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ON R_0 SPACE IN L-TOPOLOGICAL SPACES

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ABSTRACT

R_0 space in L-topological spaces are defined and studied. The authors give eight definitions of R_0 space in L-topological spaces and discuss certain relationship among them. They showed that all of these satisfy 'good extension' property. Moreover, some of their other properties are obtained.

Key words: L-fuzzy set, L-fuzzy topology

INTRDUCTION

The concept of R_0-property first defined by Shanin (1943) and there after Naimpally (1967), Dude (1974), Hutton (1975 a, b), Dorsett (1978), Ali (1990) and Caldas (2000). Khedr (2001) and Ekici (2005) defined and studied many characterizations of R_0-properties. Later, this concept was generalized to 'fuzzy R_0-propertise' by Zhang (2008), Keskin (2009) and Roy (2010) and many other fuzzy topologists. In this paper the workers defined possible eight notions of R_0 space in L-topological spaces and they showed that this space possesses many nice properties which are hereditary productive and projective.

Definition: Let X be a non-empty set and I = [0, 1]. A fuzzy set in X is a function $u: X \to I$ which assigns to each element $x \in X$, a degree of membership, $u(x) \in I$ (Zadeh 1965).

Definition: Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. An L-fuzzy set in X is a function $\alpha: X \to L$ which assigns to each element $x \in X$, a degree of membership, $\alpha(x) \in L$ (Goguen 1967).

Definition: Let α be an L-fuzzy set in X .Then $1 - \alpha = \alpha'$ is called the complement of α in X (Goguen 1967).

Definition: If $r \in L$ and α is an L-fuzzy sets in X defined by $\alpha(x) = r \forall x \in X$ then the authors refer to α as a constant L-fuzzy sets and denoted it by r itself (Goguen 1967).

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In particular, these are the constant L-fuzzy sets 0 and 1.

Definition: An L-fuzzy point p in X is a special L-fuzzy sets with membership function $p(x) = r \ if \ x = x_0$

$p(x) = 0 \ if \ x \neq x_0$ where $r \in L$ (Liu *et al.*1997).

Definition: An L-fuzzy point p is said to belong to an L-fuzzy set α in X $(p \in \alpha)$ if and only if $p(x) < \alpha(x)$ and $p(y) \leq \alpha(y)$. That is $x_r \in \alpha$ **implies** $r < \alpha(x)$ (Liu *et al.* 1997).

Definition: Let $I = [0, 1]$, X be a non-empty set and I^X be the collection of all mappings from X into I, i. e. the class of all fuzzy sets in X. A fuzzy topology on X is defined as a family t of members of I^X, satisfying the following conditions:

$(i) 1, 0 \in t$ (ii) if $u_i \in t$ for each $i \in \Delta$ then $\bigcup_{i \in \Delta} u_i \in t$ (iii) if $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$. The pair (X, t) is called a fuzzy topological space (fts, in short) and the members of t are called t-open (or simply open) fuzzy sets. A fuzzy set v is called a t-closed (or simply closed) fuzzy set if $1 - v \in t$ (Chang 1968).

Definition: Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. Suppose that τ be the sub collection of all mappings from X to L $i.e. \tau \subseteq L^X$. Then τ is called L-topology on X if it satisfies the following conditions:

(i) $0^*, 1^* \in \tau$

(ii) If $u_1, u_2 \in \tau$ then $u_1 \cap u_2 \in \tau$

(iii) If $u_i \in \tau$ for each $i \in \Delta$ then $\cup_{i \in \Delta} u_i \in \tau$.

Then the pair (X, τ) is called an L-topological space (lts, for short) and the members of τ are called open L-fuzzy sets. An L-fuzzy sets v is called a closed L-fuzzy set if $1 - v \in \tau$ (Jin-xuan and Bai-lin 1998).

Definition: Let λ be an L-fuzzy set in lts (X, τ). Then the closure of λ is denoted by $\bar{\lambda}$ and defined as $\bar{\lambda} = \cap \{\mu : \lambda \subseteq \mu, \mu \in \tau^c\}$.

The interior of λ written λ^0 is defined by $\lambda^0 = \cup \{\mu : \mu \subseteq \lambda, \mu \in \tau\}$ (Liu *et al.*1997).

Definition: An L-fuzzy singleton in X is an L-fuzzy set in X which is zero everywhere except at one point say x, where it takes a value say r with $0 < r \leq 1$ and $r \in L$. The authors denote it by x_r and $x_r \in \alpha$ iff $r \leq \alpha(x)$ (Zadeh 1965).

Definition: An L-fuzzy singleton x_r is said to be quasi-coincident (q-coincident, in short) with an L-fuzzy set α in X, denoted by $x_r q \alpha$ iff $r + \alpha(x) > 1$. Similarly, an L-fuzzy set α in X is said to be q-coincident with an L-fuzzy set β in X, denoted by

$\alpha q\beta$ if and only if $\alpha(x)+\beta(x)>1$ for some $x\in X$. Therefore iff $\alpha(x)+\beta(x)\leq 1$ for all $x\in X$, where denote an L-fuzzy set α in X is said to be not q-coincident with an L-fuzzy set β in X (Liu *et al.*1997).

Definition: Let $f: X\to Y$ be a function and u be fuzzy set in X. Then the image $f(u)$ is a fuzzy set in Y which membership function is defined by

$$(f(u))(y)=\{\sup(u(x))\,[f(x)=y\}\text{ if } f^{-1}(y)= \emptyset, x\in X$$

$$(f(u))(y)=0 \text{ if } f^{-1}(y)=\emptyset, x\in X \text{ (Chang 1968).}$$

Definition: Let f be a real-valued function on an L-topological space. If $\{x: f(x)>\alpha\}$ is open for every real α, then f is called lower-semi continuous function (lsc, in short) (Liu *et al.*1997).

Definition: Let (X,τ) and (Y,s) be two L-topological space and f be a mapping from (X,τ) into (Y,s) *i.e.* $f:(X,\tau)\to(Y,s)$. Then f is called

(i) Continuous iff for each open L-fuzzy set $u\in s\Rightarrow f^{-1}(u)\in\tau$.

(ii) Open iff $f(\mu)\in s$ for each open L-fuzzy set $\mu\in\tau$.

(iii) Closed iff $f(\lambda)$ is s-closed for each $\lambda\in\tau^c$ where τ^c is closed L-fuzzy set in X .

(iv) Homeomorphism iff f is bijective and both f and f^{-1} are continuous (Liu *et al.*1997).

Definition: Let X be a nonempty set and T be a topology on X. Let $\tau=\omega(T)$ be the set of all lower semi continuous (lsc) functions from (X,T) to L (with usual topology). Thus $\omega(T)=\{u\in L^X: u^{-1}(\alpha,1]\in T\}$ for each $\alpha\in L$. It can be shown that $\omega(T)$ is a L-topology on X . Let "P" be the property of a topological space (X,T) and LP be its L-topological analogue. Then LP is called a "good extension" of P "if the statement (X,T) has P iff $(X,\omega(T))$ has LP" holds good for every topological space (X,T) (Liu *et al.* 1997).

Definition: Let $\{(X_i,\tau_i): i\in\Delta\}$ be a family of L-topological space. Then the space $(\Pi X_i,\Pi\tau_i)$ is called the product lts of the family $\{(X_i,\tau_i): i\in\Delta\}$ where $\Pi\tau_i$ denote the usual product L-topologies of the families $\{\tau_i: i\in\Delta\}$ of L-topologies on X (Li 1998).

R_0 SPACES IN L-TOPOLOGY

The authors now give the following definitions of R_0 L-topological spaces.

Definition: An lts (X,τ) is called

(a) $L-R_0(i)$ if $\forall x,y\in X, x\neq y$ whenever $\exists u\in\tau$ with $u(x)\neq u(y)$ then

$\exists\, v \in \tau$ such that $v(x) \neq v(y)$.

(b) $L - R_0(ii)$ if $\forall\, x, y \in X, x \neq y$ whenever $\exists\, u \in \tau$ with $u(x) = 1, u(y) = 0$ then $\exists\, v \in \tau$ such that $v(x) = 0, v(y) = 1$.

(c) $L - R_0(iii)$ if for any pair of distinct L-fuzzy points $x_r, y_s \in S(X)$ whenever $\exists\, u \in \tau$ with $x_r \in u, y_s \notin u$ then $\exists\, v \in \tau$ such that $x_r \notin v, y_s \in v$.

(d) $L - R_0(iv)$ if for all pairs of distinct L-fuzzy singletons $x_r, y_s \in S(X)$ and $x_r \bar{q} y_s$ whenever $\exists\, u \in \tau$ with then $\exists v \in \tau$ such that .

(e) $L - R_0(v)$ if for any pair of distinct L-fuzzy points $x_r, y_s \in S(X)$ whenever $\exists\, u \in \tau$ with $x_r \in u, u\bar{q}y_s$ then $\exists v \in \tau$ such that $y_s \in v, v\bar{q}x_r$.

(f) $L - R_0(vi)$ if for any pair of distinct L-fuzzy points $x_r, y_s \in S(X)$ whenever $\exists\, u \in \tau$ with $x_r \in u, y_s \cap u = 0$ then $\exists v \in \tau$ such that $y_s \in v, x_r \cap v = 0$.

(g) $L - R_0(vii)$ if $\forall\, x, y \in X, x \neq y$ whenever $\exists\, u \in \tau$ with $u(x) > 0, u(y) = 0$ then $\exists v \in \tau$ such that $v(x) = 0, v(y) > 0$.

(h) $L - R_0(viii)$ if $\forall\, x, y \in X, x \neq y$ whenever $\exists\, u \in \tau$ with $u(x) > u(y)$ then $\exists v \in \tau$ such that $v(y) > v(x)$.

"GOOD EXTENSION", HEREDITARY PROPERTY AND PRODUCT L-TOPOLOGY

Now all the definitions $L - R_0(i)$, $L - R_0(ii), L - R_0(iii), L - R_0(iv), L - R_0(v), L - R_0(vi), L - R_0(vii)$ and $L - R_0(viii)$ are 'good extensions' of R_0, as shown below:

Theorem: Let (X, T) be a topological space. Then (X, T) is R_0 iff $(X, \omega(T))$ is $L - R_0(j)$, for $j = i, ii, iii, iv, v, vi, vii, viii.$

Proof: Let the topological space (X, T) be R_0, The workers shall prove that the fuzzy topological $(X, \omega(T))$ is $L - R_0(ii)$. Choose $x, y \in X$ with $x \neq y$. Let $u \in \omega(T)$ with $u(x) = 1,$ $u(y) = 0$, then it is clear that $u^{-1}(r, 1] \in T$, for any $r \in I_1$ and $x \in u^{-1}(r, 1]$, $y \notin u^{-1}(r, 1]$. Since (X, T) is R_0, then there exist $V \in T$ with $x \notin V, y \in V$. Now consider the characteristics function 1_V . The authors see that $1_V \in \omega(T)$ with $1_V(x) = 0, 1_V(y) = 1$. Thus $(X, \omega(T))$ is $L - R_0(ii)$.

Conversely, let $(X, \omega(T))$ be $L - R_0(ii)$, the workers shall prove that (X, T) is R_0. Choose $x, y \in X$, $x \neq y$ and $U \in T$, with $x \in U, y \notin U$, but we know that the characteristic function $1_U \in \omega(T)$. Also it is clear that $1_U(x) = 1, 1_U(y) = 0$. Since $(X, \omega(T))$ is $L - R_0(ii)$, then $\exists v \in \omega(T)$ such that $v(x) = 0, v(y) = 1.$ Again since v

is lower semi continuous function then $v^{-1}(0,1] \in T$ and from above, we get $x \notin v^{-1}(0,1]$, $y \in v^{-1}(0,1]$. Hence (X, T) is R_0 .

Similarly the authors showed that $L-R_0(i), L-R_0(iii),$ $L-R_0(iv),$ $L-R_0(v), L-R_0(vi), L-R_0(vii)$ and $L-R_0(viii)$ are also hold 'good extension' property.

Theorem: Let (X,τ) be an lts, $A \subseteq X$ and $\tau_A = \{u|A : u \in \tau\}$, then

(a) (X,τ) is $L-R_0(i) \Rightarrow (A,\tau_A)$ is $L-R_0(i)$.

(b) (X,τ) is $L-R_0(ii) \Rightarrow (A,\tau_A)$ is $L-R_0(ii)$.

(c) (X,τ) is $L-R_0(iii) \Rightarrow (A,\tau_A)$ is $L-R_0(iii)$.

(d) (X,τ) is $L-R_0(iv) \Rightarrow (A,\tau_A)$ is $L-R_0(iv)$.

(e) (X,τ) is $L-R_0(v) \Rightarrow (A,\tau_A)$ is $L-R_0(v)$.

(f) (X,τ) is $L-R_0(vi) \Rightarrow (A,\tau_A)$ is $L-R_0(vi)$.

(g) (X,τ) is $L-R_0(vii) \Rightarrow (A,\tau_A)$ is $L-R_0(vii)$.

(h) (X,τ) is $L-R_0(viii) \Rightarrow (A,\tau_A)$ is $L-R_0(viii)$.

Proof: The authors proved only (b). Suppose (X,τ) is L-topological space and $L-R_0(ii)$. The workers shall prove (A,τ_A) is $L-R_0(ii)$. Let $x,y \in A$, $x \neq y$, and $w \in \tau_A$ with $w(x)=1,\quad w(y)=0$. Then $x,y \in X$ with $x \neq y$ as $A \subseteq X$. Consider u be the extension function of w on the set X, then it is clear that $u(x)=1, u(y)=0$. Since (X,τ) is $L-R_0(ii)$. Then $\exists\, v \in \tau$ such that $v(x)=0, v(y)=1$. For $A \subseteq X$, we find $\exists\, v|A \in \tau_A$ such that $v|A(x)=0, v|A(y)=1$ as $x,y \in A$. Hence it is clear that the subspace (A,τ_A) is $L-R_0(ii)$. Similarly, (a), (c), (d), (e), (f), (g), (h) can be proved.

Theorem: Given $\{(X_i,\tau_i) : i \in \Lambda\}$ be a family of L-topological space. Then the product of L-topological space $(\prod X_i,\quad \prod \tau_i)$ is $L-R_0(j)$ iff each coordinate space (X_i,τ_i) is $L-R_0(j)$ where $j = i, ii, iii, iv, v, vi, vii, viii$.

Proof: Let each coordinate space $\{(X_i,\tau_i) : i \in \Lambda\}$ be $L-R_0(ii)$. Then the authors show that the product space is $L-R_0(ii)$. Suppose $x,y \in \prod X_i$, $x \neq y$, and $u \in \prod \tau_i$ with $u(x)=1, u(y)=0.$ Choose $x = \prod x_i,\quad y = \prod y_i$, but they have $u(x) = \min\{u_i(x_i), \textbf{for } i \in \Lambda \textbf{ and } u_i \in \tau_i\}$, $u(y) = \min\{u_i(y_i), \textbf{for } i \in \Lambda \textbf{ and } u_i \in \tau_i\}$, then there exist at least one $j \in \Lambda,$ such that $x_j \neq y_j$ and $u_j(x_j)=1$, $u_j(y_j)=0$. Since (X_j,τ_j) is $L-R_0(ii)$ for each $j \in \Lambda$, then $\exists\, v_j \in \tau_j$ such that $v_j(x_j)=0, v_j(y_j)=1$. Now take $v = \prod v'_j$ where $v'_j = v_j$ and $v_i = 1$ for $i \neq j$, then $\exists v \in \prod \tau_i$ such that $v(x)=0, v(y)=1$. Hence the product L-topological space $(\prod X_i, \prod \tau_i)$ is $L-R_0(ii)$.

Conversely, let the product of L-topological space $(\prod X_i, \prod \tau_i)$ is $L-R_0(ii)$. The workers shall prove each coordinate space (X_j,τ_j) is also $L-R_0(ii)$. Choose $x_j, y_j \in X_j, x_j \neq y_j$ and $u_j \in \tau_j$ with $u_j(x_j)=1, u_j(y_j)=0$. Now, construct $x,y \in X$ such that $x = \prod x'_i, y = \prod y'_i$ where $x'_i = y'_i$ for $i \neq j$ and $x'_j = x_j, y'_j = y_j$. Then $x \neq y$. Further, let $\pi_j : X \to X_j$ be a projection map from X into X_j. Now, the workers observe that

$u_j\left(\left((\pi_j)(x)\right)\right) = u_j(x_j) = 1, u_j\left(\left((\pi_j)(y)\right)\right) = u_j(y_j) = 0$, i.e for $u_j o\pi_j \in \Pi\tau_i$, with $(u_j o\pi_j)(x) = 1, (u_j o\pi_j)(y) = 1$. Since the product space $(\Pi X_i, \Pi\tau_i)$ is $L-R_0(ii)$. Then $\exists v \in \Pi\tau_i$ such that $v(x) = 0, v(y) = 1$. Now choose any L-fuzzy point y_r in v. Then $\exists$ a basic open L-fuzzy set $\Pi v_j^r \in \Pi\tau_j$ such that $y_r \in \Pi v_j^r \subseteq v$ which implies that $r < \Pi v_j^r(y)$ or that $r < inf_j v_j^r\left(y_j'\right)$ and hence $r < \Pi v_j^r\left(y_j'\right) \forall j \in \Lambda \ldots\ldots(i)$ and $v(x) = 0 \Rightarrow \Pi v_j(x) = 0 \ldots\ldots(ii)$. Further, $\Pi v_j^r(x) = 0 \Rightarrow v_i^r(x_i) = 0$, since for $j \neq i, x_j' = y_j'$ and hence from (i), $u_j^r(x_j) = v_j^r(y_j) > r$. Thus we have $v_i^r(y_i) > r, v_i^r(x_i) = 0$. Now consider $sup_r v_i^r = v_i \in \tau_i$ then $v_i \in \tau_i$ with $v_i(y_i) = 1, v_i(x_i) = 0$. This showing that (X_i, τ_i) is $L-R_0(ii)$.

Moreover one can verify that

$(X_i, \tau_i), i \in \Lambda$ is $L-R_0(i) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L-R_0(i)$

$(X_i, \tau_i), i \in \Lambda$ is $L-R_0(iii) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L-R_0(iii)$

$(X_i, \tau_i), i \in \Lambda$ is $L-R_0(iv) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L-R_0(iv)$

$(X_i, \tau_i), i \in \Lambda$ is $L-R_0(v) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L-R_0(v)$

$(X_i, \tau_i), i \in \Lambda$ is $L-R_0(vi) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L-R_0(vi)$

$(X_i, \tau_i), i \in \Lambda$ is $L-R_0(vii) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L-R_0(vii)$

$(X_i, \tau_i), i \in \Lambda$ is $L-R_0(viii) \Leftrightarrow (\Pi X_i, \Pi\tau_i)$ is $L-R_0(viii)$.

Hence, the workers see that $L-R_0(i), L-R_0(ii), L-R_0(iii), L-R_0(iv), L-R_0(v), L-R_0(vi), L-R_0(vii), L-R_0(viii)$ properties are productive and projective.

MAPPING IN L-TOOLOGICAL SPACES

The present authors show that $L-R_0(j)$ property is preserved under one-one, onto and continuous maps for $j = i, ii, iii, iv, v, vi, vii, viii$.

Theorem: Let (X, τ) and (Y, s) be two L-topological space and

$f:(X,\tau) \to (Y, s)$ be one-one, onto, L-continuous and L-open map, then-

(a) (X, τ) is $L-R_0(i) \Rightarrow (Y, s)$ is $L-R_0(i)$.

(b) (X, τ) is $L-R_0(ii) \Rightarrow (Y, s)$ is $L-R_0(ii)$.

(c) (X, τ) is $L-R_0(iii) \Rightarrow (Y, s)$ is $L-R_0(iii)$.

(d) (X, τ) is $L-R_0(iv) \Rightarrow (Y, s)$ is $L-R_0(iv)$.

(e) (X, τ) is $L-R_0(v) \Rightarrow (Y, s)$ is $L-R_0(v)$.

(f) (X,τ) is $L-R_0(vi) \Rightarrow (Y,s)$ is $L-R_0(vi)$.

(g) (X,τ) is $L-R_0(vii) \Rightarrow (Y,s)$ is $L-R_0(vii)$.

(h) (X,τ) is $L-R_0(viii) \Rightarrow (Y,s)$ is $L-R_0(viii)$.

Proof: Suppose (X,τ) is $L-R_0(ii)$. The authors shall prove that (Y,s) is $L-R_0(ii)$. Let $y_1, y_2 \in Y$, $y_1 \neq y_2$ and $u \in s$ with $u(y_1)=1, u(y_2)=0$. Since f is onto, $\exists\, x_1, x_2 \in X$ such that $[f(x]_1) = y_1, f(x_2) = y_2$ and $x_1 \neq x_2$ as f is one-one.

Now $f^{-1}(u)(x_1) = u(f(x_1)) = u(y_1) = 1$ and

$$f^{-1}(u)(x_2) = u(f(x_2)) = u(y_2) = 0$$

Since f is L-continuous then $f^{-1}(u) \in \tau$ and $f^{-1}(u)(x_1) = 1$, $f^{-1}(u)(x_2) = 0$.

Since (X,τ) is $L-R_0(ii)$, then $\exists\, v \in \tau$ such that $v(x) = 0, v(y) = 1$. Now

$$f(v)(y_1) = \{supv(x_1): [f(x]_1) = y_1\} = 0$$

$$f(v)(y_2) = \{supv(x_2): [f(x]_2) = y_2\} = 1.$$

Since f is L-open, $f(u) \in s$. Now, it is clear that $\exists\, f(v) \in s$ such that $f(v)(y_1) = 0,\ f(v)(y_2) = 1$. Hence, it is clear that the L-topological space (Y,s) is $L-R_0(ii)$. Similarly (a), (c), (d), (e), (f), (g), (h) can be proved.

Theorem: Let (X,τ) and (Y,s) be two L-topological spaces and $f:(X,\tau) \to (Y,s)$ be one-one, L-continuous and L-open map, then-

(a) (Y,s) is $L-R_0(i) \Rightarrow (X,\tau)$ is $L-R_0(i)$.

(b) (Y,s) is $L-R_0(ii) \Rightarrow (X,\tau)$ is $L-R_0(ii)$.

(c) (Y,s) is $L-R_0(iii) \Rightarrow (X,\tau)$ is $L-R_0(iii)$.

(d) (Y,s) is $L-R_0(iv) \Rightarrow (X,\tau)$ is $L-R_0(iv)$.

(e) (Y,s) is $L-R_0(v) \Rightarrow (X,\tau)$ is $L-R_0(v)$.

(f) (Y,s) is $L-R_0(vi) \Rightarrow (X,\tau)$ is $L-R_0(vi)$.

(g) (Y,s) is $L-R_0(vii) \Rightarrow (X,\tau)$ is $L-R_0(vii)$.

(h) (Y,s) is $L-R_0(viii) \Rightarrow (X,\tau)$ is $L-R_0(viii)$.

Proof: Suppose (Y,s) is $L-R_0(ii)$. The workers shall prove that (X,τ) is $L-R_0(ii)$. Let $x_1, x_2 \in X$, $x_1 \neq x_2$ and $u \in \tau$ with $u(x_1) = 1, u(x_2) = 0$. Since f is one-one map then $f(x_1) \neq f(x_2)$.

Now $f(u)(f(x_1)) = \sup\{u(x_1)\} = 1$ as f is one-one

And $f(u)(f(x_2)) = \sup\{u(x_2)\} = 0.$

So, the authors have $f(u) \in s$, with $f(u)(f(x_1)) = 1,$ $f(u)(f(x_2)) = 0$, as f is L-open map. Since (Y, s) is $L - R_0(ii),$ $\exists v \in s$ such that $v(f(x_1)) = 0, v(f(x_2)) = 1$. This implies that $f^{-1}(v)(x_1) = 0, f^{-1}(v)(x_2) = 1$ and $f^{-1}(v) \in \tau$ as f is L-continuous and $v \in s$. Now it is clear that $\exists\, f^{-1}(v) \in \tau$ such that $f^{-1}(v)(x_1) = 0$, $f^{-1}(v)(x_2) = 1$. Hence the L-topological space (X, τ) is $L - R_0(ii)$. Similarly (a), (c), (d), (e), (f), (g), (h) can be proved.

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