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NON-EXISTENCE OF AN INVISCID FLUID MOTION BETWEEN TWO FIXED CYLINDERS

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ABSTRACT

A problem on the plane inviscid and irrotational fluid motion due to the presence of a line source and sink in the region between two fixed co-axial circular cylinders is considered in terms of the stream function. It has been shown that the solution of the problem is not possible in the light of the Eulerian theory of inviscid fluid motion.

Key words: Non-existence, Inviscid fluid motion, Fixed cylinder

INTRODUCTION

In the paper (Sen and Ahammad 2009), it has been solved the problem of the two dimensional slow viscous fluid motion induced by a line source and sink of equal strength in the region between two co-axial fixed circular cylinders. Ranger (1961) has considered the problems of the viscous and non-viscous fluid flows between the same cylinders, when a line source and sink are positioned on the outer boundary, and determined the stream functions for the flow pattern between the cylinders and the force exerted by the fluid on the inner cylinder, in infinite series. A study in detail for the viscous fluid motions with cylinders and singularities is yet to be made. When the hydrodynamical singularities in a non-viscous fluid with no rigid boundaries are known in terms of the complex potential, the stream function for the flow pattern due to their presence outside a single circular cylinder is usually found by applying the Milne-Thomson circle theorem (Milne 1940, Milne 1972) or the method of the potential theory (Lamb 1932) for the same singularities outside the cylinder. The stream function for the flow pattern inside a single circular cylinder is derived by using Chorlton's extension (Chorlton 1967) of Milne–Thomson's circle theorem or Sen's circle theorem for interior flow (Sen 1963) for the same singularities inside the cylinder. Here it is noteworthy that the above circle theorems do not apply in general for the determination of the stream function for the flow system due to a number of hydrodynamical singularities within a circular cylinder in the presence of one or more rigid boundaries in an inviscid irrotational fluid motion within the same cylinder. In an attempt towards our aim to obtain, in future, solutions of the problems of non-viscous fluid flow due to the singularities in the assemblage of a number of boundaries within a circular cylinder, here we first present an analytical method of solving the problem of a two-dimensional irrotational non-viscous fluid flow due to a line source and sink in the region between two con-centric circular boundaries.

The inviscid fluid flow within a circular cylinder

At the outset we need to determine the flow due to the combination of a source of strength m at the point $A(a_1, \alpha)$ and a sink of the same strength at the point $B(a_1, -\alpha)$, where $a_1 < a$, a being the radius of the circular cylinder referred below. The basic stream function due to these singularities (Milne 1940) in the absence of rigid boundaries is given by

$$\psi_0(r,\theta) = -m\left\{\tan^{-1}\left(\frac{r\sin\theta - a_1\sin\alpha}{r\cos\theta - a_1\cos\alpha}\right) - \tan^{-1}\left(\frac{r\sin\theta + a_1\sin\alpha}{r\cos\theta - a_1\cos\alpha}\right)\right\}. \tag{2.1}$$

We may expand the stream function (2.1) as

$$\psi_0(r,\theta) = -m\left\{\theta - \sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{a_1}{r}\right)^n \sin n(\alpha-\theta)\right\} + m\left\{\theta + \sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{a_1}{r}\right)^n \sin n(\alpha+\theta)\right\}. \tag{2.2}$$

or,

$$\psi_0(r,\theta) = 2m\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{a_1}{r}\right)^n \sin n\alpha \cos n\theta\,. \tag{2.3}$$

Here we observe that $\psi_0(r,\ \theta) \sim O\left(1/r\right)$ for large r. Thus the circle theorem I for the interior potential flow in Sen (Chorlton 1967) applies here. If the cylinder $r = a$ is introduced into the flow field of the singularities, the stream function for the flow interior to the cylinder is given by the formula,

$$\psi_1(r,\theta) = \psi_0(r,\theta) - \psi_0\left(\frac{a^2}{r},\theta\right)\,. \tag{2.4}$$

Then substituting the basic stream function (2.1) in (2.4) yields the necessary stream function for the flow interior to the circle $r = a$ and this is,

$$\psi_1(r,\theta) = -m\left\{\tan^{-1}\left(\frac{r\sin\theta - a_1\sin\alpha}{r\cos\theta - a_1\cos\alpha}\right) - \tan^{-1}\left(\frac{r\sin\theta + a_1\sin\alpha}{r\cos\theta - a_1\cos\alpha}\right)\right\}$$
$$-m\left\{\tan^{-1}\left(\frac{r\sin\theta - \frac{a^2}{a_1}\sin\alpha}{r\cos\theta - \frac{a^2}{a_1}\cos\alpha}\right) - \tan^{-1}\left(\frac{r\sin\theta + \frac{a^2}{a_1}\sin\alpha}{r\cos\theta - \frac{a^2}{a_1}\cos\alpha}\right)\right\}. \tag{2.5}$$

It is here important to note that the last two terms of the stream function (2.5) constitute singularities which are outside the boundary $r = a$.

Now to use this stream function in the next section we present it in an appropriate infinite series form as

$$\psi_1(r,\theta) = 2m\sum_{n=1}^{\infty}\left\{-\frac{1}{n}\left(\frac{a_1}{r}\right)^n + \frac{1}{n}\left(\frac{ra_1}{a^2}\right)^n\right\}\sin n\alpha \cos n\theta, \tag{2.6}$$

$$i.e., \psi_1(r,\theta) = 2m\sum_{n=1}^{\infty} h_n(r)\cos n\theta\,, \tag{2.7}$$

where

$$h_n(r) = \frac{a_1^n}{na^{2n}}\left(\frac{r^{2n}-a^{2n}}{r^n}\right)\sin n\alpha\,. \tag{2.8}$$

Non-existence of the inviscid fluid motion between two co-axial circular cylinders

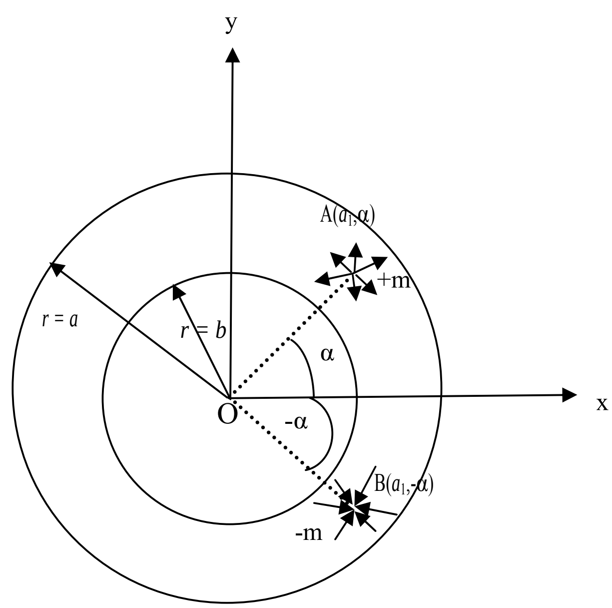

Fig. 1. Sketch for the inviscid fluid bed between two co-axial circular cylinders with the singularities.

Now we are interested in introducing a cross-section of a circular cylinder with radius b (< a) into the flow and is fixed in a co-axial position with respect to the outer cylinder. As a result of this situation, the Earnshaw stream function for a new flow field in the region common to the cylinders becomes, say

$$\psi = \psi_1 + \psi_2\,, \tag{3.1}$$

where ψ_2 is also a suitable general solution of Laplace's equation

$$\nabla_1^2\psi_2 = 0\,. \tag{3.2}$$

and it is given by

$$\psi_2(r,\theta) = 2m\sum_{n=1}^{\infty} g_n(r)\cos n\theta , \tag{3.3}$$

where $g_n(r) = A_n r^n + B_n r^{-n}$, (3.4)

A_n and B_n being relevant real constants to be determined.

The stream function ψ_1 referred to in (3.1) satisfies the condition of the flow interior to the outer boundary $r = a$.

Hence it is sufficient that the stream function ψ_2 must also satisfy the following condition on $r = a$; that is,

$$\text{on } r = a ,\ \psi_2 = 0. \tag{3.5}$$

Again, on the inner boundary, the stream function ψ , represented by equation (3.1) must satisfy the following boundary condition,

$$\text{on } r = b ,\ \psi = 0 . \tag{3.6}$$

Now using the boundary condition (3.5) we have,

$$g_n(a) = 0, \quad n = 1, 2, 3, 4,... \tag{3.7}$$

In order to use the boundary condition (3.6), we express the stream function (3.1) in the following form, by incorporating the stream functions (2.8) and (3.3).

$$\psi(r,\theta) = 2m\sum_{n=1}^{\infty} \{h_n(r) + g_n(r)\}\cos n\theta \ . \tag{3.8}$$

Now the boundary condition (3.6) yields, the result

$$h_n(b) + g_n(b) = 0 ;\ n=1, 2, 3 , \dots \ . \tag{3.9}$$

From equations (3.7) and (3.9), we get

$$A_n a^n + B_n a^{-n} = 0 \ \text{ and } A_n b^n + B_n b^{-n} = -h_n(b) , \tag{3.10}$$

which implies that

$$A_n = \frac{b^n}{(a^{2n} - b^{2n})} h_n(b) , \quad \text{and} \quad B_n = \frac{-a^{2n} b^n}{(a^{2n} - b^{2n})} h_n(b) . \tag{3.11}$$

$$\text{where } h_n(b) = \frac{a_1^n}{n a^{2n}} \left(r^{2n} - a^{2n} \right) \frac{1}{r^n} \sin n\alpha , \tag{3.12}$$

Now the stream function (3.8) on simplification, takes the simple form

$$\psi(r,\theta) = 2m\sum_{n=1}^{\infty} \left[\left\{ \frac{a_1^{2n}(r^{2n} - a^{2n})}{n a^{2n} r^n} \right\} - \frac{r^n a_1^{2n}}{n a^{2n}} + \frac{a_1^n}{n r^n} \right] \sin n\alpha \cos n\theta \tag{3.13}$$

Therefore $\psi(r,\theta) = 0$ (3.14)

Which satisfies that the fluid motion between the cylinders does not exist.

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