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# STUDIES ON FUZZY CONNECTEDNESS

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ABSTRACT

The aim of this paper is to study the fuzzy connectedness in Raja-Sethupathy-Lakshmivarahan's sense. Some characterizations and the properties of this type of fuzzy connectedness on fuzzy topological spaces are studied. Interrelationships among fuzzy connectedness are examined. We prove that fuzzy connectedness is preserved under fuzzy continuous mapping.

Key words: Fuzzy topological spaces, Fuzzy clopen sets, Fuzzy connectedness, Fuzzy continuous functions

## INTRODUCTION

Following the introduction of fuzzy sets by L. A. Zadeh (1965), C. L. Chang (1968) defined fuzzy topological space which is a successful generalization of general topological space. Since then extensive work on fuzzy topological spaces has been carried out by authors like Wong (1974), Warren (1974), Hutton (1975), Lowen (1976), Shahjahan and Amin (2017) and many others. The concepts of fuzzy connectedness have been introduced earlier by Lowen and Srivastava (1992), Wuyts (1987), D.M.Ali (1992), Tapi and Deole (2014), Fatteh and Bassan(1985), G. Jagor (1998) and others. Also there are many articles on fuzzy connectedness by researchers like Raja-Sethupathy and Lakshmivarahan (1977), Ali and Srivastava (1988), Zheng (1984), Amin and Hossain (2017), and others.

The purpose of this paper is to further contribute to the development of fuzzy topological spaces especially on fuzzy connectedness. In the present paper, we study the fuzzy connectedness in Raja Sethupathy-Lakshmivarahan's (1977) sense. Some characterizations and the effects of this type of fuzzy connectedness on fuzzy topological spaces are studied. Also we observe that some properties of connectedness on topological

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spaces hold as fuzzy connectedness on fuzzy topological spaces. Interrelationships among fuzzy connectedness are examined. In addition, we discuss about image of fuzzy connectedness space under a fuzzy continuous mapping.

## BASIC NOTIONS AND RESULTS

In this section, we recall some concepts occurring in the cited which will be needed in the sequel. In the present paper $X$ amd $Y$ always denote non-empty sets and $I = [0, 1]$.

**Definition:** A function $u$ from $X$ into the unit interval $I$ is called a fuzzy set in $X$. For every $x \in X$, $u(x) \in I$ is called the grade of membership of $x$ in $u$. Some authors say that $u$ is a fuzzy subset of $X$ instead of saying that $u$ is a fuzzy set in $X$ . The class of all fuzzy sets from $X$ into the closed unit interval $I$ will be denoted by $I^X$ (Zadeh 1965).

**Definition:** Suppose $f \in I^X$ be a fuzzy set. Then the complement of $f$ is the fuzzy set $f : X \rightarrow I$, defined by $f\text{â}(x) = 1 - f(x)$, $\forall x \in X$ (Zadeh 1965).

**Definition:** Suppose $f, g \in I^X$ are fuzzy sets. Then the union of $f$ and $g$ is the fuzzy set in $I^X$ denoted by $f \cup g$ and defined by

$$(f \cup g)(x) = \max\{f(x), g(x)\} = f(x) \vee g(x), \forall x \in X \text{ (Zadeh 1965).}$$

**Definition:** Suppose $f, g \in I^X$ are fuzzy sets. Then the intersection of $f$ and $g$ is the fuzzy set in $I^X$ denoted by $f \cap g$ and defined by

$$(f \cap g)(x) = \max\{f(x), g(x)\} = f(x) \vee g(x), \forall x \in X \text{ (Zadeh 1965).}$$

**Definition:** Let $f$ be a mapping from a set $X$ into a set $Y$ and $u$ be a fuzzy subset of $X$. Then $f$ and $u$ induce a fuzzy subset $v$ of $Y$ defined by (Chang 1968)

$$v(y)\begin{cases} \sup\{u(x)\} & \text{if } x \in f^{-1}(y) \neq \emptyset, x \in X \\ & 0, \textit{otherwise} \end{cases}$$

**Definition:** Let $f$ be a mapping from a set $X$ into a set $Y$ and $u$ be a fuzzy subset of $X$. Then the inverse of $v$ written as $f^{-1}(v)$ is the fuzzy subset of $X$ defined by

$(f^{-1}(v))(x) = v(f(x))$, for all $x \in X$ (Chang 1968).

**Definition:** Let X be a set and $I$ be the unit interval. A fuzzy topology on $X$ is a subset $t \subset I^X$ such that

(i) $0, 1 \in t$

(ii) $\forall \mu, \lambda \in t \Rightarrow \mu \wedge \lambda \in t$

(iii) $\forall (\mu_i)_{i \in I} \subset t \Rightarrow \sup \mu_i \in t$

Then $(X, t)$ is called a fuzzy topological space or in short fts If $\mu \in t$ then $\mu$ is said to be fuzzy $t$-open set. A fuzzy set $\mu$ is $t$-closed if $\mu^c$ is fuzzy open (Chang 1968).

**Definition:** The closure of a fuzzy set λ, denoted by $\overline{a}$ and the interior of a fuzzy set λ, denoted by $\lambda^0$ are defined respectively by

$\overline{a} = \cap \{\mu : \mu \in t^c$ and $\lambda \subseteq \mu\}$, $\lambda^0 = \cup \{\mu : \mu \in t)$ and $\mu \subseteq \lambda\}$ (Chang 1968).

**Definition:** A function $f : (X, t) \rightarrow (Y, s)$ is called fuzzy continuous iff for every $v \in s$, $f^{-1}(v) \in t$; $f$ is called a fuzzy homeomorphism iff $f$ is bijective and both $f$ and $f^{-1}$ are fuzzy continuous (P. Ming and Y. Ming 1980).

**Definition:** A function $f : (X, t) \rightarrow (Y, s)$ is called fuzzy open iff for every open fuzzy set $v \in t$, $f(v) \in s$ is open; $f$ is called fuzzy closed iff for every closed $v \in t^c$, $f(v) \in s^c$ (Malghan and Benchalli 1984).

**Definition:** Let $(X, t)$ be a fuzzy topological space. A fuzzy set μ in $X$ is called fuzzy clopen if it is both fuzzy open and fuzzy closed (Ali and Srivastava.1988).

**Definition:** A fuzzy set in $X$ is called a fuzzy point iff it takes the value 0 for all y in $X$ except one, say $x \in X$. If its value at $x$ is $p(0 < p \leq 1)$, we denote this fuzzy point by $x_p$, where $x$ is called its support (Pu and Liu 1980).

**Definition:** A fuzzy point $x_p$ is said to be contained in a fuzzy set μ or to belong to μ, denoted by $x_p \in \mu$, iff $p \leq \mu(x)$.

Evidently every fuzzy set can be expressed as the union of all the fuzzy points which belong to μ (Pu and Liu 1980).

**Theorem:** A bijective mapping from a fts $(X, t)$ to a fts $(Y, s)$ preserves the value of a fuzzy singleton (fuzzy point).

**Note:** Preimage of any fuzzy singleton (fuzzy point) under bijective mapping preserves its value (Amin *et al.* 2014).

## MAIN RESULTS

**Definition:** A fuzzy topological space $X$ is said to be connected if and only if there does not exists two non-empty open fuzzy sets μ and λ in $X$ such that $\mu \cup \lambda = X$ and $\mu \cap \lambda = \varnothing$ (Sethupathy and Lakshmivarahan 1977).

**Theorem:** If the fuzzy sets μ and λ form a separation of fts $X$ and if $Y$ is a connected fuzzy subspace of $X$, then $Y$ lies entirely either in μ or in λ.

**Proof:** Since the fuzzy sets μ and λ form a separation of fts $X$, then by definition, μ and λ are non-empty open fuzzy sets such that $\mu \cup \lambda = X$ and $\mu \cup \lambda = \varnothing$. Then $\mu \cup Y = X$ and $\mu \cup Y$ and are open fuzzy set in $Y$. Now

$(\mu \cap Y) \cup (\lambda \cap Y) = (\mu \cup \lambda) \cap Y = X \cap Y = Y$.

Also $(\mu \cap Y) \cap (\lambda \cap Y) = (\mu \cap \lambda) \cap Y = \varnothing \cap Y = \varnothing$.

Now, if $\mu \cap Y$ and $\lambda \cap Y$ are both non-empty, then $\mu \cap Y$ and $\lambda \cap Y$ form a separation of *Y*, which contradicts the fact that *Y* is fuzzy connected. Therefore, one of $\mu \cap Y$ or $\lambda \cap Y$ must be empty, say $\mu \cap Y = \varnothing$.

So that $Y = (\mu \cap Y) \cup (\lambda \cap Y) = \varnothing \cup (\mu \cap Y) = \lambda \cap Y$ which implies that $Y \subset \lambda$.

Hence *Y* lies entirely in $\lambda$. Similarly, if $(\lambda \cap Y) = \varnothing$, then *Y* lies entirely in $\mu$.

**Theorem:** The union of a collection of fuzzy connected subspace of fts *X* that have a fuzzy point in common is fuzzy connected.

**Proof:** Let $\{u_i\}$be a collection of fuzzy connected subspace of fts *X* and let $x_p \in \cap \mu_i$. Now, we have to prove that $Y = \cup\mu_i$ is fuzzy connected. Suppose that *Y* is not fuzzy connected. Then there exists two non-empty open fuzzy sets $\lambda_1$ and $\lambda_2$ in *Y* such that $\lambda_1 \cup \lambda_2 = Y$ and $\lambda_1 \cap \lambda_2 = \varnothing$. Since $x_p \in Y$, which gives that either $x_p \in \lambda_1$ or $x_p \in \lambda_2$. Consider $x_p \in \lambda_1$. Since $\mu_i$ is fuzzy connected subspace of *Y*, then by previous theorem, $\mu_i$ lies entirely either in $\lambda_1$ or in $\lambda_2$, we get $\mu_i \subset \lambda_1$ or $\mu_i \subset \lambda_2$. Since $x_p$ is not in $\lambda_2$ and $x_p\ \lambda\ \mu_i$, so that $\mu_i$ lies entirely in $\lambda_1$, i.e., $\mu_i \subset \lambda_1$ for every $i \in I$. Thus

$\cup\ \mu_i \subset \lambda_1 \Rightarrow Y \subset \lambda_1$. Then $Y \subset \lambda_1$ and $\lambda_1 \cup \lambda_2 = Y$ implies that $\lambda_2 = \varnothing$ (as *Y* is not in $\lambda_2$), which is a contradiction as $\lambda_1$ and $\lambda_2$ form a separation of *Y*. Therefore, *Y* is fuzzy connected.

**Theorem:** A fuzzy topological space *X* is connected if and only if the only fuzzy clopen subsets of *X* that are the empty and *X* itself.

**Proof:** Suppose *X* is connected and $\mu$ be a non-empty proper fuzzy subset of *X* which is both fuzzy open and fuzzy closed in *X*. Now, let $\lambda_1 = \mu$ and $\lambda_2 = X \setminus \mu$. Thus $\lambda_1$ and $\lambda_2$ are complement of each other, so they are both open and closed. For $\lambda_1$ and $\lambda_2$ are non-empty open fuzzy sets in *X*, we have $\lambda_1 \cap \lambda_2 = \mu \cap (X\backslash\mu) = \varnothing$.

Also $\lambda_1 \cup \lambda_2 = \mu \cup (X\backslash\mu) = X$. So they form a separation of *X*, which contradicts the fact that *X* is fuzzy connected.

Conversely, suppose that *X* is not fuzzy connected. Then there exists two non-empty open fuzzy sets $\mu$ and $\lambda$ in *X* such that $\mu \cup \lambda = X$ and $\mu \cap \lambda = \varnothing$. Here $\mu^c = \lambda$ so $\mu$ is a fuzzy clopen set which contradicts the that *X* has only two fuzzy clopen sets which are empty set and *X* itself. Hence, *X* is connected.

**Lemma:** Let $(X, t)$ be an fts. Then the following conditions are equivalent:

(i) *X* is disconnected.

(ii) There exists a non-empty proper fuzzy subset of *X* which is both open and closed.

**Remark:** In a fuzzy topological space, the number of clopen fuzzy sets is even.

**Theorem:** If $Y$ is a subspace of fts $X$, a separation of $Y$ is a pair of disjoint non-empty fuzzy sets μ and λ, whose union is $Y$, neither of which contains a limit point of the other. In other words, μ and λ, form a separation of $Y$ iff $\mu \cup \lambda = Y$ and «̄ ∩ ª = 0 = « ∩ ª̄ = 0 .

**Proof:** Suppose that μ and λ form a separation of $Y$. Then by definition, μ and λ are non-empty open fuzzy sets in $Y$ such that $\mu \cup \lambda = Y$ and $\mu \cap \lambda = \varnothing$. Since μ and λ are complement of each other, so they are both open and closed. We know that, the closure of a closed set is itself i.e. $\bar{\mu} = \mu$ and $\bar{\lambda} = \lambda$. Therefore, $\bar{\mu} \cap \lambda = \mu \cap \lambda = 0$ 0. Also $\mu \cap \bar{\lambda} = \mu \cap \lambda = 0$.

Conversely, Suppose that μ and λ are disjoint non-empty fuzzy sets, whose union is $Y$, neither of which contains a limit point of the other. Then, we have $\bar{\mu} \cap \lambda = 0$ and $\mu \cap \bar{\lambda} = 0$. Now, we have to show that μ and λ are both open and closed. We conclude that $\bar{\mu} \cap \lambda = \mu$ and $\bar{\lambda} \cap Y = \lambda$. This gives that both μ and λ are closed in $Y$. Since $\mu = \lambda^c$ and $\lambda = \mu^c$, so they are both open in $Y$ as well. Hence μ and λ are both open and closed.

**Theorem:** Let μ be a connected fuzzy subset of fuzzy topological space $X$. If $\mu \subset \lambda \subset \bar{\mu}$ then λ is fuzzy connected.

**Proof:** Let μ be a connected and $\mu \subset \lambda \subset \bar{\mu}$. Suppose λ is not connected, then there exists two non-empty open fuzzy sets $\lambda_1$ and $\lambda_2$ such that $\bar{\lambda}_1 \cap \lambda_2 = 0$ and $\lambda_1 \cap \bar{\lambda}_2 = 0$.

By previous theorem, μ lies entirely either in $\lambda_1$ or in $\lambda_2$, i.e., $\mu \subset \lambda_1$ or $\mu \subset \lambda_2$.

Consider $\mu \subset \lambda_1 \subset \bar{\mu} \subset \bar{\lambda}_1$. Now, $\mu \subset \lambda \subset \bar{\mu}$ and $\bar{\mu} \subset \bar{\lambda}_1$ implies that

$\mu \subset \lambda \subset \bar{\mu} \subset \bar{\lambda}_1 \Rightarrow \lambda \subset \bar{\mu} \subset \bar{\lambda}_1 \Rightarrow \lambda \subset \bar{\lambda}_1$.

Since $\bar{\lambda}_1 \cap \lambda_2 = 0$ so that $\lambda \cap \lambda_2 = 0$. Since $\lambda 2 \subset \lambda$ and $\lambda 2 \cap \lambda = \varnothing \Rightarrow \lambda_2 = \varnothing$, which is a contradiction as $\lambda_2$ is non-empty open fuzzy subset of λ. Hence λ is fuzzy connected.

**Theorem:** Let $(X, t)$ be a fts. Then the following conditions are equivalent;

(i) $(X, t)$ is fuzzy connected.

(ii) $\mu, \lambda \in t$ with $\mu \cup \lambda = X$ and $\mu \cap \lambda = 0$ imply one of μ or λ is 0.

(iii) $\mu, \lambda \in t^c$ with $\mu \cup \lambda = X$ and $\mu \cap \lambda = 0$ imply one of μ or λ is 0.

**Proof of (i) Ĝ (ii):** Suppose (ii) is not true. That is $\mu \neq 0 \neq \lambda$ and $\mu \cup \lambda = X$, $\mu \cap \lambda = 0$. Now, by De Morgan's rule, $(\mu \cup \lambda)^c = X^c$ Ĝ $\mu^c \cap \lambda^c = 0$ and $(\mu \cap \lambda)^c = 0^c$ Ĝ $\mu^c \cup \lambda^c = X$. Now, $(\mu^c)^- \cap \lambda^c = \mu^c \cap \lambda^c = 0$ Ĝ $(\mu^c)^- \cap \lambda^c = 0$.

Also $\mu^c \cap (\lambda^c)^- = \mu^c \cap \lambda^c = 0$ Ĝ $\mu^c \cap (\lambda^c)^- = 0$.

Since $\mu \neq 0 \neq \lambda$ and $\mu \cup \lambda = X$, $\mu \cap \lambda$ Ĝ $\mu \neq 0 \neq \lambda$.

Hence $\mu^c \neq 0 \neq \lambda^c$. So that $(\mu^c)^- \cap \lambda^c = 0 = \mu^c \cap (\lambda^c)^-$ gives that $\mu^c$ and $\lambda^c$ form a separation. Hence $(X, t)$ is not fuzzy connected, which is a contradiction.

**Proof of (ii) Ĝ (iii):** The proof is a straight forward application of De Morgan's rule.

**Proof of (iii) Ĝ (i):** Suppose (i) is not true. Then there exists two non-zero open fuzzy sets λ and μ such that $\overline{\mu} \cap \lambda = 0 = \mu \cap \overline{\lambda}$. This implies that (iii) is not true as $0 \in \{\mu , \lambda\}$.

**Theorem:** Let $(X, t)$ be an fts. If the fuzzy sets μ and λ form a separation of $X$. Then for any fuzzy set α in $X$, $\alpha \cap \mu$ and $\alpha \cap \lambda$ are not fuzzy connected.

**Proof:** Since μ and λ are separated. Then, μ and λ are non-zero open fuzzy sets such that

$\overline{\mu} \cap \lambda = 0$ and $\mu \cap \overline{\lambda} = 0$. Now, $\alpha \cap \mu \subseteq \mu \Rightarrow (\alpha \cap \mu)^- \subseteq \overline{\mu}$. Also $\alpha \cap \mu \subseteq \lambda \Rightarrow (\alpha \cap \lambda)^- \subset \overline{\lambda}$.

Now, $(\alpha \cap \mu)^- \cap (\alpha \cap \lambda) \subseteq \overline{\mu} \cap \lambda = 0$. Hence $(\alpha \cap \mu)^- \cap (\alpha \cap \lambda) = 0$. Similarly, we get

$(\alpha \cap \mu) \cap (\alpha \cap \lambda)^- = 0$. Hence $\alpha \cap \mu$ and $\alpha \cap \lambda$ form a separation. Therefore $\alpha \cap \mu$ and $\alpha \cap \lambda$ are not fuzzy connected.

**Theorem:** Let $(X, t)$ be an fts, α in $X$. Then the following conditions are equivalent:

(i) α is fuzzy connected.

(ii) $\lambda_1, \lambda_2 \in t$ are separated, $\alpha \subseteq \lambda_1 \cup \lambda_2 \Rightarrow \alpha \subseteq \lambda_1$ or $\alpha \subseteq \lambda_2$.

(iii) $\lambda_1, \lambda_2 \in t$ are separated, $\alpha \subseteq \lambda_1 \cup \lambda_2 \Rightarrow \alpha \cap \lambda_1 = 0$ or $\alpha \subseteq \lambda_2 = 0$.

**Proof of (i) ⇒ (iii):** Since α is fuzzy connected and $\lambda_1, \lambda_2 \in t$ are separated. Then by previous theorem, we have $\alpha \cap \lambda_1$ and $\alpha \cap \lambda_2$ are not connected. Since $\alpha \subseteq \lambda_1 \cup \lambda_2$ then we have,

$\alpha \cap (\lambda_1 \cup \lambda_2) = \alpha \Rightarrow (\alpha \cap \lambda_1) \cup (\alpha \cap \lambda_2) = \alpha$. Since α is connected, so that one of $\alpha \cap \lambda_1$ or $\alpha \cap \lambda_2$ equal to 0. Hence $\alpha \cap \lambda_1 = 0$ or $\alpha \cap \lambda_2 = 0$.

**Proof of (iii) ⇒ (ii):** Suppose $\alpha \cap \lambda_1 = 0$. Since $\alpha \subseteq \lambda_1 \cup \lambda_2$ then $\alpha \cap (\lambda_1 \cup \lambda_2) = \alpha$

$\Rightarrow (\alpha \cap \lambda_1) \cup (\alpha \cap \lambda_2) = \alpha \Rightarrow 0 \cup (\alpha \cap \lambda_2) = \alpha \Rightarrow \alpha \cap \lambda_2 = \alpha \Rightarrow \alpha \subseteq \lambda_2$.

Similarly, if $\alpha \cap \lambda_2 = 0$, we get $\alpha \subseteq \lambda_1$.

**Proof of (ii) ⇒ (i):** Suppose $\lambda_1$ and $\lambda_2$ are separated and α is not fuzzy connected. Let $\alpha = \lambda_1 \cup \lambda_2$. By (ii), $\alpha \subseteq \lambda_1$ or $\alpha \subseteq \lambda_2$. If $\alpha \subseteq \lambda_1$ and $\lambda_1^- \cap \lambda_2 = 0 = \lambda_1 \cap \lambda_2^-$

(as α not connected).

Now $\alpha \subseteq \lambda_1 \subseteq \overline{\lambda_1} \Rightarrow \alpha \cap \lambda_2 \subseteq \lambda_1 \cap \lambda_2 \subseteq \overline{\lambda_1} \cap \gamma_2 = 0 \Rightarrow \alpha \cap \lambda_2 = 0$

$\Rightarrow \lambda_2 = 0$ (as $\alpha = \lambda_1 \cup \lambda_2$). Similarly, if $\alpha \subseteq \lambda_2$, we get $\lambda_1 = 0$. This is a contradiction as $\lambda_1$ and $\lambda_2$ are separated. Hence α is fuzzy connected.

**Theorem:** Let $(X, t)$ be an fts and μ and λ are fuzzy connected sets which are not separated, then $\mu \cup \lambda$ is fuzzy connected.

**Proof:** Suppose $\alpha = \mu \cup \lambda$ is not fuzzy connected. Then there exists two non-zero open fuzzy sets $\lambda_1$ and $\lambda^2$ such that $\overline{\lambda_1} \cap \lambda_2 = \varnothing = \lambda_1 \cap \overline{\lambda_2}$. Since $(X, t)$ be an fts, then $\lambda_1, \lambda_2 \in t$ are separated and $\alpha = \lambda_1 \cup \lambda_2$, then by previous theorem we have $\alpha \subseteq \lambda_1$ or $\alpha \subseteq \lambda_2$. If $\alpha \subseteq \lambda_1 \Rightarrow \alpha \subseteq \lambda_1 \subseteq \overline{\lambda_1} \Rightarrow \alpha \cap \lambda_2 \subseteq \lambda_1 \cap \lambda_2 \subseteq \overline{\lambda_1} \cap \lambda_2 = 0 \Rightarrow \alpha \cap \lambda_2 = 0 \Rightarrow \lambda_2 = 0$.

Similarly, if $\alpha \subseteq \lambda_2$, then we get $\lambda_1 = 0$. But $\lambda_1 \neq \varnothing$, $\lambda_2 \neq \varnothing$. This contradiction proves that α is fuzzy connected.

**Remark**: Let $\{\mu_i\}$ be a collection of fuzzy connected subsets of fts $(X, t)$ such that no two members are separated. Then $\mu = \cap_i \mu_i$ is fuzzy connected.

**Theorem:** The image of a fuzzy connected space under a fuzzy continuous map is fuzzy connected.

**Proof:** Let $f : X \to Y$ be fuzzy continuous map and let $X$ be fuzzy connected topological space. We have to prove that $W = f(X)$ is fuzzy connected. Suppose $W$ is not fuzzy connected; then there exists two non-empty open fuzzy sets μ and λ in $W$ such that $\mu \cup \lambda = W$ and $\mu \cup \lambda = \varnothing$. Since μ and λ are open in $W$, by definition of fuzzy continuous, we have $f^{-1}(\mu)$ and $f^{-1}(\lambda)$ are open in $W$ and they are non-empty. Now, $f^{-1}(\mu) \cup f^{-1}(\lambda) = f^{-1}(\mu \cup \lambda) = f^{-1}(W) = X$ and $f^{-1}(\mu) \cup f^{-1}(\lambda) = f^{-1}(\mu \cap \lambda) = f^{-1}(\varnothing) = \varnothing$.

Therefore, $f^{-1}(\mu)$ and $f^{-1}(\lambda)$ form a separation of $W$, which is a contradiction.

Hence $W = f(X)$ is fuzzy connected.

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