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LEVEL SEPARATION ON INTUITIONISTIC FUZZY T_1 SPACES

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ABSTRACT

Intuitionistic fuzzy T_1 spaces are defined and studied in this paper. Eight new notions of intuitionistic fuzzy T_1 spaces are defined and some relationship among them has been investigated. Some relations between the defined notions and other given notions of intuitionistic fuzzy T_1 spaces has also been investigated. Under some conditions it is observed that image and preimage preserve intuitionistic fuzzy T_1 spaces. Hereditary and productive property of such spaces has been also investigated.

Key words: Fuzzy set, Intuitionistic set, Intuitionistic fuzzy set, Intuitionistic topological space, Intuitionistic fuzzy topological space, Intuitionistic fuzzy T_1space

INTRODUCTION

After introducing fuzzy set by Zadeh (Zadeh 1965), the concept of fuzzy topological spaces was introduced by Chang (Chang, 1968). Later, the notion of an intuitionistic fuzzy set which is a generalization of fuzzy sets and take into account both the degrees of membership and nonmembership subject to the condition that their sum does not exceed 1 was introduced by Atanassov (Atanassov 1986). Coker and coworker (Coker 1996,1997, Bayhan and Coker 1996) introduced the idea of the topology of intuitionistic fuzzy sets. Since then, D. Coker and S. Bayhan (Coker and Bayhan 2003), A. K. Singh and R. Srivastava (Singh and Srivastava 2012), S. J. Lee and E. P. Lee. (Lee and Lee 2000), R. Saadati and J. H. Park (Saadati and Park 2006), Estiaq Ahmed et al. (Ahmed et al. 2014, 2014, 2015, 2015, 2015) subsequently initiated a study of intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In this paper, we investigate the properties and features of intuitionistic fuzzy T_1 Space.

NOTATIONS AND PRELIMINARIES

Through this paper, X be a nonempty set. α, r and s are constants in $(0,1)$. T is a topology, t is a fuzzy topology, $\mathcal{T}$ is an intuitionistic topology and τ is an intuitionistic fuzzy topology. λ and μ are fuzzy sets, $A = (\mu_A, \nu_A)$ is intuitionistic fuzzy set . By $\underline{0}$ and $\underline{1}$, we denote constant fuzzy sets taking values 0 and 1 respectively.

Definition (Coker 1996): Let X be a non empty set. A family t of fuzzy sets in X is called a fuzzy topology (FT in short) on X if the following conditions hold.

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1. $\underline{0}, \underline{1} \in t$,
2. $\lambda \cap \mu \in t$, for all $\lambda, \mu \in t$,
3. $\cup \lambda_j \in t$, for any arbitrary family $\{\lambda_j \in t,\ j \in J\}$.

The above definition is in the sense of C. L. Chang (Chang 1968). The pair (X, t) is called a fuzzy topological space (FTS in short), members of t are called fuzzy open sets (FOS in short) in X and their complements are called fuzzy closed sets (FCS in short) in X.

Definition (Coker 1996): Suppose X is a non empty set. An intuitionistic set A on X is an object having the form $A = (X, A_1, A_2)$ where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A. In this paper, we use the simpler notation $A = (A_1, A_2)$ instead of $A = (X, A_1, A_2)$ for an intuitionistic set.

Remark (Coker 1996): Every subset A of a nonempty set X may obviously be regarded as an intuitionistic set having the form $A = (A, A^c)$ where $A^c = X - A$.

Definition (Coker 1996): Let the intuitionistic sets A and B in X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j, j \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_j = \left(A_j^{(1)}, A_j^{(2)}\right)$. Then

1. $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$,
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
3. $\bar{A} = (A_2, A_1)$, denotes the complement of A,
4. $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$,
5. $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$,
6. $\phi_\sim = (\phi, X)$ and $X_\sim = (X, \phi)$.

Definition (Coker and Bayhan 2001): Let X be a non empty set. A family $\mathcal{T}$ of intuitionistic sets in X is called an intuitionistic topology (IT in short) on X if the following conditions hold.

1. $\phi_\sim, X_\sim \in \mathcal{T}$,
2. $A \cap B \in \mathcal{T}$ for all $A, B \in \mathcal{T}$,
3. $\cup A_j \in \mathcal{T}$ for any arbitrary family $\{A_j \in \mathcal{T}, j \in J\}$.

The pair $(X, \mathcal{T})$ is called an intuitionistic topological space (ITS in short), members of $\mathcal{T}$ are called intuitionistic open sets (IOS in short) in X and their complements are called intuitionistic closed sets (ICS in short) in X.

Definition (Atanassov 1986): Let X be a non empty set. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where μ_A and ν_A are fuzzy sets in X denote the degree of membership and the degree of non-membership respectively with $\mu_A(x) + \nu_A(x) \leq 1$.

Throughout this paper, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ for intuitionistic fuzzy sets.

Remark: Obviously every fuzzy set λ in X is an intuitionistic fuzzy set of the form $(\lambda, 1 - \lambda) = (\lambda, \lambda^c)$ and every intuitionistic set $A = (A_1, A_2)$ in X is an intuitionistic fuzzy set of the form $\left(1_{A_1}, 1_{A_2}\right)$.

Definition (Atanassov 1986): Let X be a nonempty set and A, B are intuitionistic fuzzy sets on X be given by (μ_A, ν_A) and (μ_B, ν_B) respectively, then

1. $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
2. $A = B$ if $A \subseteq B$ and $B \subseteq A$,
3. $\bar{A} = (\nu_A, \mu_A)$,
4. $A \cap B = (\mu_A \cap \mu_B,\ \nu_A \cup \nu_B)$,
5. $A \cup B = (\mu_A \cup \mu_B,\ \nu_A \cap \nu_B)$.

Definition (Coker 1997): Let $\{A_j = \left(\mu_{A_j}, \nu_{A_j}\right), j \in J\}$ be an arbitrary family of IFSs in X. Then

1. $\cap A_j = (\cap \mu_{A_j},\ \cup \nu_{A_j})$,
2. $\cup A_j = (\cup \mu_{A_j},\ \cap \nu_{A_j})$,
3. $0_\sim = (\underline{0}, \underline{1})$, $1_\sim = (\underline{1}, \underline{0})$.

Definition (Coker 1997): An intuitionistic fuzzy topology (IFT in short) on a nonempty set X is a family τ of IFSs in X, satisfying the following conditions:

1. $0_\sim, 1_\sim \in \tau$,
2. $A \cap B \in \tau$, for all $A, B \in \tau$,
3. $\cup A_j \in \tau$ for any arbitrary family $\{A_j \in \tau,\ j \in J\}$.

The pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short), members of τ are called intuitionistic fuzzy open sets (IFOS in short) in X, and their complements are called intuitionistic fuzzy closed sets (IFCS in short) in X.

Remark (Ying-Ming and Mao-Kang 1997): Let X be a non empty set and $A \subseteq X$, then the set A may be regarded as a fuzzy set in X by its characteristic function $1_A: X \to \{0,1\} \subset [0,1]$ defined by

$$1_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 \text{ if} x \notin A, \text{i.e.}, & \text{if } x \in A^c \end{cases}$$

Again, we know that a fuzzy set λ in X may be regarded as an intuitionistic fuzzy set by $(\lambda, 1 - \lambda) = (\lambda, \lambda^c)$. So every subset A of X may be regarded as intuitionistic fuzzy set by $(1_A, 1 - 1_A) = (1_A, 1_{A^c})$.

Theorem: Let (X, T) be a topological space. Then (X, τ) is an IFTS where

$$\tau = \left\{(1_{A_j}, 1_{A_j}^c), j \in J : A_j \in T\right\}.$$

Note: Above τ is the corresponding intuitionistic fuzzy topology of T.

Theorem: Let (X, t) be a fuzzy topological space. Then (X, τ) is an IFTS where

$$\tau = \{(\lambda_j,\ \lambda_j^c), j \in J: \lambda_j \in t\}.$$

Note: Above τ is the corresponding intuitionistic fuzzy topology of t.

Theorem: Let $(X, \mathcal{T})$ be an intuitionistic topological space. Then (X, τ) is an intuitionistic fuzzy topological space where

$$\tau = \left\{\left(1_{A_{j1}},\ 1_{A_{j2}}\right), j \in J :\ A_j = \left(A_{j1}, A_{j2}\right) \in \mathcal{T}\right\}.$$

Note: Above τ is the corresponding intuitionistic fuzzy topology of $\mathcal{T}$.

Definition (Coker and Bayhan 2001): An intuitionistic topological space $(X, \mathcal{T})$ is called T1 if for all $x, y \in X$ with $x \neq y$, there exists intuitionistic sets $A = (A_1, A_2)$, $B = (B_1, B_2) \in \mathcal{T}$ such that $x \in A_1,\ y \notin A_1$ and $y \in B_1,\ x \notin B_1$.

Definition (Atanassov 1987): Let X and Y be two nonempty sets and $f: X \to Y$ be a function. If $A = \left\{\left(x, \mu_A(x),\ \nu_A(x)\right): x \in X\right\}$ and $B = \left\{\left(y, \mu_B(y),\ \nu_B(y)\right): y \in Y\right\}$ are IFSs in X and Y respectively, then the preimage of B under f, denotedby $f^{-1}(B)$ is the IFS in X defined by

$f^{-1}(B) = \left\{\left(x,\ \left(f^{-1}(\mu_B)\right)(x), \left(f^{-1}(\nu_B)\right)(x)\right): x \in X\right\} = \{(x, \mu_B(f(x)), \nu_B(f(x))): x \in X\}$

and the image of A under f, denoted by $f(A)$ is the IFS in Y defined by

$f(A) = \{(y,\ (f(\mu_A))(y), (f(\nu_A))(y)) : y \in Y\}$, where for each $y \in Y$

$$(f(\mu_A))(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

$$(f(\nu_A))(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

Definition(Bayhan and Coker 1996): Let $A = (x, \mu_A, \nu_A)$ and $B = (y, \mu_B, \nu_B)$ be IFSs in X and Y respectively. Then the product of IFSs A and B denoted by $A \times B$ is defined by $A \times B = \{(x, y), \mu_A \times \mu_B, \nu_A \times \nu_B)\}$ where $(\mu_A \times \mu_B)(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $(\nu_A \times \nu_B)(x, y) = \max\{\nu_A(x), \nu_B(y)\}$ for all $(x, y) \in X \times Y$.

Obviously $0 \le (\mu_A \times \mu_B) + (\nu_A \times \nu_B) \le 1$. This definition can be extended to an arbitrary family of IFSs.

Definition (Bayhan and Coker 1996): Let (X_j, τ_j), $j = 1,2$ be two IFTSs. The product topology $\tau_1 \times \tau_2$ on $X_1 \times X_2$ is the IFT generated by $\{\rho_j^{-1}(U_j) : U_j \in \tau_j, j = 1,2\}$, where $\rho_j : X_1 \times X_2 \to X_j$, $j = 1,2$ are the projection maps. IFTS $\{X_1 \times X_2, \tau_1 \times \tau_2\}$ is called the product IFTS of (X_j, τ_j), $j = 1,2$. In this case $\mathcal{S} = \{\rho_j^{-1}(U_j), j \in J : U_j \in \tau_j\}$ is a sub base and $\mathcal{B} = \{U_1 \times U_2 : U_j \in \tau_j, j = 1,2\}$ is a base for $\tau_1 \times \tau_2$ on $X_1 \times X_2$.

Definition (Coker 1997): Let (X, τ) and (Y, δ) be IFTSs. A function $f : X \to Y$ is called continuous if $f^{-1}(B) \in \tau$ for all $B \in \delta$ and f is called open if $f(A) \in \delta$ for all $A \in \tau$.

Definition (Lipschutz 1965): A topological space (X, T) is called T_1 if for all $x, y \in X$ with $x \neq y$, there exists $U, V \in T$ such that $x \in U, y \notin U$ and $y \in V, x \notin V$.

Definition (Srivastava *et al.* 1988): A fuzzy topological space (X, t) is called T_1 if for all $x, y \in X$ with $x \neq y$, there exists $u, v \in$ t such that $u(x) = 1, u(y) = 0$ and $v(y) = 1, v(x) = 0$.

Definition (Singh and Srivastava 2012): Let $A = (\mu_A, \nu_A)$ be a IFS in X and U be a non empty subset of X. The restriction of A to U is a IFS in U, denoted by $A|U$ and defined by $A|U = (\mu_A|U, \nu_A|U)$.

Definition: Let (X, τ) be an intuitionistic fuzzy topological space and U is a non empty sub set of X then $\tau_U = \{A|U : A \in \tau\}$ is an intuitionistic fuzzy topology on U and (U, τ_U) is called sub space of (X, τ).

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Definition: Let $r \in (0,1)$. An intuitionistic fuzzy topological space $(X, \tau)$is called.

1. IF-T$_1$(r-i) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > r, \nu_A(x); \mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r$; $\mu_B(x) < r, \nu_B(x) > r$.

2. IF-T$_1$(r-ii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > r, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > 0$ and $\mu_B(y) > r, \nu_B(y) < r; \mu_B(x) < r, \nu_B(x) > 0$.

3. IF-T$_1$(r-iii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > 0, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > 0, \nu_B(y) < r$; $\mu_B(x) < r, \nu_B(x) > r$.

4. IF-T_1(r-iv) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > 0, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < r$; $\mu_B(x) < r, \nu_B(x) > 0$.

5. IF-T_1(r-v) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > r, \nu_A(x) < 1; \mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < 1$; $\mu_B(x) < r, \nu_B(x) > r$.

6. IF-T_1(r-vi) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > r, \nu_A(x) < r; \mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r$; $\mu_B(x) < 1, \nu_B(x) > r$.

7. IF-T_1(r-vii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > r, \nu_A(x) < 1; \mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > r$.

8. IF-T_1(viii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > 0, \nu_A(x) < 1; \mu_A(y) < 1, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > 0$.

Theorem: Let (X, T) be a topological space and (X, τ) be its corresponding IFTS where $\tau = \left\{(1_{A_j}, 1_{A_j}^c), j \in J : A_j \in T\right\}$. Then (X, T) is $T_1 \Leftrightarrow (X, \tau)$ is IF- T_1(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X, T) is $T_1 \Leftrightarrow (X, \tau)$ is IF- T_1(viii).

Proof: Suppose (X, T) is T_1 space. Let $x, y \in X$ with $x \neq y$.Since $(X, T)$is T_1, then there exists $A, B \in T$ such that $x \in A, y \notin A$ and $y \in B, x \notin B$. By the definition of τ, we get $(1_A, 1_{A^c}), (1_B, 1_{B^c}) \in \tau$ as $A, B \in T$.

Now, $1_A(x) = 1, 1_A(y) = 0, 1_B(y) = 1$, $1_B(x) = 0$ as $x \in A, y \notin A$ and $y \in B$; $x \notin B$.

And clearly $1_{A^c}(x) = 0$, $1_{A^c}(y) = 1$, $1_{B^c}(y) = 0$, $1_{B^c}(x) = 1$.

That is, $1_A(x) = 1, 1_{A^c}(x) = 0$; $1_A(y) = 0$, $1_{A^c}(y) = 1$ and $1_B(y) = 1, 1_{B^c}(y) = 0$; $1_B(x) = 0$, $1_{B^c}(x) = 1$.

This implies

$1_A(x) > r, 1_{A^c}(x) < r; 1_A(y) < r$, $1_{A^c}(y) > r$ and $1_B(y) > r$, $1_{B^c}(y) < r$; $1_B(x) < r$, $1_{B^c}(x) > r$.

So (X, τ) is IF-T_1(r-i).

Conversely suppose (X, τ) is IF-T_1(r-i). Let $x, y \in X$ with $x \neq y$. Since (X, τ) is IF-T_1(r-i), then there exists $(1_A, 1_{A^c}), (1_B, 1_{B^c}) \in \tau$ such that $1_A(x) > r, 1_{A^c}(x) < r$; $1_A(y) < r$, $1_{A^c}(y) > r$ and $1_B(y) > r, 1_{B^c}(y) < r$; $1_B(x) < r, 1_{B^c}(x) > r$.

Since $r \in (0,1)$; we can write $1_A(x) = 1$, $1_{A^c}(x) = 0; 1_A(y) = 0, 1_{A^c}(y) = 1$ and $1_B(y) = 1, 1_{B^c}(y) = 0; 1_B(x)$

$$= 0, 1_{B^c}(x) = 1.$$

This implies $x \in A, y \notin A$and $y \in B, x \notin B$.

Clearly $A, B \in T$ as $(1_A, 1_{A^c}), (1_B, 1_{B^c}) \in \tau$. Therefore (X, T) is T_1. Space.

Similarly we can show the other implications.

Theorem: Let $(X,\mathcal{T})$ be an intuitionistic topological space and (X,τ) be its corresponding IFTS where $\tau = \left\{\left(1_{A_{j1}},\ 1_{A_{j2}}\right), j \in J : A_j = \left(A_{j1}, A_{j2}\right) \in \mathcal{T}\right\}$. Then $(X,\mathcal{T})$ is $T_1 \Leftrightarrow (X,\tau)$ is IF-T_1(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and $(X,\mathcal{T})$ is $T_1 \Leftrightarrow (X,\tau)$ is IF-T_1 (viii).

Proof: The proof of all implications are similar. As an example we prove $(X,\mathcal{T})$ is $T_1 \Leftrightarrow (X,\tau)$ is IF-T_1 (r-iv).

Suppose $(X,\mathcal{T})$ is T_1.Let $x, y \in X$ with $x \neq y$. Since $(X,\mathcal{T})$ is T_1, then there exists $A = (A_1, A_2)$, $B = (B_1, B_2) \in \mathcal{T}$ such that $x \in A_1,\ y \notin A_1$ and $y \in B_1,\ x \notin B_1$ .By the definition of τ , it is clear that $\left(1_{A_1}, 1_{A_2}\right), \left(1_{B_1},\ 1_{B_2}\right) \in \tau$. Clearly $1_{A_1}(x) = 1$, $1_{A_1}(y) = 0$; $1_{B_1}(y) = 1$, $1_{B_1}(x) = 0$.

Since $A_1 \cap A_2 = B_1 \cap B_2 = \phi$ and $x \in A_1, y \notin A_1$, $y \in B_1, x \notin B_1$ .So $1_{A_2}(x) = 0$, $1_{A_2}(y) = 1$; $1_{B_2}(y) = 0$, $1_{B_2}(x) = 1$. That is, $1_{A_1}(x) = 1$, $1_{A_2}(x) = 0$; $1_{A_1}(y) = 0$, $1_{A_2}(y) = 1$ and $1_{B_1}(y) = 1$, $1_{B_2}(y) = 0$; $1_{B_1}(x) = 0$, $1_{B_2}(x) = 1$.

Therefore we can write, $1_{A_1}(x) > 0$,
$1_{A_2}(x) < r$; $1_{A_1}(y) < r$, $1_{A_2}(y) > 0$ and
$1_{B_1}(y) > 0$, $1_{B_2}(y) < r$; $1_{B_1}(x) < r$,
$1_{B_2}(x) > 0$ as $r \in (0,1)$.

So (X,τ) is IF-T_1(r-iv).

Conversely suppose (X,τ) is IF-T_1(r-iv). Let $x, y \in X$ with $x \neq y$. Since (X,τ) is IF-T_1(r-iv), then there exists $\left(1_{A_1},\ 1_{A_2}\right), \left(1_{B_1},\ 1_{B_2}\right) \in \tau$ such that $1_{A_1}(x) > 0$, $1_{A_2}(x) < r$; $1_{A_1}(y) < r$, $1_{A_2}(y) > 0$ and $1_{B_1}(y) > 0$, $1_{B_2}(y) < r$; $1_{B_1}(x) < r$, $1_{B_2}(x) > 0$.

This implies $1_{A_1}(x) = 1$, $1_{A_1}(y) = 0$;
$1_{B_1}(y) = 1, 1_{B_2}(x) = 0$.

This implies $x \in A_1, y \notin A_1$ and $y \in B_1, x \notin B_1$. By definition of τ, $A = (A_1, A_2) \in \mathcal{T}$.

So $(X,\mathcal{T})$ is T_1 space.

Theorem: Let (X,t) be a fuzzy topological space and (X,τ) be its corresponding IFTS where $\tau = \{(\lambda, \lambda^c), j \in J : \lambda \in t\}$. (X,t) is $T_1 \Rightarrow (X,\tau)$ is IF-T_1(r-k) for $k = i, ii, iii, iv, v, vi, vii$ and (X,t) is $T_1 \Rightarrow (X,\tau)$ is IF-T_1(viii) where $r \in (0,1)$.

Proof: The proofs of all implications are similar. As an example we prove that (X,t) is $T_1 \Rightarrow (X,\tau)$ is IF-T_1(r-i).

Suppose (X,t) is T_1. Let $x, y \in X$ with $x \neq y$. then there exists $u, v \in t$ such that $u(x) = 1$, $u(y) = 0$ and $v(y) = 1$, $v(x) = 0$.

Now by the definition of τ, $(u, u^c), (v, v^c) \in \tau$ *as* $u, v \in t$.

Clearly $u^c(x) = 0, u^c(y) = 1$ and $v^c(y) = 0, v^c(x) = 1$

That is, we get $u(x) = 1$, $u^c(x) = 0$;
$u(y) = 0$, $u^c(y) = 1$ and $v(y) = 1$,
$v^c(y) = 0$; $v(x) = 0$, $v^c(x) = 1$.

So for $r \in (0,1)$, we can write

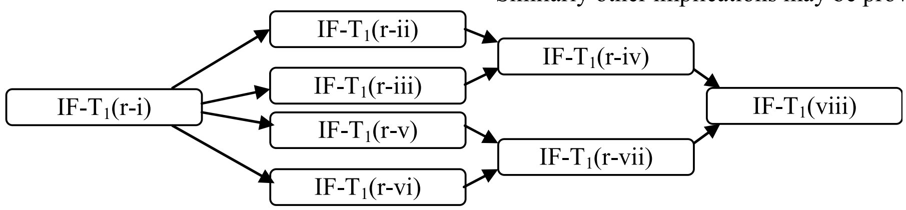
$u(x) > r, u^c(x) < r;\ u(y) < r,\ u^c(y) > r$
and $v(y) > r, v^c(y) < r; v(x) < r$,
$v^c(x) > r$.

Therefore (X,τ) is IF-T_1(r-i).

Theorem: Let (X,τ) be a IFTS. Then we have the following implications.

Proof: Suppose (X,τ) is IF-T_1(r-i). Let $x, y \in X$ with $x \neq y$. Since (X,τ) is IF-T_1(r-i), then there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > r, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r; \mu_B(x) < r, \nu_B(x) > r$.....(1)



Now, from (1) we can write,

$\mu_A(x) > r, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > 0$ and $\mu_B(y) > r, \nu_B(y) < r; \mu_B(x) < r, \nu_B(x) > 0$. ....(2)

Again from (2) we get,

$\mu_A(x) > 0, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < r; \mu_B(x) < r, \nu_B(x) > 0$. ....(3)

And finally from (3),

$\mu_A(x) > 0, \nu_A(x) < 1; \mu_A(y) < 1, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < 1; \mu_B(x) < 1, \nu_B(x) > 0$.

Therefore IF-T_1(r-i) ⇒ IF-T_1(r-ii) ⇒ IF-T_1(r-iv) ⇒IF-T_1(viii).

Again from (1) we get

$\mu_A(x) > r, \nu_A(x) < r; \mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r; \mu_B(x) < 1, \nu_B(x) > r$.

This implies

$\mu_A(x) > r, \nu_A(x) < 1; \mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > r$.

This implies

$\mu_A(x) > 0, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > 0$.

Therefore IF-T_1(r-i) ⇒ IF-T_1(r-vi) ⇒ IF-T_1(r-vii) ⇒ IF-T_1(viii).

Similarly other implications may be proved.

The reverse implications are not true in general as can be seen from the following examples:

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.7, 0.1), (y, 0.2, 0.3)\}, B = \{(x, 0.4, 0.1), (y, 0.6, 0.3)\}$ .If $r = 0.5$, then clearly (X, τ) is IF-T_1(r-ii) but not IF-T_1(r-i).

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.2, 0.1), (y, 0.2, 0.3)\}, B = \{(x, 0.2, 0.3), (y, 0.3, 0.4)\}$. If $r = 0.5$, then clearly (X, τ) is IF-T_1(r-iv) but not IF-T_1(r-i), IF-T_1(r-ii) and IF-T_1(r-iii)

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.6, 0.1), (y, 0.2, 0.6)\}$, $B = \{(x, 0.2, 0.6), (y, 0.2, 0.3)\}$. If $r = 0.5$, then clearly (X, τ) is IF-T_1(r-iii) but not IF-T_1(r-i).

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.6), (y, 0.1, 0.6)\}$, $B = \{(x, 0.1, 0.3), (y, 0.4, 0.5)\}$. If $r = 0.2$, then clearly (X, τ) is IF-T_1(r-v) but not IF-T_1(r-i).

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.1), (y, 0.4, 0.5)\}$

$B = \{(x, 0.3, 0.6), (y, 0.4, 0.1)\}$. If $r = 0.2$, then clearly (X, τ) is IF-T_1(r-vi) but not IF-T_1(r-i).

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.6), (y, 0.5, 0.3)\}$, $B = \{(x, 0.4, 0.5), (y, 0.4, 0.3)\}$. If $r = 0.2$, then clearly (X, τ) is IF-T_1(r-vii) but not IF-T_1(r-i), IF-T_1(r-v) and IF-T_1(r-vi)

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.3, 0.5), (y, 0.2, 0.6)\}$, $B = \{(x, 0.3, 0.2), (y, 0.7, 0.3)\}$. If $r = 0.5$, then clearly (X, τ) is IF-T_1(viii) but not IF-T_1(r-iv) and IF-T_1(r-vii)

Theorem: Let (X, τ) be a IFTS and $r, s \in (0,1)$ with $r < s$, then (X, τ) is IF-T_1(r-iv) $\Rightarrow$ (X, τ) is IF-T_1(s-iv) and (X, τ) is IF-T_1(s-vii) $\Rightarrow$ (X, τ) is IF-T_1(r-vii).

Proof: IF-T_1(r-iv) $\Rightarrow$ IF-T_1(s-iv): Suppose (X, τ) is IF-T_1(r-iv). Let $x, y \in X$ with $x \neq y$.

Since (X, τ) is IF-T_1(r-iv), then there exists intuitionistic fuzzy sets $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > 0, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < r; \mu_B(x) < r,$ $\nu_B(x) > 0.$

Since $r < s$, we can write

$\mu_A(x) > 0, \nu_A(x) < s; \mu_A(y) < s, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < s; \mu_B(x) < s,$ $\nu_B(x) > 0.$

Therefore (X, τ) is IF-T_1(s-iv).

IF-T_1(s-vii) $\Rightarrow$ IF-T_1(r-vii): Suppose (X, τ) is IF-T_1(s-vii).

Let $x, y \in X$ with $x \neq y$. Since (X, τ) is IF-T_1(s-vii), then there exists intuitionistic fuzzy set $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > s, \nu_A(x) < 1; \mu_A(y) < 1, \nu_A(y) > s$ and $\mu_B(y) > s, \nu_B(y) < 1; \mu_B(x) < 1,$ $\nu_B(x) > s.$

Since $r < s$, we can write

$\mu_A(x) > r, \nu_A(x) < 1; \mu_A(y) < 1, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < 1; \mu_B(x) < 1,$ $\nu_B(x) > r.$

So (X, τ) is IF-T_1(r-vii).

The reverse implications are not true in general as can be seen from the following examples:

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.2, 0.4), (y, 0.4, 0.3)\}$, $B = \{(x, 0.1, 0.4), (y, 0.2, 0.4)\}$. If $r = 0.3$ and $s = 0.5$ then clearly (X, τ) is IF-T_1(s-iv) but not IF-T_1(r-iv).

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.4, 0.4), (y, 0.2, 0.4)\}, B = \{(x, 0.1, 0.4), (y, 0.5, 0.4)\}$. If $r = 0.3$ and $s = 0.5$ then clearly (X, τ) is IF-T_1(r-vii) but not IF-T_1(s-vii).

Theorem: Let (X, τ) and (Y, δ) be IFTSs and $f: X \to Y$ is one-one and continuous. Then

(Y, δ) is IF-T_1(r-k) $\Rightarrow$ (X, τ) is IF-T_1(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and

(Y, δ) is IF-T_1(viii) $\Rightarrow$ (X, τ) is IF-T_1(viii).

Proof: Suppose (Y, δ) is IF-T_1(r-i). Let $x, y \in X$ with $x \neq y$. Since f is one-one, then $f(x), f(y) \in Y$ with $f(x) \neq f(y)$. Again, since (Y, δ) is IF-T_1(r-i), there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \delta$ such that $\mu_A(f(x)) > r, \nu_A(f(x)) < r$; $\mu_A(f(y)) < r, \nu_A(f(y)) > r$ and

$\mu_B(f(y)) > r, \nu_B(f(y)) < r; \mu_B(f(x)) < r, \nu_B(f(x)) > r$.

Since f is continuous,
$f^{-1}(A)=(f^{-1}(\mu_A), f^{-1}(\nu_A)) \in \tau$ and
$f^{-1}(B)=(f^{-1}(\mu_B), f^{-1}(\nu_B)) \in \tau$.

Now we have $f^{-1}(\mu_A)(x) = \mu_A(f(x)) > r$, $f^{-1}(\nu_A)(x) = \nu_A(f(x)) < r$; $f^{-1}(\mu_A)(y) = \mu_A(f(y)) < r$, $f^{-1}(\nu_A)(y) = \nu_A(f(y)) > r$.

and $f^{-1}(\mu_B)(y) = \mu_B(f(y)) > r$,
$f^{-1}(\nu_B)(y) = \nu_B(f(y)) < r$;

$f^{-1}(\mu_B)(x) = \mu_B(f(x)) < r$,
$f^{-1}(\nu_B)(x) = \nu_B(f(x)) > r$.

Therefore (X, τ) is IF-T_1(r-i).

Similarly we can show the other implications.

Theorem: Let (X, τ) and (Y, δ) be IFTSs and $f: X \to Y$ is one-one, onto and open. Then

(X, τ) is IF-T_1(r-k)$\Rightarrow$ (Y, δ) is IF-T_1(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and

(X, τ) is IF-T_1(viii) $\Rightarrow$ (Y, δ) is IF-T_1(viii).

Proof: Suppose (X, τ) is IF-T_1(r-i).

Let $x, y \in Y$ with $x \neq y$. Since f is onto, then there exists some $p, q \in X$ such that $f(p) = x$ and $f(q) = y$. Since f is one-one, these p and q are unique and $p \neq q$. i.e., $f^{-1}(x) = \{p\}$ and $f^{-1}(y) = \{q\}$.

Now since (X, τ) is IF-T_1(r-i), there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(p) > r, \nu_A(p) < r$; $\mu_A(q) < r, \nu_A(q) > r$ and $\mu_B(q) > r, \nu_B(q) < r$; $\mu_B(p) < r, \nu_B(p) > r$.

Further since f is open, so $f(A) = (f(\mu_A), f(\nu_A)) \in \delta$, $f(B) = (f(\mu_B), f(\nu_B)) \in \delta$

Now we have

$f(\mu_A)(x) = \sup_{a \in f^{-1}(x)} \mu_A(a) = \mu_A(p) > r$,
$f(\nu_A)(x) = \inf_{a \in f^{-1}(x)} \nu_A(a) = \nu_A(p) < r$;

$f(\mu_A)(y) = \sup_{a \in f^{-1}(y)} \mu_A(a) = \mu_A(q) < r$,
$f(\nu_A)(y) = \inf_{a \in f^{-1}(y)} \nu_A(a) = \nu_A(q) > r$.

And $f(\mu_B)(y) = \sup_{a \in f^{-1}(y)} \mu_B(a) = \mu_B(q) > r$, $f(\nu_B)(y) = \inf_{a \in f^{-1}(y)} \nu_B(a) = \nu_B(q) < r$;

$f(\mu_B)(x) = \sup_{a \in f^{-1}(x)} \mu_B(a) = \mu_B(p) < r$,
$f(\nu_B)(x) = \inf_{a \in f^{-1}(x)} \nu_B(a) = \nu_B(p) > r$.

Therefore (Y, δ) is IF-T_1(r-i).

Similarly we can show the other implications.

From above two theorems we have the following corollary:

Corollary: If (X, τ) and (Y, δ) are IFTSs and $f: X \to Y$ is a homeomorphism then (X, τ) is IF-T_1(r-k) if and only if (Y, δ) is IF-T_1(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X, τ) is IF-T_1(viii) if and only if (Y, δ) is IF-T_1(viii).

Remarks: IF-T_1(r-k) for k= i, ii, iii, iv, v, vi, vii and IF-T_1(viii) are topological properties.

Theorem: Let (X, τ) be an intuitionistic fuzzy topological space and U is a non-empty sub set of X. Then (X, τ) is IF-T_1(r-k) $\Rightarrow (U, \tau_U)$ is IF-T_1(r-k) for any

$k \in \{i, ii, iii, iv, v, vi, vii\}$ and (X, τ) is IF-T_1(viii) $\Rightarrow (U, \tau_U)$ is IF-T_1(viii).

Proof: Suppose (X, τ) is IF-T_1(r-i). Let $x, y \in U$ with $x \neq y$. $\Rightarrow x, y \in X$ with $x \neq y$ as $U \subseteq X$.

Since (X, τ) is IF-T_1(r-i), then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > r, \nu_A(x) < r; \mu_A(y) < r, \nu_A(y) > r$ and $\mu_B(y) > r, \nu_B(y) < r; \mu_B(x) < r,$ $\nu_B(x) > r$.

Clearly $A|U = (\mu_A|U, \nu_A|U) \in \tau_U$.

Now we have$\mu_A|U(x) = \mu_A(x) > r,$ $\nu_A|U(x) = \nu_A(x) < r; \mu_A|U(y) = \mu_A(y) < r, \nu_A|U(y) = \nu_A(y) > r$.

And $\mu_B|U(y) = \mu_B(y) > r, \nu_B|U(y) = \nu_B(y) < r; \mu_B|U(x) = \mu_B(x) < r,$ $\nu_B|U(x) = \nu_A(x) > r$.

Therefore (U, τ_U) is IF-T_1(r-i).

Similarly, we can show the other implications.

Remarks: The properties IF-T_1(r-k) for k= i, ii, iii, iv, v, vi, vii and IF-T_1(viii) are hereditary.

Definition (Ahmed *et al.* 2014): An intuitionistic fuzzy topological space (X, τ) is called

1. IF-T_1(i) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) = 1, \nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) = 1$ and $\mu_B(y) = 1, \nu_B(y) = 0;$ $\mu_B(x) = 0, \nu_B(x) = 1$.

2. IF-T_1(ii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) = 1, \nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) > 0$ and $\mu_B(y) = 1, \nu_B(y) = 0;$ $\mu_B(x) = 0, \nu_B(x) > 0$.

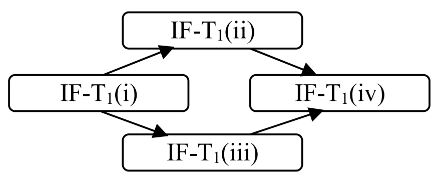
3. IF-T_1(iii) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > 0, \nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) = 1$ and $\mu_B(y) > 0, \nu_B(y) = 0;$ $\mu_B(x) = 0, \nu_B(x) = 1$.

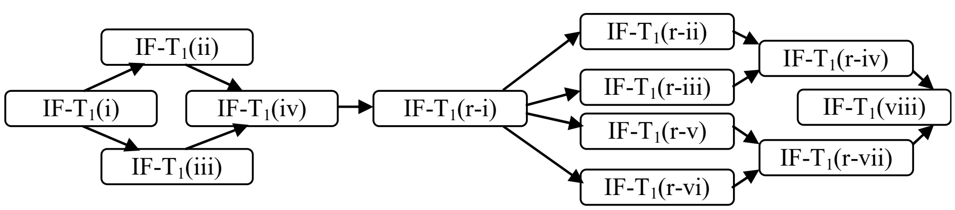
4. IF-T_1(iv) if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > 0, \nu_A(x) = 0;\ \mu_A(y) = 0, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) = 0;$ $\mu_B(x) = 0, \nu_B(x) > 0$.

Theorem (Ahmed *et al.* 2014): Let (X, τ) be a IFTS. Then the following implications hold.



Theorem: If (X, τ) bea IFTS, then the following implications hold.



Proof: To prove this theorem we only have to prove that (X, τ) is IF-T_1(iv) $\Rightarrow (X, \tau)$ is IF-T_1(r-i).

Let (X, τ) be IF-T_1(iv) and $x, y \in X$ with $x \neq y$. Then there exists $A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > 0, \nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) > 0$. So for $r \in (0,1)$, $\mu_A(x) > r$, $\nu_A(x) < r; \mu_A(y) < r, \nu_A(y) >$ and $\mu_B(y) > r, \nu_B(y) < r; \mu_B(x) < r, \nu_B(x) > r$. Therefore$(X, \tau)$ is IF-T_1(r-i).

The reverse implication is not necessarily true. For this we only have to show that (X, τ) is IF-T_1(r-i) $\nRightarrow (X, \tau)$ is IF-T_1(iv), which is shown by the following example.

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.6, 0.4), (y, 0.2, 0.6)\}$, $B = \{(x, 0.2, 0.8), (y, 0.7, 0.1)\}$. If $r = 0.5$ then clearly (X, τ) is IF-T_1(r-i) but not IF-T_1(iv).

Definition (Ahmed *et al.* 2014): Let $\in (0,1)$. An intuitionistic fuzzy topological space (X, τ) is called

1. α-IF-T_1(i)if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) = 1, \nu_A(x) = 0; \mu_A(y) = 0, \nu_A(y) \geq \alpha$ and $\mu_B(y) = 1, \nu_B(y) = 0;$ $\mu_B(x) = 0, \nu_B(x) \geq \alpha$.

2. α-IF-T_1(ii)if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) \geq \alpha, \nu_A(x) = 0;$ $\mu_A(y) = 0, \nu_A(y) \geq \alpha$ and $\mu_B(y) \geq \alpha, \nu_B(y) = 0;$ $\mu_B(x) = 0, \nu_B(x) \geq \alpha$.

3. α-IF-T_1(iii)if for all $x, y \in X$ with $x \neq y$, there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

   $\mu_A(x) > 0, \nu_A(x) = 0;$ $\mu_A(y) = 0, \nu_A(y) \geq \alpha$ and $\mu_B(y) > 0, \nu_B(y) = 0;$ $\mu_B(x) = 0, \nu_B(x) \geq \alpha$.

Theorem (Ahmed *et al.* 2014): Let $\alpha \in (0,1)$ and (X, τ) be an intuitionistic fuzzy topological space. Then the following implications hold.

α-IF-T_1(i) → α-IF-T_1(ii) → α-IF-T_1(iii)

Theorem: Let $\alpha \in (0,1)$ and (X, τ) be an intuitionistic fuzzy topological space. Then (X, τ) is α-IF-T_1 (k) ⇒ (X, τ) is IF-T_1(viii) for k=i, ii, iii. i.e., the following implications hold.

α-IF-T_1(i) → α-IF-T_1(ii) → α-IF-T_1(iii) → IF-T_1(viii)

Proof: We only have to prove that (X, τ) is α-IF-T_1(iii) ⇒ (X,τ) is IF-T_1(viii). Suppose (X, τ) is α-IF-T_1(iii). Let $x, y \in X$ with $x \neq y$. Since (X, τ) is is α-IF-T_1(iii), then there exists $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $\mu_A(x) > 0, \nu_A(x) = 0$; $\mu_A(y) = 0, \nu_A(y) \geq \alpha$ and $\mu_B(y) > 0, \nu_B(y) = 0$; $\mu_B(x) = 0, \nu_B(x) \geq \alpha$.

Since $\alpha \in (0,1)$, we can write $\mu_A(x) > 0, \nu_A(x) < 1$; $\mu_A(y) < 1, \nu_A(y) > 0$ and $\mu_B(y) > 0, \nu_B(y) < 1$; $\mu_B(x) < 1, \nu_B(x) > 0$. Therefore (X, τ) is IF-T_1(viii).

The reverse implications are not necessarily true. For this we only have to show that (X, τ) is IF-T_1(viii) ⇏α-IF-T_1(iii) which is shown as the following example.

Example: Let $X = \{x, y\}$ and τ be an intuitionistic fuzzy topology on X generated by $A = \{(x, 0.6, 0.4), (y, 0.2, 0.6)\}$, $B = \{(x, 0.2, 0.7), (y, 0.3, 0.5)\}$. If $\alpha = 0.5$ then clearly (X, τ) is IF-T_1(viii) but not α-IF-T_1(iii).

Theorem: Let $(X_j, \tau_j), j = 1,2$ be two IFTSs and $(X, \tau) = (X_1 \times X_2, \tau_1 \times \tau_2)$. If each (X_j, τ_j), $j = 1,2$ is IF-T_1(r-k), then (X, τ) is IF-T_1(r-k) for any $k \in \{i, ii, iii, iv, v, vi, vii\}$;

where $r \in (0,1)$ and If each $(X_j, \tau_j), j = 1,2$ are IF-T_1(viii), then (X, τ) is IF-T_1(viii).

Proof: The proofs of all implications are similar. As an example, we prove that if each $(X_j, \tau_j), j = 1,2$ are IF-T_1(r-i), then (X, t) is IF-T_1(r-i).

Let each $(X_j, \tau_j), j = 1,2$ are IF-T_1(r-i).

Suppose $x, y \in X$ with $x \neq y$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Then at least$x_1 \neq y_1$ or $x_2 \neq y_2$.

Consider$x_1 \neq y_1$. Clearly $x_1, y_1 \in X_1$. Since (X_1, τ_1) is IF-T_1(r-i), then there exists $A_1 = (\mu_{A_1}, \nu_{A_1})$, $B_1 = (\mu_{B_1}, \nu_{B_1}) \in \tau_1$ such that $\mu_{A_1}(x_1) > r, \nu_{A_1}(x_1) < r$; $\mu_{A_1}(y_1) < r$, $\nu_{A_1}(y_1) > r$ and

$\mu_{B_1}(y_1) > r, \nu_{B_1}(y_1) < r$; $\mu_{B_1}(x_1) < r$, $\nu_{B_1}(x_1) > r$.

Choose $A_2 = B_2 = 1_\sim = (\underline{1}, \underline{0})$ and Clearly $A_2, B_2 \in \tau_2$

Let $A = A_1 \times A_2 = (\mu_{A_1} \times \underline{1}, \nu_{A_1} \times \underline{0}) = (\mu_A, \nu_A)$ (say) and

$$B = B_1 \times B_2 = (\mu_{B_1} \times \underline{1}, \nu_{B_1} \times \underline{0})$$

$$= (\mu_B, \nu_B) \text{ (say)}$$

By the definition of product IFT; $A, B \in \tau$.

Now we have

$\mu_A(x) = (\mu_{A_1} \times \underline{1})(x_1, x_2) =$

$\min(\mu_{A_1}(x_1), \underline{1}(x_2)) = \min(\mu_{A_1}(x_1), 1) > r$ as $\mu_{A_1}(x_1) > r$,$\nu_A(x) = (\nu_{A_1} \times \underline{0})(x_1, x_2)$

$=\max(\nu_{A_1}(x_1), \underline{0}(x_2))$=$\max(\nu_{A_1}(x_1), 0) < r$ as $\nu_{A_1}(x_1) < r$;$\mu_A(y) = (\mu_{A_1} \times \underline{1})(y_1, y_2)$

$=\min(\mu_{A_1}(y_1), \underline{1}(y_2))$=$\min(\mu_{A_1}(y_1), 1) < r$ as $\mu_{A_1}(y_1) < r$,$\nu_A(y) = (\nu_{A_1} \times \underline{0})(y_1, y_2)$

$=\max(\nu_{A_1}(y_1), \underline{0}(y_2))$=$\max(\nu_{A_1}(y_1), 0) > r$ as $\nu_{A_1}(y_1) > r$

And $\mu_B(y) = (\mu_{B_1} \times \underline{1})(y_1, y_2)$

$=\min(\mu_{B_1}(y_1), \underline{1}(y_2))$=$\min(\mu_{B_1}(y_1), 1) > r$ as $\mu_{B_1}(y_1) > r$,$\nu_B(y) = (\nu_{B_1} \times \underline{0})(y_1, y_2)$

$= \max(\nu_{B_1}(y_1), \underline{0}(y_2)) = \max(\nu_{B_1}(y_1), 0) < r$ as $\nu_{B_1}(y_1) < r$; $\mu_B(x) = (\mu_{B_1} \times \underline{1})(x_1, x_2)$

$=\min(\mu_{B_1}(x_1), \underline{1}(x_2))$=$\min(\mu_{B_1}(x_1), 1) < r$ as $\mu_{B_1}(x_1) < r$, $\nu_B(x) = (\nu_{B_1} \times \underline{0})(x_1, x_2)$

$=\max(\nu_{B_1}(x_1), \underline{0}(x_2))$=$\max(\nu_{B_1}(x_1), 0) > r$ as $\nu_{B_1}(x_1) > r$;

i.e., for $x, y \in X$ with $x \neq y$, we get $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that

$\mu_A(x) > r$, $\nu_A(x) < r$;$\mu_A(y) < r$,$\nu_A(y) > r$ and $\mu_B(y) > r$, $\nu_B(y) < r$; $\mu_B(x) < r$, $\nu_B(xy) > r$.

Therefore (X, τ) is IF-T_1(r-i).

Remarks: The properties IF-T_1(r-k) for k= i, ii, iii, iv, v, vi, vii and IF-T_1(viii) are productive.

CONCLUSION

In this paper we see that our eight definitions are more general than those of Estiaq Ahmed *et al.* (Ahmed *et al.* 2014). Also we see that our definitions satisfy hereditary and productive properties. Moreover the definitions are preserved under one-one and open mapping. As far we know our definition (viii) is the most general among all given definitions of intuitionistic fuzzy T_1 topological spaces.

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