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CALCULATION OF RADIATION AND MATTER DOMINANT UNIVERSE BY APPLYING THE POWER LAW INFLATION

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ABSTRACT

The homogeneous and isotropic Brans-Dicke cosmological solutions satisfying Whitrow-Randall's relation which have been discussed by Bermann and Som were examined. After correcting an error in their work, we extended and examined their results obtaining the radiation and matter dominant universe applying the power law.

INTRODUCTION

In 1961 Brans and Dicke [4] provided an interesting alternative to general relativity that was based on Mach's [5] principle. Over the years there has been a great deal of interest in Mach's principle, viz, that the inertia of a body and hence its gravitational properties ought to arise from the rest of the matter in the universe. To understand the reasons leading to their field equations, we first note that the concept of a variable inertial mass arrived here itself leads to a problem of interpretation. For how do we compare masses at two different points in space-time? Masses are measured in certain units, such as, masses of elementary particles which are themselves subject to change. We need an independent unit of mass against which an increase or decrease in mass of a particle can be measured. Such a unit is provided by gravity, by the so-called Plank mass encountered earlier:

$$\left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} \equiv 2.16 \times 10^{-5}\,\text{gm} \tag{1}$$

Thus the dimensionless quantity,

$$\chi = m\left(\frac{G}{\hbar c}\right)^{\frac{1}{2}} \tag{2}$$

measured at different points in space-time can tell us whether masses m are changing. Or alternately if we insist on using mass units that are the same everywhere, a change of χ would tell us that the gravitational constant G is changing. This is the conclusion Brans and Dicke drew from the approach to Mach's principle. They looked for a frame work in which the gravitational constant G arises from the structure of the universe, so that a changing G could be looked upon as the Machian consequence of a changing universe.

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We have already come across one example of such a relation in the Friedmann [6] cosmologies:

$$\rho_0 = \frac{3H_0^2 q_0}{4\pi G} \tag{3}$$

If we write $R_0 = \frac{C}{H_0}$ as a characteristic length of the universe and $M_0 = \frac{4\pi G R_0^3}{3}$ as the characteristic mass of the universe and q_o is the deceleration parameter, then the above relation becomes,

$$\frac{1}{G} = \frac{M_0 q_0^{-1}}{R_0 C^2} \sim \frac{M_0}{R_0 C^2} \sim \Sigma \frac{m}{rc^2} \tag{4}$$

Given a dynamical coupling between the inertia and gravity, a relation of the above type is expected to hold. Brans-Dicke took this relation as one that determines G^{-1} from a linear super position of inertial contributions $\frac{m}{rc^2}$, the typical one being from a mass m at a distance r from the point, where G is measured. Since $\frac{m}{r}$ is a solution of a scalar wave equation with a point source of strength m, Brans-Dicke postulated that G behaves as the reciprocal of a scalar field φ

$$\text{i.e, } G \sim \varphi^{-1} \tag{5}$$

where φ is expected to satisfy a scalar wave equation, where source is all the matter in the universe.

There are many different formulations of Mach's principle and one of these is the Whitrow-Randall relation (WRR) given by Bermann and Som [3] as,

$$\frac{GM}{R} \sim 1 \tag{6}$$

Here M and R stand for the mass and radius respectively of the visible universe. The relation (6) suggests, either that $\frac{M}{R}$ is constant, or that the gravitational constant G should vary.

In this paper we try to correct an error in [1]. We then calculate the final results for radiation and matter dominant universe appropriately by applying the power law inflation calculated by [1] .

Solutions of Berman and Som

The Robertson-Walker model can be written as follows:

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\theta^2)\right] \tag{7}$$

where R(t) is an unknown function of time t and k is a constant, which by a suitable choice of units for r can be chosen to have the value +1, 0 or -1.

The Brans-Dicke formula reads as follows:

$$\square^2 = \frac{8\pi}{3+2\omega} T^{\mu}_{M,\mu} \tag{8}$$

where, ω is a convenient dimensionless constant and we also have,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi}{\varphi} T_{M\mu\nu} - \frac{\omega}{\varphi^2}\left(\varphi_{;\mu}\varphi_{;\nu} - \frac{1}{2} g_{\mu\nu}\varphi_{;\rho}\varphi_{;}^{\rho}\right) - \frac{1}{\varphi}(\varphi_{;\mu;\nu} - g_{\mu\nu}\square^2\varphi) \tag{9}$$

The field equation in BDT for the Robertson-Walker Metric for a perfect fluid are [7] in suitable units,

$$\ddot{\varphi} + 3\dot{\varphi}\frac{\dot{R}}{R} = \frac{4-3\gamma}{3+2\omega}\rho \tag{10}$$

Due to the conservation $T^{\mu\nu}_{;\nu} = 0$ gives the following relation:

$$\dot{\rho} + 3\frac{\dot{R}}{R}(p+\rho) = 0 \tag{11}$$

Also, the other equation is given by,

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} = \frac{\rho}{3\varphi} - \frac{\dot{\varphi}}{\varphi}\frac{\dot{R}}{R} + \frac{\omega}{6}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 \tag{12}$$

Now, the gravitational parameter G is related to Φ by the equation,

$$G = \frac{2\omega+4}{2\omega+3}.\frac{1}{\varphi} = \frac{A}{\varphi} \tag{13}$$

where $A = \frac{2\omega+4}{2\omega+3}$

So, the WRR relation $\frac{GM}{R} \sim 1$, may also be written in the form [3]

$$G\rho = \frac{6}{t^2} \tag{14}$$

Here, the deceleration parameter q is defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{15}$$

Berman and Som [3] assumed that q is constant. This leads to the solution

$$R = (mDt)^{\frac{1}{m}}$$

where m and D are constants.

Now, from (16), $\dot{R} = \frac{R}{mt}$ (17)

So, the equation (12) becomes,

$$\frac{\omega}{6}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - \frac{1}{mt}\left(\frac{\dot{\varphi}}{\varphi}\right) + \left(2a - \frac{1}{m^2}\right)\frac{1}{t^2} = 0 \tag{18}$$

Now, Berman and Som state that

$$\varphi = Bt, \quad B = \text{Constant}$$

is a solution of equation (15).

Now, using (10), (11) and (14)-(19), the equation for m is given by,

$$m^5 - 8m^4 - 2m^3 + 54m^2 - 24m - 24 = 0$$

and then listed the following five different solutions in terms of m and the coupling constant w:

$$\omega_1 \sim -18.3, \quad \omega_2 \sim -0.34, \quad \omega_3 \sim 1.12, \quad \omega_4 \sim 1.69, \quad \omega_5 \sim -26.5$$
$$m_1 \sim -2.5, \quad m_2 \sim 0.45, \quad m_3 \sim 1.06, \quad m_4 \sim 2.57, \quad m_5 \sim 7.34 \tag{21}$$

THE WRR AND THE EUCLIDEAN CASE

Here Berman and Som [3] derive their solutions by assuming a constant deceleration parameter as[3] well as WRR. In this section, we consider the Euclidean case k=0. We must show that by assuming only the WRR, we obtain solutions in which the deceleration parameter is automatically constant.

Now equation (11) can be written as [8]

$$\rho = \frac{J}{R^{3\gamma}} \tag{22}$$

where J= Constant.

The equation of state $p = (\gamma - 1)\rho$, for $\gamma = 1$, $p = 0$; the dust like universe, the equation (11) becomes, $\rho \propto R^{-3}$ (23)

Which is matter dominant universe.

The equation of state $p = (\gamma - 1)\rho$, for $\gamma = \frac{4}{3}$, $p = \frac{1}{3}\rho$; the dust like universe, the equation (11) becomes, $\rho \propto R^{-4}$ (24)

which is radiation dominant universe. Similarly, the equation (22) is a general solution for both matter and radiation dominant universe.

Now substituting equation (13) & equation (22) into equation (14) we have,

$$\varphi = \frac{AJt^2}{6R^{3\gamma}} \tag{25}$$

Now substituting equation (22) & equation (25) with the derivatives into equation (12) with k = 0, we have

$$H^2(1 - 3\gamma - \frac{3\gamma^2\omega}{2}) + \frac{H}{t}(2 + 2\gamma\omega) - \frac{2}{t^2}(\frac{1}{A} + \frac{\omega}{3}) = 0 \tag{26}$$

Equation (26) may be solved for H to yield, $H = \frac{\alpha \pm \beta}{\sigma}.\frac{1}{t}$ (27)

where H=$\frac{\dot{R}}{R}$ is the Hubble parameter and also we consider,

$$\left.\begin{aligned} \alpha &= -2 - 2\gamma w \\ \beta^2 &= 4 + \frac{8w}{3} + \frac{1}{A}(8 - 24\gamma - 12\gamma^2 w) \\ \sigma &= 2(1 - 3\gamma - \frac{3\gamma^2 w}{2}) \end{aligned}\right] \tag{28}$$

Now let us consider, $\xi = \frac{\alpha \pm \beta}{\sigma}$. (29)

So, the equation (27) can be written as,

$$R = \varsigma t^{\xi} \tag{30}$$

where ς = constant

We thus obtain a power-law solution (30). From definition of the deceleration parameter, it then follows that the deceleration parameter is constant. Such power-law solutions appear to have been first found by Nariai [9].

EXTENDED CALCULATIONS FOR NEW RESULTS

4.1 Verification of Radiation Dominant Universe by putting different values of γ & ω in our calculations as follows:

when $\gamma = 4/3, \omega = -1.41$ then from (28),

$$\left.\begin{aligned} \alpha &= 1.76 \\ \beta^2 &= 1.167 \quad \therefore \beta = \pm 1.08 \\ \sigma &= 1.52 \end{aligned}\right] \tag{31}$$

So from (29) we have, $\xi = 0.45$, then also equation (30) becomes,

$R\infty^{0.45}$, which is not the exact result for radiation dominant universe.

Again,

when $\gamma = 4/3, \omega = -1.45$ then from (28)

$$\left.\begin{aligned} \alpha &= 1.87 \\ \beta^2 &= 0.76 \qquad \therefore \beta = \pm 0.87 \\ \sigma &= 1.74 \end{aligned}\right] \tag{32}$$

So from (29) we have, $\xi = 0.57$, then also equation (30) becomes,

$R\infty^{0.57}$, which is not the exact result for radiation dominant universe.

Again,

when $\gamma = 4/3, \omega = -1.43$ then from (28)

$$\left.\begin{aligned} \alpha &= 1.81 \\ \beta^2 &= 0.989 \quad \therefore \beta = \pm 0.87 \\ \sigma &= 1.63 \end{aligned}\right] \tag{33}$$

So from (29) we have, $\xi = 0.5$, then also equation (30) becomes,

$R\infty^{0.5}$ i.e $R\infty^{\frac{1}{2}}$ which is the *appropriate result for radiation dominant universe*.

This means that numerical calculating values of equation (33) are an appropriate relation for radiation dominant universe.

4.2 Verification of Matter Dominant Universe by putting different values of γ & ω in our calculations:

when $\gamma = 1$, $\omega = -1.32$ then from (28)

$$\left.\begin{aligned} &\alpha = 1.64 \\ &\beta^2 = 0.4377 \quad \therefore \beta = \pm\, 0.66 \\ &\sigma = 1.04 \end{aligned}\right] \tag{34}$$

So from (29) we have, $\xi = 0.50$, then also equation (30) becomes, $R\infty^{0.50}$, which is not the exact result for Matter Dominant Universe.

Again,

when $\gamma = 1$ $\omega - 1.30$ then, from (28),

$$\left.\begin{aligned} &\alpha = 1.60 \\ &\beta^2 = 0.416 \quad \therefore \beta = \pm\, 0.645 \\ &\sigma = 0.1 \end{aligned}\right] \tag{35}$$

So from (29) we have, $\xi = 0.45$, then also equation (30) becomes,

$R\alpha t^{0.45}$, which is not the exact result for Matter Dominant Universe.

Again,

when, $\gamma = 1$, $\omega = -1.31$ then from (28),

$$\left.\begin{aligned} &\alpha = 1.62 \\ &\beta^2 = 0.433 \quad \therefore \beta = \pm\, 0.658 \\ &\sigma = 1.070 \end{aligned}\right] \tag{36}$$

So from (29) we have, $\xi = 0.57$, then also equation (30) becomes,

$R\infty^{0.57}$ i.e $R\infty^{\frac{2}{3}}$**,** which is the *appropriate result for matter dominant universe.*

This means that numerical calculating values of equation (36) is an appropriate relation for matter dominant universe.

CONCLUSION

We have re-examined the Robertson-Walker cosmological models in Brans-Dicke theory that satisfies Mach's principle in the form of the Whitrow-Randall Relation (WRR). The error in the paper of Bermann and Som has been corrected and we have extended their results by solving the power law into the radiation and matter dominant universe by applying the numerical values of γ, ω in equation (25), which is the power law expansion of the universe, sometimes it is regarded as the power law inflation. As a result the cosmic scale factor grows as a large power of time rather than exponentially (De-Sitter model).

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