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MAPPINGS IN FUZZY HAUSDORFF SPACES IN QUASI-COINCIDENCE SENSE

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ABSTRACT

In this paper, we introduce two notions of property in fuzzy topological spaces by using quasi-coincidence sense and we establish relationship among our and other such notions. We also show that all these notions satisfy good extension property. Also hereditary, productive and projective properties are satisfied by these notions. We observe that all these concepts are preserved under one-one, onto, fuzzy open and fuzzy continuous mappings. Finally, we discuss initial and final fuzzy topologies on our second notion.

Key words: Fuzzy Topological Space, Quasi-coincidence, Fuzzy Hausdorff Topological Spaces, Good Extension, Mappings

INTRODUCTION

Chang (1968) defined fuzzy topological spaces by using fuzzy sets introduced by Zadeh (1965). Since then extensive work on fuzzy topological spaces has been carried out by many researchers like Gouguen (1973), Wong (1974), Warren (1974), Hutton (1975), Lowen (1976), and others. Fuzzy Hausdorff (topologicalspace has been already introduced in the literature. There are many articles on fuzzy topological space which are created by many authors like Wuyts and Lowen (1983), Srivastava *et al.* (1981), Ali (1990) and others.

The purpose of this paper is to further contribute to the development of fuzzy topological spaces especially on fuzzy topological spaces. In the present paper, fuzzy topological space is defined by using quasi-coincidence senseand it is showed that the good extension property is satisfied by our notions. In the next section of this paper, it is also shown that the hereditary, order preserving, productive, and projective properties hold on the new concepts. Finally, we discuss initial and final fuzzy topologies on our second notion.

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PRELIMINARIES

In this section, we recall some concepts occurring in the cited papers whichwill be needed in the sequel. In the present paper X and Y always denote non-empty sets.

Definition: A function u from X into the unit interval I is called afuzzy set in X. For every is called the grade of membership of x in u. Some authors say that u is a fuzzy subset of instead of saying that u is a fuzzy set in . The class of all fuzzy sets from into the closed unit interval will be denoted by (Zadeh 1965).

Definition: A fuzzy set in is called a fuzzy singleton if and only if for a certain for all points of except . The fuzzy singleton is denoted by and is its support. The class of all fuzzy singletons in X will be denoted by . If and , then we say that if and only if (Ming 1980).

Definition: A fuzzy set u in X is called a fuzzy point if and only if, for a certain and for all points y of X except x. The fuzzy point is denoted by and x is its support (Wong 1974).

Definition: A fuzzy singleton is said to be quasi-coincidence with u, denoted by if and only if . If is not quasi-coincidencewith u, we write and defined as (Kandil 1991).

Definition: Let f be a mapping from a set X into a set Y and u be a fuzzy subset of X. Then f and u induce a fuzzy subset v of Y defined by

if otherwise (Chang 1968).

Definition: Let f be a mapping from a set X into a set Y and v be a fuzzy subset of Y. Then the inverse of v written as is a fuzzy subset of X defined by , for $x \in X$ (Chang 1968).

Definition: Let $I = [0, 1]$, X be a non empty set and be the collection of all mappings from X into I, *i.e.* the class of all fuzzy sets in X.A fuzzy topology on X is defined as a family t of members of , satisfyingthe following conditions.

(*i*)

(*ii*) If for each , then , where Λ is an index set.

(*iii*) If $u$then.

The pair (X, t) is called a fuzzy topological space (in short fts) and members of t are called *t-open* fuzzy sets. A fuzzy set v is called a *t-closed* fuzzy set if (Chang 1968).

Definition: The function is called fuzzy continuous if and only if for every , the function f is called fuzzy homeomorphic if and only if f is bijective and both f and are fuzzy continuous(Ming 1980).

Definition: The function is called fuzzy open if and only if for every open fuzzy set u in is open fuzzy set in (Y, s) (Malghan 1984).

Definition: If u and v are two fuzzy subsets of X and Y respectively then the Cartesian product of two fuzzy subsets u and v is a fuzzy subsetof defined by , for each pair (Azad 1981).

Definition: Let *{, i ∈ Λ},* be any class of sets and let X denotes the Cartesian product of these sets, Note that X consists of all points , where . Recall that, for each , we define the projection from the product set X to the coordinate space,i.e. by .These projections are used to define the product topology (Lipschutz 1965).

Definition: Let be a family of nonempty sets. Let be the usual product of 's and let be the projection from X into . Further assume that each is an fuzzy topological space with fuzzy topology . Now, the fuzzy topology generated by as a sub basis, is called the product fuzzy topology on X. Clearly if w is a basis element in the product, then there exist , with such that (Wong 1974).

Definition: Let f be a real valued function on a topological space. If is open for every real α, then f is called lower semicontinuous function (Rudin 1966).

Definition: Let X be a nonempty set and T be a topology on X. Let be the set of all lower semi continuous functions from (X, T) to I (with usual topology). Thus for each $\alpha\in$. It can be shown that $\omega(T)$ is a fuzzy topology on X (Lowen 1976).

Let P be the property of a topological space (X, T) and FP be its topological analogue. Then FP is called a 'good extension' of P if and only if the statement (X, T) has P if and only if has FP holds good for every topological space (X, T).

Definition: The initial fuzzy topology on a set X for the family of fts and the family of functions is the smallest fuzzy topology on X making each fuzzy continuous. It is easily seen that it is generated by the family (Lowen 1977).

Definition: The final fuzzy topology on a set X for the family of fts and the family of functions is the finest fuzzy topology on X making each fuzzy continuous (Lowen 1977).

Theorem: A bijective mapping from an fts (X, t) to an fts (Y, s) preserves the value of a fuzzy singleton (fuzzy point). Note: Preimage of any fuzzy singleton (fuzzy point) under bijective mapping preserves its value (Amin *et al.* 2014).

THE MAIN RESULTS

In this section, we discuss about our notions and findings. Some well-known properties are discussed here by using our concepts.

Definition: A fuzzy topological space is called

if and only if for any pair with, there exists such that, and

if and only if for any pair with , there exists such that , and .

(c) if and only if for any pair with, there exists such that, and (Srisvastava *et al.* 1981).

Theorem: For a fuzzy topological space the following implications are true:

, . But in general the converse is not true.

Proof of Let be a fuzzy topological space and is. We have to prove that is. Let be fuzzy points in *X* with. Since is *F* fuzzy topological space, we have, there exists such that and . To prove is, it is only needed to prove that .

Now, . It follows that there exists such that , and . Hence it is clear that is .

To show is is, we give a counter example.

Counter-example: Let and be given by, , , . Let us consider the fuzzy topology on generated by . For, and, . Now, and. But . Hence it is clear that is but is not .

Proof of: Let be a fuzzy topological space and is. We have to prove that is . Let be fuzzy points in *X* with. Since is *F* fuzzy topological space, we have, there exists such that and . To prove is, it is only needed to prove that , and . Now, , for any . Similarly, . Hence it is clear that is .

To show is is, we give a counter example. **Counter-example:** Let and be given by, and , , where for and Let us consider the fuzzy topology on generated by . Then, . Similarly, . Hence it is clear that is . But and . Thus is not .

Proof of: Let be a fuzzy topological space and is. We have to prove that is . Let be fuzzy points in *X* with. Since is *F* fuzzy topological space, we have, there exists such that and . To prove is, it is only needed to prove that , and . Now, , for any . Similarly, .

Also, =0=0= 0

. Hence it is clear that is .

To show is is, we give a counter example. **Counter-example:** Let and be given by, and , , where for Let us consider the fuzzy topology on generated by . Then, . Similarly, . Also, i.e.. Hence it is clear that is . But , and 0 00. Thus is not . This completes the proof.

Now we shall show that our new notions satisfy the good extension property.

Theorem: Let be a fuzzy topological space. Consider the following statements:

(1) be a topological space.

(2)be an space.

(3)be an space. Then thesis implications are true: , .

Proof of (1) ⇔ (2): Let be a topological space and is. We have to prove that is. Let be fuzzy points in X with . Since is topological space, we have, there exists such that , and . From the definition of lower semi continuous we have and ,.Then . Similarly, . Also, If then there exists z in X such that , , a contradiction. So, It follows that there exists , such that , and Hence is . Thus holds.

Conversely, be a fuzzy topological space and is We have to prove that is . Let be points in with . Since is topological space we have, for any fuzzy points in , there exists such that , *and u.*

Now,.

Similarly, . Also,

Again, *u* We claim that For, if , then and and , a contradiction. Thus,

It follows that such that , and . Hence, is topological space. Thus holds. Similarly, we can prove that .

Now we shall show that our new notions satisfy the hereditary property.

Theorem: Let be a fuzzy topological space, , then

(*a*) is is and

(*b*) is is .

Proof of (*b*) : Let be a fuzzy topological space and is . We have to prove thatis . Let be fuzzy points in *A* with . Since, these fuzzy points are also fuzzy points in . Also since is fuzzy topological space, we have, there exists such that and u. For , we have u.

Now, . Similarly, . And. It follows that there exist u such that and . Hence, is Similarly, we can prove (*b*).

Now we shall show that our new notions satisfy the productive and projective property.

Theorem: Let be fuzzy topological spaces and and be the product topology on , then

(*a*) for all is if and only if is and

(*b*) for all is) if and only if is

Proof of (a): Let for all is space. We have to prove that is . Let be fuzzy points in with. Then are fuzzy points with for some . Since is , there exists such that and . Now, .

But we have and .

Now,

. Similarly, we can prove that .

Also,

. Hence, It follows that there exists such that and

Hence it is clear that is.

Conversely, Let be a fuzzy topological space and is . We have to prove that . Here let us consider, be a fixed element in . Let
Then is a subset of , and hence is a subspace of . Since is , so is . Now we have is homeomorphic image of . Hence it is clear that for all is space. Thus (*a*) holds. Similarly, we can prove (*b*).

Theorem: Let (X, t) and (Y, s) be two fuzzy topological spaces and be a one-one, onto and fuzzy open map then

and

Proof of (*a*): Let be a fuzzy topological space and is . We have to prove that is . Let be fuzzy points in Y with x y Since f is onto then there exist with , and , are fuzzy points in X with as f is one-one. Again since (X, t) is $F(i)$ space, there exists $u, v \in t$ such that and u . Now, and, Now,} for some x } for some y. Also we have, f is a fuzzy open mapping. Then .
Again, and . Also, u Now,

f(u f(u and

f(uf(u.

Therefore, f(u f(u. It follows that there exists such that and f(u. Hence it is clear that is space. Similarly, we can prove (*b*).

Theorem: Let (X, t) and (Y, s) be two fuzzy topological spaces and $f: X \rightarrow Y$ be a one-one, onto and fuzzy continuous mapping then,

(*a*) and

(*b*) .

Proof of (*b*): Let (Y, s) be a fuzzy topological space and (Y, s) is $F(ii)$. We have to prove that is . Let be fuzzy points in *X* with .Then are fuzzy points in *Y* with as *f* is one-one. Again since is space, there exists such that . Now, . Similarly, we can prove that .Also

Now since, is fuzzy continuous mapping and then . It follows that there existssuch that and . Hence it is clear that is space. Similarly, we can prove (a).

As our next work, here we introduce two theorems on the second notion of us. The idea of these theorems are taken from Amin (2016).

Theorem: If is a family of fts and , a family of one-one and fuzzy continuous functions, then the initial fuzzy topology on X for the family is .

Proof: Let t be the initial fuzzy topology on X for the family . Let be fuzzy points in *X* with . Then (x), (y) and (x)(y) as is one-one. Scince is , then for every two distinct fuzzy points , in , there exist fuzzy sets , such that q , and . Now,q and . That is and That is and Also, . That is . That is This is true for every . So inf , inf and inf . Let u= inf and v= inf . Then u as is fuzzy continuous.So, v(y)+s>1 and . Hence, ,and . Therefore, (*X*, *t*) is

Theorem: If is a family of fts and , a family of fuzzy open and bijective function, then the final fuzzy topology on X for the family is .

Proof: Let t be the final fuzzy topology on X for the family . Let be fuzzy points in *X* with . Then (x), (y) and (x)(y) as is bijective. Since is , then for every two distinct fuzzy points , in , there exist fuzzy sets , such that q , and . Now, q and . That is and That is and Also . That is That is This is true for every . So inf , inf and Let u= infand v= inf. Then u, v as is fuzzy open. So, , u(y)+s >1 and . Hence, , and . Therefore, (X, t) is

REFERENCES

Ali, D. M. 1990. On Certain Separation and Connectedness Concepts in Fuzzy Topology. *PhD thesis, Banaras Hindu University, India.*

Amin, M. R., D. M. Ali and M. S. Hossain. 2014. On Fuzzy Bitopological Spaces. *Journal of Bangladesh Academy of Sciences*. **32(2):** 209- 217.

Amin, M. R. and M. S. Hossain. 2016.On Concepts in Fuzzy Bitopological Spaces. *Annals of Fuzzy Mathematics and Informatics.* **11(6):** 945- 955.

Azad, K. K. 1981. On Fuzzy semi-continuity, Fuzzy almost continuity and Fuzzy weakly continuity. *J. Math. Anal. Appl* .**82(1):** 14-32.

Chang, C. L. 1968. Fuzzy Topological Spaces. *J. Math. Anal. Appl.* **24:** 182-192.

Goguen, T. A. 1973.Fuzzy Tychonoff Theorem. *J. Math. Anal. Appl.* **43:** 734-742.

Hutton, B. 1975. Normality in Fuzzy Topological Spaces.*J. Math. Anal. Appl.* **50:** 74-79.

Kandil and El-Shafee. 1991.Separation Axioms for Fuzzy Bitopological Spaces. *J. Inst. Math. Comput. Sci.* **4(3):** 373-383.

Lipschutz, S. 1965. General Topology. *The Schaum publishing company.*

Lowen, R. 1976. Fuzzy Topological Spaces and Fuzzy Compactness. *J. Math. Anal. Appl.* **56:** 621-633.

Lowen, R. 1977. Initial and Final Fuzzy Topologies and the Fuzzy Tyconoff Theorem. *J. Math. Anal. Appl.* **58:** 11-21.

Malghan, S. R. and S. S. Benchalli. 1984. On open maps, closed maps and local compactness in fuzzy topological spaces. *J. Math. Anal. Appl.* **99(2):** 338-349.

Ming, Pu. Pao and Liu Ying Ming. 1980. Fuzzy Topology I. Neighborhood structure of a fuzzy point and Moore-Smith convergence. *J. Math. Anal. Appl.* **76:** 571-599.

Ming, Pu. Pao and Liu Ying Ming. 1980. Fuzzy Topology II. Product and Quotient Spaces. *J. Math. Anal. Appl.* **77:** 20-37.

Rudin, W. Real. 1966. *Real and Complex Analysis*. McGrawHill Inc.

Srivastava, R., S. N. Lal and A. K. Srivastava. 1981. Fuzzy Hausdorff Topological Spaces. *J. Math. Anal. Appl.* **81:** 497-506.

Warren, R. H. 1974. Continuity of Mappings in Fuzzy Topological Spaces. *Notices A.M. S.* **21:** A-451.

Wong, C. K. 1974. Fuzzy Topology: Product and Quotient Theorem. *J. Math. Anal. Appl.* **45:** 512-521.

Wong, C. K. 1974. Fuzzy points and local properties of Fuzzy Topology. *J. Math. Anal. Appl.* **46:** 316-328.

Wuyts, P. and R. Lowen. 1983. On Separation Axioms in Fuzzy TopologicalSpaces, Fuzzy Neighborhood Spaces, and Fuzzy Uniform Spaces. *J. Math. Anal. Appl.* **93:** 27-41.

Zadeh, L. A. 1965. Fuzzy sets. *Information and control.* **8:** 338-353.

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