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ON Q-COMPACTNESS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, two new notions of Q-compactness in an intuitionistic fuzzy topological space has been introduced. These two notions satisfy hereditary and productive property. Also it is shown that under some conditions Q-compactness preserved under continuous, one-one and onto function. Further some of their features have been introduced.

Keywords: Fuzzy set, intuitionistic fuzzy set, intuitionistic topological space, intuitionistic fuzzy topological space, intuitionistic fuzzy compact topological spaces.

INTRODUCTION

After the emergence of the fundamental concept on fuzzy set by Zadeh (Zadeh 1965), a huge number of paper have appeared in literature featuring the application of fuzzy sets. The concept of fuzzy topology was introduced by Chang (Chang, 1968). As a generalization of fuzzy set, the concept of intuitionistic fuzzy set was introduced by Atanassov (Atanassov, 1986) which take into account both the degrees of membership and non-membership subject to the condition that their sum does not exceed 1. In the last five decades various concepts of fuzzy mathematics have been extended. Dogen Coker (Coker 1996, 1997, Bayhan and Coker 1996, Coker and Bayhan 2001, 2003) and his colleagues introduced intuitionistic fuzzy topological spaces and fuzzy compactness in intuitionistic fuzzy topological spaces. Islam (Islam *et al.* 2017, 2018, 2018), Lee (Lee *et al*. 2000. 2004), Ahmed (Ahmed *et. al.* 2014, 2015, 2015, 2014, 2014), L. Ying-Ming (Ying-Ming *et al.* 1997), Talukder (Talukder *et. al.* 2013), J. Tamilmani (Tamilmani 2015) and R. Islam (R. Islam *et al.* 2018) subsequently developed the study of intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. Mahbub (Mahbub *et al.* 2018) define compactness in several ways. In this paper, two new notions of Q-compactness in intuitionistic fuzzy topological space and some of their features are defined.

Notations and Preliminaries

Through this paper, X will be a nonempty set, T is a topology, t is a fuzzy topology, $\mathcal{T}$ is an intuitionistic topology and τ is an intuitionistic fuzzy topology. λ and μ are fuzzy sets, $A = (\mu_A, \nu_A)$ is intuitionistic fuzzy set. Particularly $\underline{0}$ and $\underline{1}$ denote constant fuzzy sets taking values 0 and 1 respectively.

Definition (Chang 1968): Let X be a non empty set. A family t of fuzzy sets in X is called a fuzzy topology on X if the following conditions hold.

(1) $\underline{0}, \underline{1} \in t$,
(2) $\lambda \cap \mu \in t$ for all $\lambda, \mu \in t$,
(3) $\cup \lambda_j \in t$ for any arbitrary family $\{\lambda_j \in t, j \in J\}$.

Definition (Coker 1996): Let X is a non empty set. An intuitionistic set A on X is an object having the form $A = (X, A_1, A_2)$ where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A. In this paper, the simpler notation $A = (A_1, A_2)$ instead of $A\ (X, A_1, A_2)$ is used for an intuitionistic set.

Remark: Every subset A of a non-empty set X may obviously be regarded as an intuitionistic set having the form $A(A, A^c)$ where $A^c = X - A$.

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Definition (Coker 1996): Let the intuitionistic sets A and B in X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j, j \in J\}$ be an arbitrary family ofintuitionistic sets in X, where $A_j = \left(A_j^{(1)}, A_j^{(2)}\right)$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$and $A_2 \supseteq B_2$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\bar{A} = (A_2, A_1)$, denotes the complement of A,
(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$,
(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$,
(f) $\phi_{\sim} = (\phi, X)$ and $X_{\sim} = (X, \phi)$.

Definition (Coker and Bayhan 2001): Let X be a non empty set. A family $\mathcal{T}$ of intuitionistic sets in X is called an intuitionistic topology on X if the following conditions hold.

(1) $\phi_{\sim}, X_{\sim} \in \mathcal{T}$,
(2) $A \cap B \in \mathcal{T}$ for all $A, B \in \mathcal{T}$,
(3) $\cup A_j \in \mathcal{T}$ for any arbitrary family $\{A_j \in \mathcal{T}, j \in J\}$.

The pair $(X, \mathcal{T})$ is called an intuitionistic topological space (ITS, in short), members of $\mathcal{T}$ are called intuitionistic open sets (IOS, in short) in X and their complements are called intuitionistic closed sets (ICS, in short) in X.

Definition (Atanassov 1986): Let X be a non empty set. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where μ_A and ν_A are fuzzy sets in X denote the degree of membership and the degree of non- membership respectively subject to the condition $\mu_A(x) + \nu_A(x) \leq 1$.

Throughout this paper, the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ for IFSs is used.

Definition (Atanassov 1986): Let X be a non-empty set and IFSs A, B in X be given by $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ respectively, then

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$,
(c) $\bar{A} = (\nu_A, \mu_A)$,
(d) $A \cap B = (\mu_A \cap \mu_B, \nu_A \cup \nu_B)$,
(e) $A \cup B = (\mu_A \cup \mu_B, \nu_A \cap \nu_B)$.

Definition (Coker 1997): Let $\{A_j = \left(\mu_{A_j}, \nu_{A_j}\right), j \in J\}$ be an arbitrary family of IFSs in X. Then

(a) $\cap A_j = (\cap \mu_{A_j}, \cup \nu_{A_j})$,
(b) $\cup A_j = (\cup \mu_{A_j}, \cap \nu_{A_j})$,
(c) $0_{\sim} = (\underline{0}, \underline{1})$, $1_{\sim} = (\underline{1}, \underline{0})$.

Definition (Coker 1997): An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(1) $0_{\sim}, 1_{\sim} \in \tau$,
(2) $A \cap B \in \tau$, for all $A, B \in \tau$,
(3) $\cup A_j \in \tau$ for any arbitrary family $\{A_j \in \tau, j \in J\}$.

The pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS, in short), members of τ are called intuitionistic fuzzy open sets (IFOS, in short).

Definition (Atanassov 1986): Let X and Y be two nonempty sets and $f: X \to Y$ be a function. If $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ and $B = \{(y, \mu_B(y), \nu_B(y)) : y \in Y\}$ are IFSs in X and Y respectively, then the pre image of B under f, denoted by $f^{-1}(B)$ is the IFS in X defined by $f^{-1}(B) = \left\{\left(x, \left(f^{-1}(\mu_B)\right)(x), \left(f^{-1}(\nu_B)\right)(x)\right) : x \in X\right\} = \{(x, \mu_B(f(x)), \nu_B(f(x))) : x \in X\}$ and the image of A under f, denoted by $f(A)$ is the IFS in Y defined by

$f(A) = \{(y, (f(\mu_A))(y), (f(\nu_A))(y)) : y \in Y\}$, where for each $y \in Y$,

$$(f(\mu_A))(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & if\ f^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$

and $$(f(\nu_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & if\ f^{-1}(y) \neq \phi \\ 1 & otherwise \end{cases}$$

Definition (Chang 1968): Let $A = (x, \mu_A, \nu_A)$ and $B = (y, \mu_B, \nu_B)$ be IFSs in X and Y respectively. Then the product of IFSs A and B denoted by $A \times B$ is defined by $A \times B = \{(x, y), \mu_A \overset{\times}{.} \mu_B, \nu_A \underset{\times}{\dot{}} \nu_B)\}$ where $\left(\mu_A \overset{\times}{.} \mu_B\right)(x, y) = \min(\mu_A(x), \mu_B(y))$ and $(\nu_A \underset{\times}{\dot{}} \nu_B)(x, y) = \max(\nu_A(x), \nu_B(y))$ for all

$(x, y) \in X \times Y$. Obviously $0 \leq \left(\mu_A \dot{\times} \mu_B\right) + \left(\nu_A \dot{\times} \nu_B\right) \leq 1$. This definition can be extended to an arbitrary family of IFSs.

Definition (Chang 1968): Let (X_j, τ_j), $j = 1,2$ be two IFTSs. The product topology $\tau_1 \times \tau_2$ on $X_1 \times X_2$ is the IFT generated by $\{\rho_j^{-1}(U_j) : U_j \in \tau_j, j = 1,2\}$, where $\rho_j : X_1 \times X_2 \to X_j$, $j = 1,2$ are the projection maps and IFTS $\{X_1 \times X_2, \tau_1 \times \tau_2\}$ is called the product IFTS of (X_j, τ_j), $j = 1,2$. In this case $\mathcal{S} = \{\rho_j^{-1}(U_j), j \in J : U_j \in \tau_j\}$ is a sub base and $\mathcal{B} = \{U_1 \times U_2 : U_j \in \tau_j, j = 1,2\}$ is a base for $\tau_1 \times \tau_2$ on $X_1 \times X_2$.

Definition (Coker 1997): Let (X, τ) and (Y, δ) be IFTSs. A function $f : X \to Y$ is called continuous if $f^{-1}(B) \in \tau$ for all $B \in \delta$ and f is called open if $f(A) \in \delta$ for all $A \in \tau$.

Definition (Singh *et al*. 2012): Let $A = (\mu_A, \nu_A)$ be an IFS in X and U be a non empty subset of X. The restriction of A to U is an IFS in U, denoted by $A|U$ and defined by $A|U = (\mu_A|U, \nu_A|U)$.

Definition (Islam *et al.* 2017): Let (X, τ) be an intuitionistic fuzzy topological space and U is a non empty sub set of X then $\tau_U = \{A|U : A \in \tau\}$ is an intuitionistic fuzzy topology on U and (U, τ_U) is called sub space of (X, τ).

Definition (Singh *et al.* 2012): Let $\alpha, \beta \in (0,1)$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP for short) $p_{(\alpha,\beta)}^x$ of X defined by $p_{(\alpha,\beta)}^x = \langle x, \mu_p, \nu_p \rangle$, for $y \in X$

$\mu_p(y) = \begin{cases} \alpha & if \quad y = x \\ 0 & if \quad y \neq x \end{cases}$ and

$$\nu_p(y) = \begin{cases} \beta & if \quad y = x \\ 1 & if \quad y \neq x \end{cases}$$

In this case, x is called the support of $p_{(\alpha,\beta)}^x$. An IFP $p_{(\alpha,\beta)}^x$ is said to belong to an IFS A= $\langle x, \mu_A, \nu_A \rangle$ of X, denoted by $p_{(\alpha,\beta)}^x \in A$, if $\alpha \leq \mu_A(x)$ and $\beta \geq \nu_A(x)$.

Proposition (Singh *et al.* 2012): An IFS A in X is the union of all its IFP belonging to A.

Definition (Das 2013): A collection $\mathcal{B}$of IFS on a set X is said to be basis (or base) for an IFT on X, if

(i) For every $p_{(\alpha,\beta)}^x \in X$, there exists$B \in \mathcal{B}$ such that $p_{(\alpha,\beta)}^x \in B$.

(ii) If $p_{(\alpha,\beta)}^x \in B_1 \cap B_2$, where $B_1, B_2 \in \mathcal{B}$ then $\exists B_3 \in \mathcal{B}$ such that $p_{(\alpha,\beta)}^x \in B_3 \subseteq B_1 \cap B_2$.

Definition (Coker 1997): Let (X, τ) be an IFTS.

(a) If a family $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ of IFOS in X satisfy the condition $\cup \{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\} = 1_\sim$ then it is called a fuzzy open cover of X. A finite subfamily of fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ of X, which is also a fuzzy open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$.
(b) A family $\{\langle x, \mu_{K_i}, \nu_{K_i} \rangle : i \in J\}$ of IFCS's in X satisfies the finite intersection property iff every finite subfamily $\{\langle x, \mu_{K_i}, \nu_{K_i} \rangle : i = 1,2, \ldots, n\}$ of the family satisfies the condition $\bigcap_{i=1}^n \{\langle x, \mu_{K_i}, \nu_{K_i} \rangle\} \neq 0_\sim$.

Definition (Coker 1997): An IFTS (X, τ) is called fuzzy compact iff every fuzzy open cover of X has a finite subcover.

Definition (Coker 1997): (a) Let (X, τ) be an IFTS and A be an IFS in X. If a family $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ of IFOS's in X satisfies the condition $A \subseteq \cup \{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$, then it is called a fuzzy open cover of A. A finite subfamily of the fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ of A, which is also a fuzzy open cover of A, is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$.

(b) An IFS $A = \langle x, \mu_{G_i}, \nu_{G_i} \rangle$ in an IFTS (X, τ) is called fuzzy compact iff every fuzzy open cover of A has a finite subcover.

Definition (Ramadan *et al*. 2005): An IFTS (X, τ) is called (α, β)-intuitionistic fuzzy compact (resp., (α, β)-intuitionistic fuzzy nearly compact and (α, β)-intuitionistic fuzzy almost compact) if and only if for every family $\{G_i : i \in J\}$ in $\{G : G \in \zeta^X, \tau(G) > \langle \alpha, \beta \rangle\}$ such that $\bigcup_{i \in J} G_i = 1_\sim$, where $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$, there exists a finite subset J_0 of J such that $\bigcup_{i \in J_0} G_i = 1_\sim$ (resp., $\bigcup_{i \in J_0} int_{\alpha,\beta}\left(cl_{\alpha,\beta}(G_i)\right) = 1_\sim$ and $\bigcup_{i \in J_0} cl_{\alpha,\beta}(G_i) = 1_\sim$).

Definition (Ramadan *et al.* 2005): Let (X, τ) be an IFTS and A be an IFS in X. A is said to be

(α, β)-intuitionistic fuzzy compact if and only if every family $\{G_i : i \in J\}$ in $\{G : G \in \zeta^X, \tau(G) > \langle\alpha, \beta\rangle\}$ such that $A \subseteq \cup_{i \in J_0} G_i$, where $\alpha \in I_0, \beta \in I_1$ with $\alpha + \beta \leq 1$.

Definition (Mahbub *et al.* 2018): Let (X, τ) be an intuitionistic fuzzy topological space. A family $\{(\mu_{G_i}, \nu_{G_i}) : i \in J\}$ of IFOS in X is called open cover of X if $\cup \mu_{G_i} = 1$and $\cap \nu_{G_i} = 0$. If every open cover of X has a finite subcover then X is said to be intuitionistic fuzzy compact (IF-compact, in short).

Definition (Mahbub *et al.* 2018): A family $\{(\mu_{G_i}, \nu_{G_i}) : i \in J\}$ of IFOS in X is called (α, β)-level open cover of X if $\cup \mu_{G_i} \geq \alpha$ and $\cap \nu_{G_i} \leq \beta$ with $\alpha + \beta \leq 1$. If every (α, β)-level open cover of X has a finite subcover then X is said to be (α, β)-level IF-compact.

Definition (Barlie): A topological space (X, T) is called T_0 if for all $x, y \in X$ with $x \neq y$, there exists $U \in T$ such that $x \in U, y \notin U$ or $y \in U, x \notin U$.

Definition (Srivastava *et al.* 1988): A fuzzy topological space (X, t) is called T_0 if for all $x, y \in X$ with $x \neq y$, there exists $U \in t$ such that $U(x) = 1, U(y) = 0$ or $U(y) = 1$, $U(x) = O$ i.e., $x \in U, y \notin U$ or $y \in U, x \notin U$.

Q-compactness in intuitionistic fuzzy topological space

In this section two new definitions of Q-compactness in intuitionistic fuzzy topological space (IFTS, in short are defined) and established several properties of such notions are established.

Definition: Let(X, τ) be an intuitionistic fuzzy topological space (IFTS) and $A = (\mu_A, \nu_A)$ be an IFS in X. Consider $\mathcal{M} = \{B_i : i \in J\}$ be a family of IFS in X, where $B_i = (\mu_{B_i}, \nu_{B_i})$. Then $\mathcal{M}$ is called Q-cover of A if $A \subseteq \cup B_i$, $\mu_A(x) + \mu_{B_i}(x) \geq 1$ for each $\mu_{B_i}$and some $x \in X$. If each B_i is open then $\mathcal{M}$ is called an open Q-cover of A. A subfamily of Q-cover of an IFS A in X which is also a Q-cover of A is called Q-subcover of A.

Definition: An IFS $A = (\mu_A, \nu_A)$ in X is said to be Q-compact if every open Q-cover of A has a finite Q-subcover i.e. $\exists B_{i1}, B_{i2}, \dots, B_{in} \in \mathcal{M}$ such that $A \subseteq \cup_{i=1}^n B_i$, $\mu_A(x) + \mu_{B_{ij}}(x) \geq 1$ for each $\mu_{B_{ij}}$ and some $x \in X$, $j = 1, 2, \dots, n$.

Example: Let $X = \{a, b\}$ and $I = [0,1]$. Let $A_1, A_2 \in I^X$ defined by $A_1(a) = (0.5, 0.2)$, $A_1(b) = (0.7, 0.2)$, $A_2(a) = (0.6, 0.3)$ and $A_2(b) = (0.8, 0.1)$. Considering, τ be an intuitionistic fuzzy topological space which is generated by $\{A_1, A_2\}$. Then (X, τ) be an intuitionistic fuzzy topological space (IFTS). Again let $A \in I^X$ with $A(a) = (0.5, 0.3)$, $A(b) = (0.3, 0.2)$. Here $A(a) \subseteq \cup A_i(a)$, $\mu_A(a) + \mu_{A_i}(a) \geq 1$. Again, $A(b) \subseteq \cup A_i(b)$, $\mu_A(b) + \mu_{A_i}(b) \geq 1$. Therefore $\{A_1, A_2\}$ is a Q-cover of A.

Theorem: Let (X, τ) be an IFTS, V is an subset of X and A be an IFS in V, where $A = (\mu_A, \nu_A)$. Then A is Q-compact in (X, τ) iff $A|V$ is Q-compact in (V, τ_V).

Proof: Let $A = (\mu_A, \nu_A)$ is Q-compact in (X, τ). Let $\mathcal{M} = \{B_i = (\mu_{B_i}, \nu_{B_i}) : i \in J\}$ be an open Q-cover of A in (V, τ_V). By the definition of subspace topology, $B_i = U_i|V$, where $U_i \in \tau$. Hence $\mu_A(x) + \mu_{B_i}(x) \geq 1$ for each $\mu_{B_i}$and some $x \in V$ and consequently $\mu_A(x) + \mu_{U_i}(x) \geq 1$ for each μ_{U_i} and some $x \in X$ as $V \subseteq X$. Now $A \subseteq \cup B_i \Rightarrow A \subseteq \cup U_i|V \Longrightarrow A|V \subseteq \cup U_i|V$. Therefore $\{U_i : i \in J\}$ is an open Q-cover of A|V in (X, τ). As A is Q-compact in (X, τ) then A has finite Q-subcover i.e. there exist $U_{ik} \in \{U_i\}, k \in j_n$ such that $\mu_A(x) + \mu_{U_{ik}}(x) \geq 1$ for each $\mu_{U_{ik}}$ and some $x \in V$. This implies that $\mu_A(x) + \mu_{(U_{ik}|V)}(x) \geq 1$ for each $x \in V$. Also $A \subseteq \cup B_{ik} \Rightarrow A \subseteq \cup U_{ik}|V \Longrightarrow A|V \subseteq \cup U_{ik}|V$. Thus $\{B_i\}$ contains a finite subcover $\{B_{i1}, B_{i2}, \dots, B_{in}\}$ and hence $A|V$ is Q-compact in (V, τ_V).

Theorem: Let (X, τ)be an IFTS. If $A = (\mu_A, \nu_A)$ and $V = (\mu_V, \nu_V)$ are Q-compact in (X, τ) then $A \cup V$ is also Q-compact in (X, τ).

Proof: Let $\mathcal{M} = \{A_i = (\mu_{A_i}, \nu_{A_i}) : i \in J\}$ be an open Q-cover of $A = (\mu_A, \nu_A)$ and $\aleph = \{B_i = (\mu_{B_i}, \nu_{B_i}) : i \in J\}$ be an open Q-cover of $V =$

(μ_V, ν_V) in (X, τ). Now $A \subseteq \bigcup_{i=1}^{n} A_i$ and $V \subseteq \bigcup_{i=1}^{n} B_i$

$\Rightarrow A \cup V \subseteq \bigcup_{i=1}^{n} A_i \cup \bigcup_{i=1}^{m} B_i =$

$$\begin{cases} \bigcup_{i=1}^{m}((A_i \cup B_i) \cup (\bigcup_{i=m+1}^{n} A_i)) & if \quad n > m \\ \bigcup_{i=1}^{n}((A_i \cup B_i) \cup (\bigcup_{i=n+1}^{mn} B_i)) & if \quad m > n \end{cases}$$

$$\Rightarrow A \cup V \subseteq \cup (A_i \cup B_i)$$

Again, by the definition of Q-compactness, $\mu_A(x) + \mu_{A_i}(x) \geq 1$ for each μ_{A_i} and some $x \in X$ and $\mu_V(x) + \mu_{B_i}(x) \geq 1$ for each μ_{B_i} and some $x \in X$.

$\Rightarrow \mu_{(A \cup V)}(x) + \mu_{(A_i \cup B_i)}(x) \geq 1$

i.e. $\mathcal{M} \cup \aleph = \{A_i \cup B_i\}$ is an open Q-cover of $A \cup V$.

Again, as A is Q-compact in (X, τ) then A has finite Q-subcover i.e. there exist $A_{ik} \in \{A_i\}, k \in j_n$ such that $A \subseteq \bigcup_{k=1}^{n} A_{ik}$ and $\mu_A(x) + \mu_{A_{ik}}(x) \geq 1$ for each $\mu_{A_{ik}}$ and some $x \in X$. Also as V is Q-compact in (X, τ) then V has finite Q-subcover i.e. there exist $B_{ik} \in \{B_i\}, k \in j_n$ such that $V \subseteq \bigcup_{k=1}^{n} B_{ik}$ and $\mu_V(x) + \mu_{B_{ik}}(x) \geq 1$ for each $\mu_{B_{ik}}$ and some $x \in X$. Now $A \subseteq \bigcup_{k=1}^{n} A_{ik}$ and $V \subseteq \bigcup_{k=1}^{n} B_{ik}$ $\Rightarrow A \cup V \subseteq \bigcup_{k=1}^{n} A_{ik} \cup \bigcup_{k=1}^{m} B_{ik}$

$$= \begin{cases} \bigcup_{k=1}^{m}((A_{ik} \cup B_{ik}) \cup (\bigcup_{i=m+1}^{n} A_{ik})) & if \quad n > m \\ \bigcup_{k=1}^{n}((A_{ik} \cup B_{ik}) \cup (\bigcup_{i=n+1}^{mn} B_{ik})) & if \quad m > n \end{cases}$$

$\Rightarrow A \cup V \subseteq \cup (A_{ik} \cup B_{ik})$

Also, $\mu_A(x) + \mu_{A_{ik}}(x) \geq 1$ and $\mu_V(x) + \mu_{B_{ik}}(x) \geq 1$

$\Rightarrow \mu_{(A \cup V)}(x) + \mu_{(A_{ik} \cup B_{ik})}(x) \geq 1$.

i.e. $\{A_{ik} \cup B_{ik}\}$ is an open Q-subcover of $A \cup V$. Hence $A \cup V$ is also Q-compact in (X, τ).

Theorem: Let (X, τ)be an IFTS and $A = (\mu_A, \nu_A)$ be an IFS in X. If every $\{F_i\}$ where $F_i = (\nu_{F_i}, \mu_{F_i})$ of closed subset of X with $\cap F_i = (0,1)$ implies $\{F_i\}$ contains finite subclass $\{F_{i1}, F_{i2}, \dots, F_{im}\}$ with $F_{i1} \cap F_{i2} \cap \dots \cap F_{im} = (0,1)$ then A is Q-compact in(X, τ).

Proof: Given $\cap F_i = (0,1)$ where $F_i = (\nu_{F_i}, \mu_{F_i})$, then by De Morgan's law $(\cap F_i)^c = ((0,1))^c \Rightarrow \cup F_i{}^c = (1,0)$

$$\Rightarrow \cup (\nu_{F_i}, \mu_{F_i})^c = (1,0) \Rightarrow \cup (\mu_{F_i}, \nu_{F_i}) = (1,0)$$

$\Rightarrow (\cup \mu_{F_i}, \cap \nu_{F_i}) = (1,0)$.

Let $\mathcal{M} = \{B_i = (\mu_{B_i}, \nu_{B_i}) : i \in J\}$ be an open Q-cover of A in(X, τ), so $A \subseteq \cup B_i$, $\mu_A(x) + \mu_{B_i}(x) \geq 1$ for each μ_{B_i} and some $x \in X$. Since each B_i is open, so $\{B_i{}^c\}$ is a class of closed sets and by given condition$\exists B_{i1}{}^c, B_{i2}{}^c, \dots, B_{im}{}^c \epsilon \{B_i{}^c\}$ such that $B_{i1}{}^c \cap B_{i2}{}^c \cap \dots \cap B_{im}{}^c = (0,1)$. So by De Morgan's law $(1,0) = (0,1)^c = (B_{i1}{}^c \cap B_{i2}{}^c \cap \dots \cap B_{im}{}^c)^c = B_{i1} \cup B_{i2} \cup \dots \cup B_{im}$, hence$A \subseteq \bigcup_{i=1}^{n} B_{ij}$, $\mu_A(x) + \mu_{B_{ij}}(x) \geq 1, j = 1,2, \dots, n$ for each $\mu_{B_{ij}}$ and some $x \in X$. So, A is Q-compact in (X, τ).

Theorem: Let (X, τ) and (Y, δ) be two IFTS and $f : X \to Y$ is bijective, open and continuous. If $f(A) = (f(\mu_A), f(\nu_A))$ is Q-compact in (Y, δ) then A is Q-compact in (X, τ).

Proof: Let $A = (\mu_A, \nu_A) \in \tau$. Consider $\mathcal{M} = \{B_i \in \tau\}$ where $B_i = (\mu_{B_i}, \nu_{B_i}), i \in J$ with $A \subseteq \cup B_i$ and $\mu_A(x) + \mu_{B_i}(x) \geq 1$ for each μ_{B_i} and some $x \in X$ i.e. $\mathcal{M}$ is a Q-open cover of A, then $f(A) = (f(\mu_A), f(\nu_A))$ is an IFS of Y. Since $B_i \in \tau$ then $f(B_i) \in \delta$ as f is open. But $f(B_i) = (f(\mu_{B_i}), f(\nu_{B_i}))$. Now we have $A \subseteq \cup B_i \Rightarrow f(A) \subseteq f(\cup_i B_i) = \cup_i f(B_i)$ i.e. $f(A) \subseteq \cup_i f(B_i)$. For any $y \in Y, f(\mu_A)(y) + f(\mu_{B_i})(y) = sup\mu_A(x) + sup\mu_{B_i}(x)$, where $x \in f^{-1}(y)$

$\geq \mu_A(x) + \mu_{B_i}(x) \forall i \in J$, since f is onto and $f(y) \neq \emptyset \geq 1$

$$\Rightarrow \quad f(\mu_A)(y) + f(\mu_{B_i})(y) \geq 1$$

i.e. $\mathcal{H} = \{f(B_i) : i \in J\}$ is Q-open cover of $f(A)$. Since $f(A)$ is Q-compact then $\exists f(B_{i1}), f(B_{i2}), \dots, f(B_{in}) \in \delta \ni f(A) \subseteq \bigcup_{k=1}^{n} f(B_{ik})$ and

$$f(\mu_A)(y) + f(\mu_{B_{ik}})(y) \geq 1$$

$$\Rightarrow f^{-1}\left(f(\mu_A)(y) + f(\mu_{B_{ik}})(y)\right) \geq f^{-1}(1)$$

$$\Rightarrow f^{-1} f(\mu_A)(y) + f^{-1} f(\mu_{B_{ik}})(y) \geq 1$$

$\Rightarrow \mu_A(x) + \mu_{B_{ik}}(x) \geq 1$ as f is continuous and so $\forall y \in Y \Rightarrow \exists$ unique $x \in X$ since $f(x) = y$. Again $f(A) \subseteq \bigcup_{k=1}^{n} f(B_{ik})$ $\Rightarrow f^{-1} f(A) \subseteq f^{-1}(\bigcup_{k=1}^{n} f(B_{ik}))$ $\Rightarrow A \subseteq \bigcup_{k=1}^{n} f^{-1} f(B_{ik})$ $\Rightarrow A \subseteq \bigcup_{k=1}^{n} B_{ik}$. It is clear that $B_{ik} \in \tau \ni A \subseteq \bigcup_{k=1}^{n} B_{ik}$ and $\mu_A(x) + \mu_{B_{ik}}(x) \geq 1$. Hence A is Q-compact in (X, τ).

Theorem: Let (X, τ) and (Y, δ) be two IFTS and $f : X \to Y$ is bijective, open and continuous. If $A =$

(μ_A, ν_A) is Q-compact in (X, τ) then $f(A) = (f(\mu_A), f(\nu_A))$ is Q-compact in (Y, δ).

Proof: Let $\mathcal{M} = \{B_i \in \delta\}$ where $B_i = (\mu_{B_i}, \nu_{B_i}), i \in J$ be an open Q-cover of $f(A)$ with $f(A) \subseteq \cup B_i$ and $\mu_{f(A)}(y) + \mu_{B_i}(y) \geq 1$ for each $\mu_{B_i}$and some $y \in Y$. Since $B_i \in \delta$ then $f^{-1}(B_i) \in \tau$ but $f^{-1}(B_i) = (f^{-1}(\mu_{B_i}), f^{-1}(\nu_{B_i}))$. Again $f(A) \subseteq \cup B_i => A \subseteq f^{-1}(\cup B_i)$ i.e. $A \subseteq \cup f^{-1}(B_i)$. For any $x \in X$, $\mu_A(x) + \mu_{f^{-1}(B_i)}(x) \geq 1$. Again since A is Q-compact. i.e. $\mathcal{H} = \{f^{-1}(B_i): i \in J\}$ is Q-open cover of A. Further since A is Q-compact in (X, τ) then $\exists f^{-1}(B_{i1}), f^{-1}(B_{i2}), \ldots, f^{-1}(B_{in}) \in \tau \ni A \subseteq \cup_{k=1}^{n} f^{-1}(B_{ik})$ and

$$\mu_A(x) + \mu_{f^{-1}(B_{ik})}(x) \geq 1$$

$$=> f(\mu_A)(x) + f(\mu_{f^{-1}(B_{ik})})(x) \geq f(1)$$

$$=> \mu_{f(A)}(y) + \mu_{ff^{-1}(B_{ik})}(y) \geq 1$$

$=> \mu_{f(A)}(y) + \mu_{B_{ik}}(y) \geq 1$ as f is continuous. But $A \subseteq \cup_{k=1}^{n} f^{-1}(B_{ik})$ $=> f(A) \subseteq f(\cup_{k=1}^{n} f^{-1}(B_{ik}))$ $=> f(A) \subseteq \cup_{k=1}^{n} ff^{-1}(B_{ik})$ $=> f(A) \subseteq \cup_{k=1}^{n} B_{ik}$. Hence $B_{ik} \in \delta \ni f(A) \subseteq \cup_{k=1}^{n} B_{ik}$ and $\mu_{f(A)}(x) + \mu_{B_{ik}}(x) \geq 1$. Hence $f(A)$ is Q-compact in (Y, δ).

Theorem: Let A and V be Q-compact IFS in an IFTS (X, τ). Then $A \times V$ is also Q-compact in $(X \times X, \tau \times \tau)$.

Proof: Let $\mathcal{M} = \{B_i: B_i = (\mu_{B_i}, \nu_{B_i}) \in \tau \times \tau, i \in J\}$ be a Q-cover of $A \times V$ in $(X \times X, \tau \times \tau)$. Then $A \times V \subseteq \cup B_i$ and $\mu_{A \times V}(x, y) + \mu_{B_i}(x, y) \geq 1$ for each $\mu_{B_i}$and some $(x, y) \in X \times X$. Now can it be written, $B_i = U_i \times W_i$, where $U_i, W_i \in \tau$. Thus $A \times V \subseteq \cup B_i => A \times V \subseteq \cup(U_i \times W_i) => A \subseteq \cup U_i, V \subseteq \cup W_i$. Also $\mu_{A \times V}(x, y) + \mu_{U_i \times W_i}(x, y) \geq 1$ for each $\mu_{U_i \times W_i}$ and some $(x, y) \in X \times X$. Hence it is clear that $\mu_A(x) + \mu_{U_i}(x) \geq 1$ for each μ_{U_i} and some $x \in X$ and $\mu_V(y) + \mu_{W_i}(y) \geq 1$ for each μ_{W_i} and some y$\in X$. Therefore $\{U_i: i \in J\}$ and $\{W_i: i \in J\}$ are open Q-cover of A and V respectively. Since A and V are Q-compacts then $\{U_i: i \in J\}$ and $\{W_i: i \in J\}$ have finite Q-subcovers, say $\{U_{ik}: k \in J_n\}$ and $\{W_{ik}: k \in J_n\}$ respectively such that $A \subseteq \cup U_{ik}, \mu_A(x) + \mu_{U_ik}(x) \geq 1$ for each $\mu_{U_{ik}}$and some $x \in X$ and $V \subseteq \cup W_{ik}, \mu_V(y) + \mu_{W_{ik}}(y) \geq 1$ for each $\mu_{W_{ik}}$and some y$\in X$. Thus $A \times V \subseteq \cup(U_{ik} \times W_{ik}) => A \times V \subseteq \cup B_{ik}$ and $\mu_{A \times V}(x, y) + \mu_{B_{ik}}(x, y) \geq 1$ for each $\mu_{B_{ik}}$ and some $(x, y) \in X \times X$. Hence Then $A \times V$ is Q-compact in $(X \times X, \tau \times \tau)$.

CONCLUSION

In this paper, two notions introduced satisfy the basic properties of general topology in intuitionistic fuzzy topological space. The two notions satisfy hereditary and productive property and the notions are under one-one, onto and continuous mapping.

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