]\lambda\,cn\left(2jK(m)\tilde{x}-K(m)|\,m\right)$$

$$=\sqrt{2m}\,\left[2jK(m)\right]\lambda\,cn\left(K(m)(2j\tilde{x}-1)|\,m\right) \tag{19}$$

This leaves the chemical potential (17) and the wave function (19) determined up to the parameter *m* and the scale λ.

In Fig.1(a) graphical solution of Eq.(18) is shown. The plot demonstrates that the solutions are unique. These solutions are in one-to-one correspondence with those of the 1D particle-in-a-box problem in linear quantum mechanics,

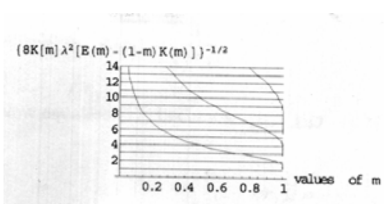

Fig. 1(a) : Graphical solution of equation (18)

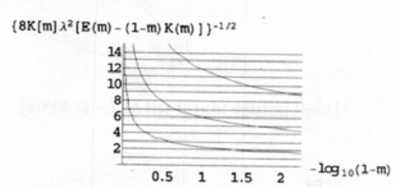

Fig. 1(b) : Graphical solution of equaion (18)

Fig. 1(a) Graphical solution of equation (18)In figure: 1(a) & (b): In this graphical solution of Eq.(18) λ is the scale and j-1 with $j\in\{1,2,3,........\}$ or j with $j\in\{2,4,6,........\}$ is the number of nodes respectively. The three curved lines are plots of Eq.

(18) solved for the number of nodes *j*, with $\lambda^{-1} = L/\xi = 10{,}25$. The left-hand side of the plot is the *m* =0 linear limit, while the right-hand side exponentially approaches the *m*=1 bright soliton limit. Solutions are found where these lines intersect with the horizontal lines of *j*. Note the rapid convergence to *m* = 0 in the high-*j* limit, so that for large *j* the solutions are in the linear regime.

From figure 1(a) we can determine the values of "*m*" for different values of "*j*" easily. But for ground states and lower excited states the values of "*m*" will be found by using figure 1(b)

BRIGHT SOLITON LIMIT AND FORM OF THE CHEMICAL POTENTIAL

One may add a peak without disturbing another peak, provided that adjacent peaks have opposite phases and that the overlap between them is exponentially small in the ratio of their separation to their healing length. In this limit we ought to recover a series of equally spaced *sech* solutions of alternating phase to within exponentially small factors. These solutions are also called bright solitons. The *sech* function is the $m \to 1^-$ limit of the *cn* function in the solution (Eq. (13))

Bright solitons solve the free NLSE. Thus we should find that the wave function and chemical potential no longer depend on the box length L. In this limit the solitons must have a different length scale than λ, which depends on L. The soliton width is proportional to the parameter

$$\eta \equiv \frac{A_t}{8\pi N|a|} \tag{20}$$

where, A_t is the transverse area of the box, η is usually set to 1 by renormalizing the wave function[8]

We now consider the limit $\lambda \to 0$, which corresponds to $m \to 1^-$. Physically, this means that the peaks become highly separated and the interaction between them becomes exponentially small. By using Taylor expansions in 1- m of the complete elliptic integrals, we find that,

$$2(2j)^2 \lambda^2 K(m)\left[E(m) - (1-m)K(m)\right] = 1$$

$$\text{or, } 8j^2\lambda^2 K(m)\left[E(m) - K(m) + mK(m)\right] = 1$$

$$\text{or, } 8j^2\lambda^2 K(m)E(m) - 8j^2\lambda^2 K^2(m) + 8j^2\lambda^2 mK^2(m) = 1$$

For bright soliton limit *m* =1, and from Jacobian elliptical function $E(m) = 1$. So we get from above given equations, $K(m) = \dfrac{1}{8j^2\lambda^2}$

Now Eq. (17) becomes, $\tilde{\mu} = -\lambda^2(2-1)\left[2j\dfrac{1}{8j^2\lambda^2}\right]^2$ [since *m* =1 in the bright soliton limit]

$$or, \tilde{\mu} = -\frac{1}{16 j^2 \lambda^2} \tag{21}$$

and Eq.(19) becomes

$$f(\tilde{x}) = \sqrt{2}\left[2j\frac{1}{8j^2\lambda^2}\right]\lambda\, cn\left(\frac{1}{8j^2\lambda^2}(2j\tilde{x}-1)\Big|m\right)$$

$$or, f(\tilde{x}) = \frac{1}{2^{3/2}\, j}\,\lambda^{-1}\, cn\left(\frac{1}{8j^2\lambda^2}(2j\tilde{x}-1)\Big|m\right) \tag{22}$$

Again we know that,

$$\tilde{\mu} = \frac{2M\xi^2}{\hbar^2}\mu$$

$$or, \mu = -\frac{\hbar^2}{2M}\,\frac{1}{16j^2\lambda^2}\,\frac{1}{\xi^2} \qquad \text{[Using Eq. (21)]}$$

Putting back in the units we find

$$\mu = -\frac{\hbar^2}{2M}\,\frac{1}{16\eta^2}\,\frac{1}{j^2} \tag{23}$$

$$\frac{f(\tilde{x})}{\sqrt{L}} = \frac{1}{2^{3/2}\, j}\,\frac{1}{\eta}\, cn\left[\left(\frac{1}{4j\eta}\right)x - \delta(L)\Big|m\right] \tag{24}$$

where $\delta(L)$ is an offset which depends on the box length. But as $L \rightarrow \infty$ the offset becomes arbitrary, so that we can set it to zero. Note that we put back in the units of the wave function $f(\tilde{x})$ which we took out in making the separation of variables in the quasi-1D approximation.

$$\frac{f(\tilde{x})}{\sqrt{L}} = \frac{1}{2^{3/2}\, j}\,\frac{1}{\eta}\, cn\left[\left(\frac{1}{4j\eta}\right)x\Big|m\right]$$

As may be seen for the case in which $j = 1$ and using the Jacobian elliptic function for the case $m = 1$, we get

$$\frac{f(x)}{\sqrt{L}} = \frac{1}{2^{3/2}}\,\frac{1}{\eta}\sec h\left[\left(\frac{1}{4\eta}\right)x\right] \tag{25}$$

This is the wave function of the free 1D nonlinear Schrödinger equation.

Graph:

1. Ground and Excited States for the Scaling Parameter λ^{-1}=10:

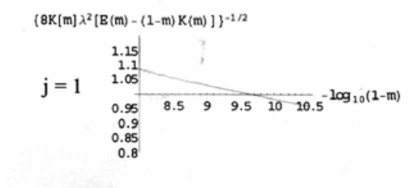

Fig 2(a): value of m for j = 1; (λ^{-1} = 10)

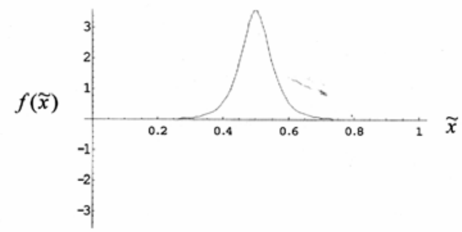

Fig 2(b): plot of ground state, $\mu \cong -6.362$

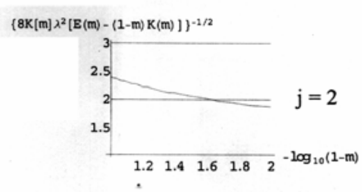

Fig 3(a): value of m for j = 2; (λ^{-1} = 10)

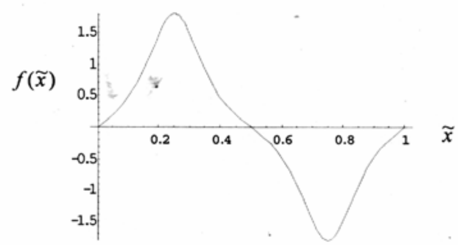

Fig 3(b): plot of 1st excited state, $\mu \cong -1.598$

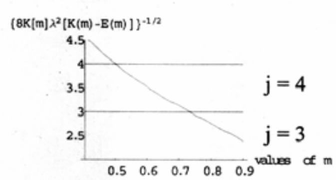

Fig 4(a): value of m for j = 3 & 4; (λ^{-1} = 10)

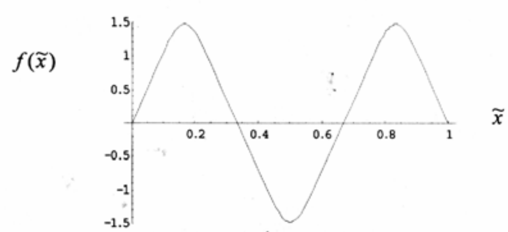

Fig 4(b): plot of 2nd excited state, $\mu \cong -0.620$

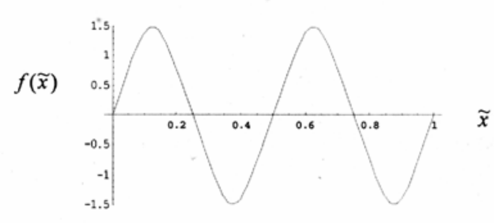

Fig 4(c): plot of 3rd excited state, $\mu \cong 0$

Fig. 2-4 (a) : Values of m for ground and different excited states (j=1,23....) with scaling parameter λ^{-1}=10

(b) Plots of corresponding wave function and values of their chemical potentional μ.

1. Excited States for the Scaling Parameter λ^{-1}=25:

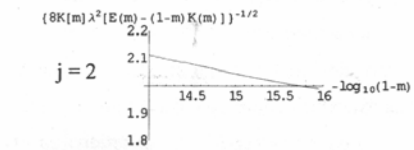

Fig 5(a) value of m for j =2; (λ = 1/25)

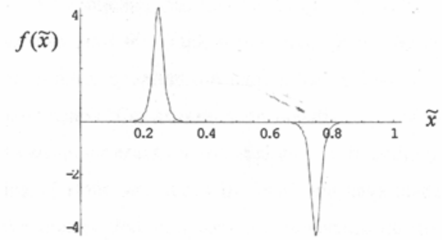

Fig 5(b): plot of 1st excited state, $\mu \cong -9.643$

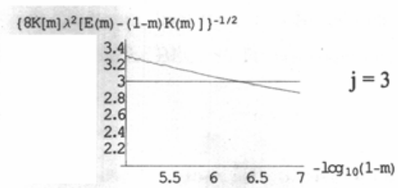

Fig 6(a): value of m for j = 3; (λ^{-1} = 25)

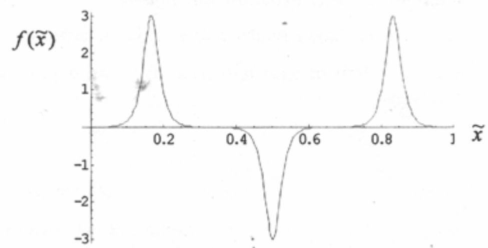

Fig 6(b): plot of 2nd excited state, $\mu \cong -4.531$

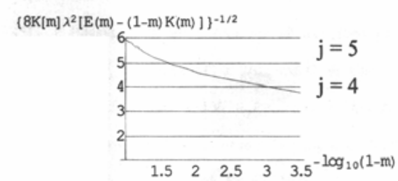

Fig 7(a): value of m for j= 4 &5 ;(λ^{-1} = 25)

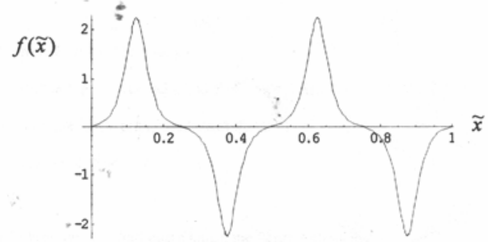

Fig 7(b): plot of 3rd excited state, $\mu \cong -2.511$

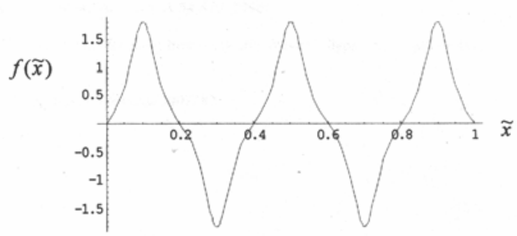

Fig 7(c): plot of 4th excited state, $\mu \cong -1.598$

Fig. 5-7 (a) : Values of m for different excited states (j=1,23....) with scaling parameter λ^{-1}=25

(b) Plots of corresponding wave function and values of their chemical potentional μ.

RESULT AND DISCUSSION

Plots have been obtained for the different values of the nodes j and the values of m have been obtained from the graphs. Using Eq. (19) we also draw the graph which gives somes peaks that are analogous to the particle-in-a box problem in linear quantum mechanics. These peaks can be characterized as the bright soliton trains. Bose-Einstein Condensate with negative scattering length is only stable due to the finite size and the zero-point energy associated with it. In addition to describing the properties and physical meaning of stationary states in detail, we have made experimental prediction specific to the BEC. We predict that in quasi-one-dimension, i.e. for transverse dimension on the order of ξ , a slow enough accretion of particles will gives the attractive condensates. If it can be shown that the NLSE models the attractive BEC in the quasi-one-dimension, then there is a lot of rich phenomena in fiber optics which could have a direct analogue in the BEC. So far we know this technique has been used first time to find out soliton from BEC.

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