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Research Article

Mathematically forecasting for generalized business by using fuzzy trapezoidal numbers

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ABSTRACT

In this paper, we forecasted the future of a business by using fuzzy trapezoidal numbers and the fuzzy Delphi method. This result is compared with another result obtained using the fuzzy triangular numbers and Delphi method. At last, we see that our method is more general than others.

Introduction

The main theme of these Fuzzy trapezoidal numbers defined by Abbasbandy and Hajjari (2010) model construction is to predict the time duration for future forecasting of a business basis on the past historical observation, which is introduced by Ali et al. (2016) and Mutalib et al. (2018). To solve the future forecasting problem of a business, such a way is an appropriate way based on the fuzzy set theory defined by Zadeh (1996 and 1965). Based on the fuzzy set theory, many future forecasting models have been established for businesses using the fuzzy Delphi method introduced by Kuo and Chen (2008).

In this paper, we propose a trapezoidal model based on the fuzzy number and fuzzy Delphi method. Fuzzy numbers are introduced by Bellman and Zadeh (1970) and Kaufmann and Gupta (1985 and 1988). And Delphi method was developed by Roy and Garai (2012). California in the 1940s.

Preliminaries

2.1 Fuzzy set: A fuzzy set A is defined by a set, A=$\{(x, \mu_A(x) | x \in A, \mu_A(x) \in [0,1]\}$, where $\mu_A(x)$ is a membership function belonging to [0,1] (Mohanpriya and Jeyanthi, 2016)

2.2 Fuzzy number: A fuzzy number is defined as a convex and normalized fuzzy set on the universe R (Mohanpriya and Jeyanthi, 2016).

2.3 Triangular fuzzy number: A triangular fuzzy number A with membership function $\mu_A(x)$ is defined on R by (Gani & Assarudeen, 2012)

$$A \triangleq \mu_A(x) = \begin{cases} \frac{x-a_1}{a_M-a_1} & for \quad a_1 \le x \le a_M \\ \frac{x-a_2}{a_M-a_2} & for \quad a_M \le x \le a_2 \\ 0 & otherwise. \end{cases}$$

2.4 Trapezoidal fuzzy number: A trapezoidal fuzzy number A with membership function $\mu_A(x)$ is defined on R by (Mohanpriya and Jeyanthi, 2016).

$$A \triangleq \mu_A(x) = \begin{cases} \frac{x-a_1}{b_1-a_1} & for \quad a_1 \le x \le b_1 \\ 1 & for \quad b_1 \le x \le b_2 \\ \frac{x-a_2}{b_2-a_2} & for \quad b_2 \le x \le a_2 \\ 0 & otherwise. \end{cases}$$

2.5 Fuzzy averaging: (i). Triangular fuzzy average formula (Bojadziev and Bojadziev, 2007). Consider n triangular numbers

$A_i = \left(a_1^{(i)}, a_M^{(i)}, a_2^{(i)}\right),$

$where\ i = 1, 2, \dots \dots . n.$

The triangular average A_{ave} ,

$A_{ave} = (m_1, m_M, m_2) = \frac{A_1+A_2+\cdots+A_n}{n}$

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$= \frac{\left(a_1^{(1)}, a_M^{(1)}, a_2^{(1)}\right) + \cdots + \left(a_1^{(n)}, a_M^{(n)}, a_2^{(n)}\right)}{n}$

$= \frac{\left(\sum_{i=1}^{n} a_1^{(i)}, \sum_{i=1}^{n} a_M^{(i)}, \sum_{i=1}^{n} a_2^{(i)}\right)}{n}$ (2.1)

(ii). Trapezoidal fuzzy average formula (Mutalib et al., 2018)

Consider n trapezoidal numbers

$A_i = \left(a_1^{(i)}, b_1^{(i)}, b_2^{(i)}, a_2^{(i)}\right),$

Where $i = 1,2, \ldots \ldots . n.$

The trapezoidal average A_{ave},

$A_{ave} = (m_1, m_{M1}, m_{M2}, m_2) = \frac{A_1 + A_2 + \cdots + A_n}{n}$

$= \frac{\left(a_1^{(1)}, b_1^{(1)}, b_2^{(1)}, a_2^{(1)}\right) + \cdots + \left(a_1^{(n)}, b_1^{(n)}, b_2^{(n)}, a_2^{(n)}\right)}{n}$

$= \frac{\left(\sum_{i=1}^{n} a_1^{(i)}, \sum_{i=1}^{n} b_1^{(i)}, \sum_{i=1}^{n} b_2^{(i)}, \sum_{i=1}^{n} a_2^{(i)}\right)}{n}$ (2.2)

Related work

The classical method is generalized by the fuzzy Delphi method for long-range forecasting in management is known as the Delphi method (Milkovich et al. (1972) and Bojadziev and Bojadziev (2007). It can be described as follows:

Expert's responses are analyzed statistically in each round using Fuzzy numbers. A fuzzy statistical analysis is done to find out the difference between individual and mean values obtained from all experts and is communicated to experts for review. Experts' reviews are analyzed, and this process is repeated until the outcome converges to a reasonable solution.

In 1988, Kaufman and Gupta introduced the fuzzy Delphi method.

It consists of the following parts for the triangle (Bojadziev & Bojadziev, 2007):

Step 1: Experts E_i, $i = 1, \ldots, n$, are asked to provide the possible realization dates of a certain event in business. The earliest date $a_1^{(i)}$, the most plausible date $a_M^{(i)}$, and the latest date $a_2^{(i)}$. The data given by the experts E_i are presented in the form of triangular numbers

$A_i = \left(a_1^{(i)}, a_M^{(i)}, a_2^{(i)}\right),$

Where $i = 1,2, \ldots, n.$ (3.1)

Step 2: First, the average (mean) $A_{ave} = (m_1, m_M, m_2)$ of all A_i is computed (see 2.1). Then for each expert E_i the deviation between A_{ave} and A_i is computed. It is a triangular number defined by

$A_{ave} - A_i = (m_1 - a_1^{(i)}, m_M - a_M^{(i)}, m_2 - a_2^{(i)})$

$= (\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)} - a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_M^{(i)} - a_M^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)} - a_2^{(i)})$ (3.2)

The deviation $A_{ave} - A_i$ is sent back to the expert E_i for reexamination.

Step 3: Each expert E_i presents a new triangular number

$B_i = (b_1^{(i)}, b_M^{(i)}, b_2^{(i)}), \quad i = 1, \ldots, n.$ (3.3)

This process starts with Step 2 is repeated. The triangular average B_{ave} is calculated according to formula (2.1) with the difference that now $a_1^{(i)}, a_M^{(i)}, a_2^{(i)}$ are substituted correspondingly by $b_1^{(i)}, b_M^{(i)}, b_2^{(i)}$. If necessary, new triangular numbers $C_i = (c_1^{(i)}, c_M^{(i)}, c_2^{(i)})$ are generated, and their average C_i is calculated. The process could be repeated again and again until two successive means A_{ave}, B_{ave}, C_{ave},. . . become reasonably close.

Step 4: Later, the same process may be reexamine the forecasting if there is important information available due to new discoveries.

An Innovative Product Time Estimation for Technical Realization (Bojadziev and Bojadziev, 2007).

A group of 15 computer experts are asked to estimate using the Fuzzy Delphi method for the technical realization of a brand-new product, say a cognitive information processing computer. They are ranked equally; hence their opinions carry the same weight. The triangular numbers $A_i, i = 1, \ldots, 15$ (see (3.1)) presented by the experts are shown in Table 1.

Table 1. Triangular numbers A_i presented by experts (first request) (Bojadziev and Bojadziev, 2007).

E_i	A_i	Earliest date	Most plausible date	Latest date
E_1	A_1	$a_1^{(1)}$= 1995	$a_M^{(1)}$ = 2003	$a_2^{(1)}$= 2020
E_2	A_2	$a_1^{(2)}$= 1997	$a_M^{(2)}$ = 2004	$a_2^{(2)}$= 2010
E_3	A_3	$a_1^{(3)}$=2000	$a_M^{(3)}$ = 2005	$a_2^{(3)}$=2010
E_4	A_4	$a_1^{(4)}$=1998	$a_M^{(4)}$ = 2003	$a_2^{(4)}$=2008
E_5	A_5	$a_1^{(5)}$=2000	$a_M^{(5)}$ = 2005	$a_2^{(5)}$=2015
E_6	A_6	$a_1^{(6)}$=1995	$a_M^{(6)}$ = 2010	$a_2^{(6)}$=2015
E_7	A_7	$a_1^{(7)}$=2010	$a_M^{(7)}$ = 2018	$a_2^{(7)}$=2015
E_8	A_8	$a_1^{(8)}$=1995	$a_M^{(8)}$ = 2007	$a_2^{(8)}$=2013
E_9	A_9	$a_1^{(9)}$=1995	$a_M^{(9)}$ = 2002	$a_2^{(9)}$=2007
E_{10}	A_{10}	$a_1^{(10)}$=2008	$a_M^{(10)}$ = 2009	$a_2^{(10)}$=2020
E_{11}	A_{11}	$a_1^{(11)}$=2010	$a_M^{(11)}$ = 2020	$a_2^{(11)}$=2024
E_{12}	A_{12}	$a_1^{(12)}$=1996	$a_M^{(12)}$ = 2002	$a_2^{(12)}$=2006
E_{13}	A_{13}	$a_1^{(13)}$=1998	$a_M^{(13)}$ = 2006	$a_2^{(13)}$=2010
E_{14}	A_{14}	$a_1^{(14)}$=1997	$a_M^{(14)}$ = 2005	$a_2^{(14)}$=2012
E_{15}	A_{15}	$a_1^{(15)}$=2002	$a_M^{(15)}$ = 2010	$a_2^{(15)}$=2020

To find the average A_{ave} the sums of the numbers in the last three columns are calculated

$$\sum_{i=1}^{15} a_1^{(i)} = 29996, \sum_{i=1}^{15} = 30109,$$

$$\sum_{i=1}^{15} a_2^{(i)} = 30210$$

and substituted into (2.1), which gives

A_{ave} = $(\frac{29996}{15}, \frac{30109}{15}, \frac{30210}{15})$ = (1999.7, 2007.3, 2014)

or approximately, A_{ave}^{a}= (2000, 2007, 2014).

The deviations (3.2) between A_{ave}^{a} and A_i are presented in Table 2.

Table 2. Deviation $A_{ave} - A_i$.

E_i	$m_1 - a_1^{(i)}$	$m_M - a_M^{(i)}$	$m_2 - a_2^{(i)}$
E_1	5	4	-6
E_2	3	3	4
E_3	0	2	4
E_4	2	4	6
E_5	0	2	-1
E_6	5	-3	-1
E_7	-10	-11	-6
E_8	5	0	1
E_9	5	5	7
E_{10}	-8	-2	-6
E_{11}	-10	-13	-10
E_{12}	4	5	8
E_{13}	2	1	4
E_{14}	3	2	2
E_{15}	-2	-3	-6

Table 2. shows the divergence of each expert's opinion from the average. A quick glance gives that the experts E_3, E_5, E_8, E_{13}, E_{14} are close to the average while E_7, E_{11} is not.

Since the word close is fuzzy, a more detailed study requires some clarification. It can be based on distance d_{ij} between two triangular numbers A_i and A_j. If all d_{ij} are calculated and recorded in a table (in our case consisting of 15 rows and columns), we will have a better grasp of how close various pairs of A_i and A_j are. Here we do not give a formula for calculating the distance d_{ij} (there are several), 4 but refer to Kaufmann and Gupta (1988).

Suppose the manager is not satisfied with the average (2000, 2007, 2014). Then the deviation $(m_1 - a_1^{(i)}, m_M - a_M^{(i)}, m_2 - a_2^{(i)})$ is given to each expert E_i for reconsideration. The experts suggest new triangular numbers B_i (see (3.3)) presented in Table 3.

Table 3. Triangular numbers presented by experts (second request) (Bojadziev and Bojadziev, 2007).

E_i	B_i	Earliest date	Most plausible date	Latest date
E_1	B_1	$b_1^{(1)}=1996$	$b_M^{(1)}=2004$	$b_2^{(1)}=2018$
E_2	B_2	$b_1^{(2)}=1997$	$b_M^{(2)}=2004$	$b_2^{(2)}=2011$
E_3	B_3	$b_1^{(3)}=2000$	$b_M^{(3)}=2005$	$b_2^{(3)}=2011$
E_4	B_4	$b_1^{(4)}=1998$	$b_M^{(4)}=2003$	$b_2^{(4)}=2010$
E_5	B_5	$b_1^{(5)}=2000$	$b_M^{(5)}=2005$	$b_2^{(5)}=2015$
E_6	B_6	$b_1^{(6)}=1997$	$b_M^{(6)}=2009$	$b_2^{(6)}=2015$
E_7	B_7	$b_1^{(7)}=2005$	$b_M^{(7)}=2015$	$b_2^{(7)}=2016$
E_8	B_8	$b_1^{(8)}=1996$	$b_M^{(8)}=2007$	$b_2^{(8)}=2013$
E_9	B_9	$b_1^{(9)}=1997$	$b_M^{(9)}=2004$	$b_2^{(9)}=2010$
E_{10}	B_{10}	$b_1^{(10)}=2004$	$b_M^{(10)}=2009$	$b_2^{(10)}=2017$
E_{11}	B_{11}	$b_1^{(11)}=2004$	$b_M^{(11)}=2015$	$b_2^{(11)}=2016$
E_{12}	B_{12}	$b_1^{(12)}=1996$	$b_M^{(12)}=2004$	$b_2^{(12)}=2006$
E_{13}	B_{13}	$b_1^{(13)}=1998$	$b_M^{(13)}=2006$	$b_2^{(13)}=2010$
E_{14}	B_{14}	$b_1^{(14)}=1997$	$b_M^{(14)}=2004$	$b_2^{(14)}=2012$
E_{15}	B_{15}	$b_1^{(15)}=2001$	$b_M^{(15)}=2009$	$b_2^{(15)}=2015$

The experts E_5, E_{12}, and E_{13} have not changed their first estimate. Other experts, for instance, E_2, E_3, E_8, E_{14}, made minimal changes. Using again (2.1), this time to find B_{ave}, gives

B_{ave} = (1999.07, 2006.9, 2013.2)

Which is approximately,

B_{ave}= (1999, 2007, 2013).

The manager is satisfied that A_{ave} and B_{ave}, also A_{ave}^a and B_{ave}^a, are very close (see Fig. 1), stop the fuzzy Delphi process, and accepts the triangular number B_{ave}^a as a combined conclusion of experts' opinions. The interpretation is that the realization of the invention will occur in the time interval [1999, 2013], the supporting interval of the triangular number B_{ave}^a which is almost in central form.

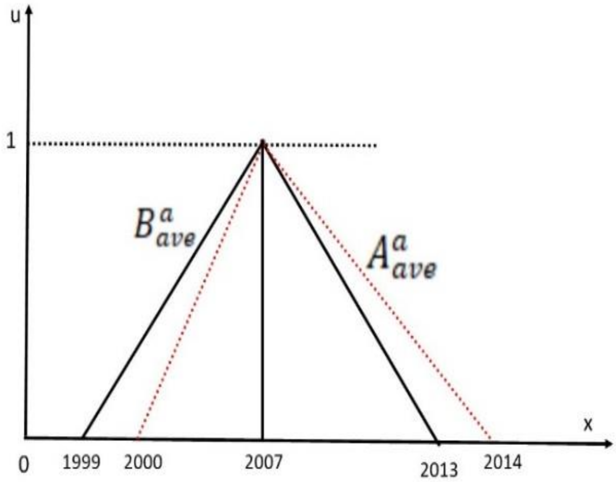

Fig. 1. Average triangular numbers A_{ave}^a and B_{ave}^a.

Materials and methods

The fuzzy Delphi method consists of the following parts for trapezoidal:

Step 1. Experts E_i, $i = 1, \ldots, n$, are asked to provide the possible realization dates of a particular event in science, technology, or business, namely: the earliest date $a_1^{(i)}$, the earliest most plausible date $a_{M1}^{(i)}$, the latest most plausible date $a_{M2}^{(i)}$, and the latest date $a_2^{(i)}$. The data given by the experts E_i are presented in the form of trapezoidal numbers

$$A_i = \left(a_1^{(i)}, a_{M1}^{(i)}, a_{M2}^{(i)}, a_2^{(i)}\right),$$

$$\text{Where } i = 1,2, \ldots, n. \quad (4.1)$$

Step 2. First, the average (mean) $A_{ave} = (m_1, m_{M1}, m_{M2}, m_2)$ of all A_i is computed (see 2.2). Then for each expert E_i the deviation between A_{ave} and A_i is computed. It is a trapezoidal number defined by

$A_{ave} - A_i = (m_1 - a_1^{(i)}, m_{M1} - a_{M1}^{(i)}, m_{M2} - a_{M2}^{(i)}, m_2 - a_2^{(i)})$

$=(\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)} - a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_{M1}^{(i)} - a_{M1}^{(i)},$

$\frac{1}{n}\sum_{i=1}^{n} a_{M2}^{(i)} - a_{M2}^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)} - a_2^{(i)})$ (4.2)

The deviation $A_{ave} - A_i$ is sent back to the expert E_i for reexamination.

Step 3. Each expert E_i presents a new trapezoidal number

$B_i = (b_1^{(i)}, b_{M1}^{(i)}, b_{M2}^{(i)}, b_2^{(i)}), i = 1, \ldots, n.$ (4.3)

This process starts with Step 2 is repeated. The trapezoidal average B_{ave} is calculated according to formula (2.2) with the difference that now $a_1^{(i)}, a_{M1}^{(i)}, a_{M2}^{(i)}, a_2^{(i)}$ are substituted correspondingly by $b_1^{(i)}, b_{M1}^{(i)}, b_{M2}^{(i)}, b_2^{(i)}$. If necessary, new trapezoidal numbers $C_i = (c_1^{(i)}, c_{M1}^{(i)}, c_{M2}^{(i)}, c_2^{(i)})$ are generated, and their average C_i is calculated. The process could be repeated again and again until two successive means $A_{ave}, B_{ave}, C_{ave}, \ldots$ become reasonably close.

Step 4. Later, the forecasting may be reexamined by the same process if there is important information available due to new discoveries.

An Innovative Product Time Estimation for Technical Realization.

A group of 15 computer experts are asked to estimate using the Fuzzy Delphi method for the technical realization of a brand-new product, say a cognitive information processing computer. They are ranked equally, hence their opinions carry the same weight. The trapezoidal numbers, A_i, $i = 1, \ldots, 15$ (see (4.1)) presented by the experts are shown in Table 4.

Table 4. Trapezoidal numbers A_i presented by experts (first request).

E_i	A_i	Earliest date	Earliest Most plausible date	Latest Most plausible date	Latest date
E_1	A_1	$a_1^{(1)}$=1995	$a_{M1}^{(1)} = 2003$	$a_{M2}^{(1)} = 2006$	$a_2^{(1)}$=2020
E_2	A_2	$a_1^{(2)}$=1997	$a_{M1}^{(2)} = 2004$	$a_{M2}^{(2)} = 2005$	$a_2^{(2)}$=2010
E_3	A_3	$a_1^{(3)}$=2000	$a_{M1}^{(3)} = 2005$	$a_{M2}^{(3)} = 2007$	$a_2^{(3)}$=2010
E_4	A_4	$a_1^{(4)}$=1998	$a_{M1}^{(4)} = 2003$	$a_{M2}^{(4)} = 2005$	$a_2^{(4)}$=2008
E_5	A_5	$a_1^{(5)}$=2000	$a_{M1}^{(5)} = 2005$	$a_{M2}^{(5)} = 2008$	$a_2^{(5)}$=2015
E_6	A_6	$a_1^{(6)}$=1995	$a_{M1}^{(6)} = 2010$	$a_{M2}^{(6)} = 2012$	$a_2^{(6)}$=2015
E_7	A_7	$a_1^{(7)}$=2010	$a_{M1}^{(7)} = 2018$	$a_{M2}^{(7)} = 2019$	$a_2^{(7)}$=2015
E_8	A_8	$a_1^{(8)}$=1995	$a_{M1}^{(8)} = 2007$	$a_{M2}^{(8)} = 2010$	$a_2^{(8)}$=2013
E_9	A_9	$a_1^{(9)}$=1995	$a_{M1}^{(9)} = 2002$	$a_{M2}^{(9)} = 2005$	$a_2^{(9)}$=2007
E_{10}	A_{10}	$a_1^{(10)}$=2008	$a_{M1}^{(10)} = 2009$	$a_{M2}^{(10)} = 2013$	$a_2^{(10)}$=2020
E_{11}	A_{11}	$a_1^{(11)}$=2010	$a_{M1}^{(11)} = 2020$	$a_{M2}^{(11)} = 2022$	$a_2^{(11)}$=2024
E_{12}	A_{12}	$a_1^{(12)}$=1996	$a_{M1}^{(12)} = 2002$	$a_{M2}^{(12)} = 2003$	$a_2^{(12)}$=2006
E_{13}	A_{13}	$a_1^{(13)}$=1998	$a_{M1}^{(13)} = 2006$	$a_{M2}^{(13)} = 2\ 008$	$a_2^{(13)}$=2010
E_{14}	A_{14}	$a_1^{(14)}$=1997	$a_{M1}^{(14)} = 2005$	$a_{M2}^{(14)} = 2008$	$a_2^{(14)}$=2012
	A_{15}	$a_1^{(15)}$=2002	$a_{M1}^{(15)} = 2010$	$a_{M2}^{(15)} = 2013$	$a_2^{(15)}$=2020

To find the average A_{ave} the sums of the numbers in the last four columns are calculated

$$9996, \sum_{i=1}^{15} a_{M1}^{(i)} = 30109,$$

$$\sum_{i=1}^{15} a_{M2}^{(i)} = 30144, \quad \sum_{i=1}^{15} a_{2}^{(i)} = 30210$$

and substituted into (2.2) which gives

$A_{ave} = (\frac{29996}{15}, \frac{30109}{15}, \frac{30144}{15}, \frac{30210}{15})$

= (1999.7, 2007.3, 2009.6, 2014)

or approximately,

$$A_{ave}^{a} = (2000, 2007, 2010, 2014).$$

The deviations (4.2) between A_{ave}^{a} and A_i are presented in Table 5.

Table 5. Deviation $A_{ave} - A_i$.

E_i	$m_1 - a_1^{(i)}$	$m_{M1} - a_{M1}^{(i)}$	$m_{M2} - a_{M2}^{(i)}$	$m_2 - a_2^{(i)}$
E_1	5	4	4	-6
E_2	3	3	5	4
E_3	0	2	3	4
E_4	2	4	5	6
E_5	0	2	2	-1
E_6	5	-3	-2	-1
E_7	-10	-11	-9	-6
E_8	5	0	0	1
E_9	5	5	5	7
E_{10}	-8	-2	-3	-6
E_{11}	-10	-13	-12	-10
E_{12}	4	5	7	8
E_{13}	2	1	2	4
E_{14}	3	2	2	2
E_{15}	-2	-3	-3	-6

Table 5. shows the divergence of each expert's opinion from the average. A quick glance gives that the experts E_3, E_5, E_8, E_{13}, E_{14} are close to the average while E_7, E_{11} is not.

Since the word close is fuzzy, a more detailed study requires some clarification. It can be based on distance d_{ij} between two trapezoidal numbers A_i and A_j. If all d_{ij} are calculated and recorded in a table (in our case consisting of 15 rows and columns), we will have a better grasp of how close various pairs of A_i and A_j are. Here we do not give a formula for calculating the distance d_{ij} (there are several), 4 but refer to Kaufmann & Gupta (1988).

Suppose the manager is not satisfied with the average (2000, 2007, 2010, 2014). Then the deviation $(m_1 - a_1^{(i)}, m_{M1} - a_{M1}^{(i)}, m_{M2} - a_{M2}^{(i)}, m_2 - a_2^{(i)})$ is given to each expert E_i for reconsideration. The experts suggest new trapezoidal numbers B_i (see (4.3) presented in Table 6.

Table 6. Trapezoidal numbers presented by experts (second request).

E_i	B_i	Earliest date	Earliest Most plausible date	Latest Most plausible date	Latest date
E_1	B_1	$b_1^{(1)}$ =1996	$b_{M1}^{(1)} = 2004$	$b_{M2}^{(1)} = 2007$	$b_2^{(1)}$ =2018
E_2	B_2	$b_1^{(2)}$ =1997	$b_{M1}^{(2)} = 2004$	$b_{M2}^{(2)} = 2006$	$b_2^{(2)}$ =2011
E_3	B_3	$b_1^{(3)}$ =2000	$b_{M1}^{(3)} = 2005$	$b_{M2}^{(3)} = 2005$	$b_2^{(3)}$ =2011
E_4	B_4	$b_1^{(4)}$ =1998	$b_{M1}^{(4)} = 2003$	$b_{M2}^{(4)} = 2005$	$b_2^{(4)}$ =2010
E_5	B_5	$b_1^{(5)}$ =2000	$b_{M1}^{(5)} = 2005$	$b_{M2}^{(5)} = 2008$	$b_2^{(5)}$ =2015
E_6	B_6	$b_1^{(6)}$ =1997	$b_{M1}^{(6)} = 2009$	$b_{M2}^{(6)} = 2011$	$b_2^{(6)}$ =2015
E_7	B_7	$b_1^{(7)}$ =2005	$b_{M1}^{(7)} = 2015$	$b_{M2}^{(7)} = 2016$	$b_2^{(7)}$ =2016

E_8	B_8	$b_1^{(8)}$ =1996	$b_{M1}^{(8)}$ = 2007	$b_{M2}^{(8)}$ = 2010	$b_2^{(8)}$ =2013
E_9	B_9	$b_1^{(9)}$ =1997	$b_{M1}^{(9)}$ = 2004	$b_{M2}^{(9)}$ = 2007	$b_2^{(9)}$ =2010
E_{10}	B_{10}	$b_1^{(10)}$ =2004	$b_{M1}^{(10)}$ = 2009	$b_{M2}^{(10)}$ = 20013	$b_2^{(10)}$ =2017
E_{11}	B_{11}	$b_1^{(11)}$ =2004	$b_{M1}^{(11)}$ = 2015	$b_{M2}^{(11)}$ = 2017	$b_2^{(11)}$ =2016
E_{12}	B_{12}	$b_1^{(12)}$ =1996	$b_{M1}^{(12)}$ = 2004	$b_{M2}^{(12)}$ = 2005	$b_2^{(12)}$ =2006
E_{13}	B_{13}	$b_1^{(13)}$ =1998	$b_{M1}^{(13)}$ = 2006	$b_{M2}^{(13)}$ = 2008	$b_2^{(13)}$ =2010
E_{14}	B_{14}	$b_1^{(14)}$ =1997	$b_{M1}^{(14)}$ = 2004	$b_{M2}^{(14)}$ = 2007	$b_2^{(14)}$ =2012
E_{15}	B_{15}	$b_1^{(15)}$ =2001	$b_{M1}^{(15)}$ = 2009	$b_{M2}^{(15)}$ = 2012	$b_2^{(15)}$ =2015

The experts E_5, E_{12}, and E_{13} have not changed their first estimate. Other experts, for instance, E_2, E_3, E_8, E_{14}, made minimal changes. Using again (2.2), this time to find B_{ave}, gives

B_{ave} = (1999.07, 2006.9, 2009.13, 2013.2)

which is approximately,

$$B_{ave} = (1999, 2007, 2009, 2013).$$

The manager is satisfied that A_{ave} and B_{ave}, also A_{ave}^a and B_{ave}^a, are very close (see Fig. 2.), stops the fuzzy Delphi process, and accepts the trapezoidal number B_{ave}^a as a combined conclusion of experts' opinions. The interpretation is that the realization of the invention will occur in the time interval [1999, 2013], the supporting interval of the trapezoidal number B_{ave}^a, which is almost in central form.

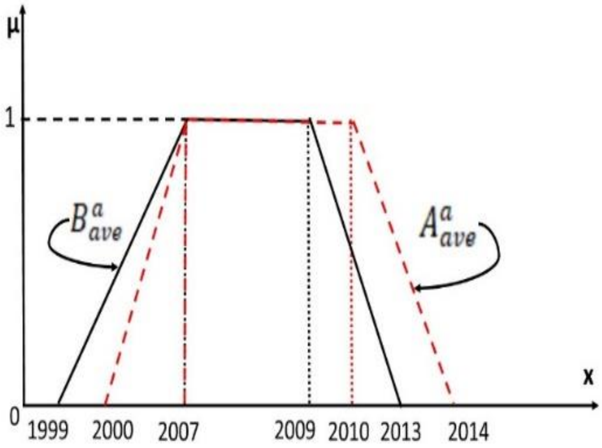

Fig. 2. Average trapezoidal numbers A_{ave}^a and B_{ave}^a.

Results and Discussion

In this article, we use triangular and trapezoidal numbers. By comparing two of these numbers, we get,

(i) From the triangle, we get one peak point from where it's not sure how many days it will run well. On the other hand, we get an interval of the peak points that define that the business will run well in this interval from trapezoidal numbers.

(ii) Also, we see from the trapezoidal numbers figure that a fast business will build up or fall. But there is no proper definition in the triangular numbers.

Conclusion

Here we see that trapezoidal numbers give better results than triangular numbers for future business forecasting since the trapezoidal numbers are more generalized than triangular numbers. So, the results we have gotten using trapezoidal numbers will be better than triangular numbers.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

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